Almost slightly-continuity, slightly open and slightly closed mappings

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ABSTRACT

In this paper we discuss new type of continuous functions called Almost slightly–continuous, slightly-open and slightly-closed functions; its properties and interrelation with other such functions are studied.

Keywords: slightly continuous functions; slightly semi-continuous functions; slightly pre-continuous; slightly β-continuous functions and slightly υ-continuous functions.

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1. INTRODUCTION

In 1995 T. M. Nour introduced slightly semi-continuous functions. After him T. Noiri and G. I. Chae further studied slightly semi-continuous functions in 2000. T. Noiri individually studied about slightly β–continuous functions in 2001. C. W. Baker introduced slightly precontinuous functions in 2002. Arse Nagli Uresin and others studied slightly δ–continuous functions in 2007. Recently S. Balasubramanian and P.A.S. Vyjayanthi studied slightly υ–continuous functions in 2011. Inspired with these developments we introduce in this paper Almost slightly–continuous, slightly-open and slightly-closed functions and study its basic properties and interrelation with other type of such functions. Throughout the paper (X, 1) and (Y, α) (or simply X and Y) represent topological spaces on which no separation axioms are assumed unless otherwise mentioned.

2. PRELIMINARIES

Definition 2.1: \( A \subset X \) is called g-closed[rg-closed] if cl \( A \subset U \) whenever \( A \subset U \) and \( U \) is open in X.

Definition 2.2: A function \( f: X \to Y \) is said to be

(i) continuous[resp: nearly-continuous; \( \kappa \)-continuous; \( \alpha \)-continuous; semi-continuous; \( \beta \)-continuous; pre-continuous] if inverse image of each open set is open[resp: regular-open; \( \tau \)-open; \( \alpha \)-open; semi-open; \( \beta \)-open; preopen].

(ii) almost continuous[resp: almost nearly-continuous; almost \( \kappa \)-continuous; almost \( \alpha \)-continuous; almost semi-continuous; almost \( \beta \)-continuous; almost pre-continuous] if for each \( x \) in \( X \) and each open set \( V \), \( \exists \) an open[resp: regular-open; \( \tau \)-open; \( \alpha \)-open; semi-open; \( \beta \)-open; preopen] set \( (U, x) \) such that \( f(U) \subset (cl(V))^2 \).

(iii) slightly continuous[resp: slightly semi-continuous; slightly pre-continuous; slightly \( \beta \)-continuous; slightly \( \alpha \)-continuous; slightly \( \tau \)-continuous; slightly \( \nu \)-continuous] at \( x \) in \( X \) if for each clopen subset \( V \) of \( Y \) containing \( f(x) \), \( \exists U \in \tau(X) \), \( \exists U \in SO(X) \), \( \exists U \in PO(X) \), \( \exists U \in BO(X) \), \( \exists U \in \alpha O(X) \), \( \exists U \in RO(X) \), \( \exists U \in \nu O(X) \) containing \( x \) such that \( f(U) \subset V \).

Lemma 2.1:

(i) Let \( A \) and \( B \) be subsets of a space \( X \), if \( A \subset \tau(X) \) and \( B \subset RO(X) \), then \( A \subset \tau(B) \).

(ii)Let \( A \subset B \subset X \), if \( A \subset \tau(B) \) and \( B \subset RO(X) \), then \( A \subset \tau(X) \).

Note 1: \( RCO(Y, f(x)) \) means regular-clopen set in \( Y \) containing \( f(x) \) and \( \tau(X, x) \) means open set in \( X \) containing \( x \).

3. ALMOST SLIGHTLY CONTINUOUS FUNCTIONS

Definition 3.1: A function \( f: X \to Y \) is said to be almost slightly continuous at \( x \) in \( X \) if for each \( V \in RCO(Y, f(x)) \), \( \exists U \in \tau(X, x) \) such that \( f(U) \subset V \) and almost slightly continuous if it is almost slightly continuous at each \( x \) in \( X \).

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Note 2: Here we call almost slightly continuous function as al.sl.c function shortly.

Example 3.1: \( X = Y = \{a, b, c\}; \tau = \{\emptyset, \{a\}, \{b, c\}, X\}\) and \( \sigma = \{\emptyset, \{a\}, \{b, c\}, Y\}\). Let \( f \) be defined as \( f(a) = b; f(b) = c \) and \( f(c) = a \), then \( f \) is al.sl.c.

Example 3.2: \( X = Y = \{a, b, c\}; \tau = \{\emptyset, \{a\}, \{b, a\}, \{b, c\}, X\}\) and \( \sigma = \{\emptyset, \{a\}, \{b, c\}, Y\}\). Let \( f \) be defined as \( f(a) = b; f(b) = c \) and \( f(c) = a \), then \( f \) is not al.sl.c.

Theorem 3.1: The following are equivalent:
(i) \( f \) is al.sl.c.
(ii) \( f^{-1}(V) \) is open for every \( r \)-clopen set \( V \) in \( Y \).
(iii) \( f^{-1}(V) \) is closed for every \( r \)-clopopen set \( V \) in \( Y \).
(iv) \( f(\text{cl}(A)) \subseteq \text{cl}(f(A)) \).

Corollary 3.1: The following are equivalent.
(i) \( f \) is al-sl.c.
(ii) For each \( x \in X \) and each \( V \in \text{RCO}(Y) \) there exists \( U \in \tau(X) \) such that \( f(U) \subseteq V \).

Theorem 3.2: Let \( X \) be any cover of \( X \) by regular open sets in \( X \). A function \( f \) is al-sl.c. if \( f(U) \) is al-sl.c. for each \( U \) in \( X \).

Proof: Let \( f : X \to Y \) be an arbitrary function and \( U \in \text{RO}(X) \). Then \( f(U) \subseteq V \). Since \( f(U) \in \text{RCO}(Y) \), \( f(U) \subseteq V \). Hence \( f(U) \) is al-sl.c.

Conversely, let \( x \in X \) and \( V \in \text{RCO}(Y) \). Then \( x \notin f(U) \) implies \( x \notin f(U) \). Since \( x \notin X \), \( x \notin f(U) \) is al-sl.c. for each \( x \) in \( X \) such that \( x \notin f(U) \).

Theorem 3.3: If \( f \) is almost continuous and \( g \) is continuous[al.sl.c.], then \( g \circ f \) is al-sl.c.

Theorem 3.4: If \( f \) is almost continuous, open and \( g \) is any function, then \( g \circ f \) is al-sl.c iff \( g \) is al-sl.c.

Proof: If part: Theorem 3.3

Only if part: Let \( A \subset X \) be a clopen subset of \( Z \). Then \( (g \circ f)^{-1}(A) \) is an open subset of \( X \). Since \( f \) is open \( (g \circ f)^{-1}(A) = f^{-1}(A) \) is open in \( Y \). Thus \( g \circ f \) is al-sl.c.

Corollary 3.2: If \( f \) is continuous, open and bijective, \( g \) is a function. Then \( g \circ f \) is al-sl.c.

Theorem 3.5: If \( g : X \to Y \), defined by \( g(x) = (x, f(x)) \) for all \( x \in X \) be the graph function of \( f : X \to Y \). Then \( g \) is al-sl.c.

Proof: Let \( V \subseteq \text{RCO}(Y) \), then \( X \times V \subseteq \text{RCO}(X \times Y) \). Since \( g \) is al-sl.c., \( f^{-1}(V) \) is al-sl.c. Thus \( f \) is al-sl.c.

Conversely, let \( x \in X \) and \( f \subseteq \text{RCO}(X \times Y) \). Then \( f(x) \subseteq V \). Since \( f(x) \) is homeomorphic to \( Y \), \( y \subseteq f(x) \subseteq f^{-1}(V) \) is al-sl.c. Therefore \( x \subseteq f^{-1}(V) \). Further \( x \subseteq f^{-1}(V) \subseteq f^{-1}(V) \). Hence \( g^{-1}(V) \) is open.

Thus \( g \) is al-sl.c.

Theorem 3.6: \( f^{-1}(x) \) is al-sl.c., \( f^{-1}(y) \) is al-sl.c.

(i) \( f^{-1}(x) \) is al-sl.c., \( f^{-1}(y) \) is al-sl.c.

(ii) \( f^{-1}(x) \) is al-sl.c., \( f^{-1}(y) \) is al-sl.c.

Remark 1: Composition, Algebraic sum, product and the pointwise limit of al-sl.c functions is not in general al-sl.c. However we can prove the following:

Theorem 3.7: The uniform limit of a sequence of al-sl.c functions is al-sl.c.

Note 3: Pasting Lemma is not true for al-sl.c functions. However we have the following weaker versions.

Theorem 3.8: Let \( X \) and \( Y \) be such that \( X = A \cup B \) and let \( f_A \) and \( f_B \) are al-sl.c maps such that \( f(x) = g(x) \) for all \( x \in A \cup B \). If \( A, B \subseteq \text{RO}(X) \) and \( RO(X) \) is closed under finite unions, then the combination \( \alpha \) is al-sl.c.

Theorem 3.9: Pasting Lemma Let \( X \) and \( Y \) be such that \( X = A \cup B \) and let \( f_A \) and \( f_B \) are al-sl.c maps such that \( f(x) = g(x) \) for all \( x \in A \cup B \). If \( A, B \subseteq \text{RO}(X) \) and \( RO(X) \) is closed under finite unions, then the combination \( \alpha \) is al-sl.c.

Proof: Let \( f \subseteq \text{RCO}(Y) \), then \( f^{-1}(F) \subseteq \text{RCO}(X) \). Since \( f^{-1}(F) \subseteq \text{RCO}(X) \), \( f^{-1}(F) \subseteq \text{RCO}(X) \). Hence \( f^{-1}(F) \subseteq \text{RCO}(X) \).

Definition 3.2: A function \( f \) is said to be almost somewhat continuous if for every \( U \subseteq \text{RO}(\sigma) \) and \( f^{-1}(U) \) is \( \emptyset \), there exists a non-empty open set \( V \) in \( X \) such that \( V \subseteq f^{-1}(U) \).

Example 3.3: Let \( X = \{a, b, c\}, \tau = \{\emptyset, \{a\}, \{b, c\}, X\} \) and \( \sigma = \{\emptyset, \{a\}, \{b, c\}, Y\} \). The function \( f \) defined by \( f(a) = b; f(b) = c \) and \( f(c) = a \) is almost somewhat continuous.

Example 3.4: Let \( X = \{a, b, c\}, \tau = \{\emptyset, \{a\}, \{b, c\}, X\} \) and \( \sigma = \{\emptyset, \{a\}, \{b, c\}, Y\} \). The function \( f \) defined by \( f(a) = b; f(b) = c \) and \( f(c) = a \) is also almost somewhat continuous.

Note 4: Every almost somewhat continuous function is almost slightly continuous.

Theorem 3.10: If \( f \) is almost somewhat continuous and \( g \) is continuous, then \( g \circ f \) is almost somewhat continuous.
Corollary 3.3: If \( f \) is almost somewhat continuous and \( g \) is \( r \)-continuous\( [r \)-irresolute], then \( g \circ f \) is almost somewhat continuous.

Theorem 3.11: For a surjective function \( f \), the following statements are equivalent:

(i) \( f \) is almost somewhat continuous.

(ii) If \( C \) is a \( r \)-closed subset of \( Y \) such that \( f^{-1}(C) = X \), then there is a proper closed subset \( D \) of \( X \) such that \( f^{-1}(C) \subset D \).

(iii) If \( M \) is a dense subset of \( X \), then \( f(M) \) is a dense subset of \( Y \).

Proof: (i) \( \Rightarrow \) (ii): Let \( C \) be a \( r \)-closed subset of \( Y \) such that \( f^{-1}(C) = X \). Then \( Y \cdot C \) is an \( r \)-open set in \( Y \) such that \( f^{-1}(Y \cdot C) = X \cdot f^{-1}(C) = X \cdot f^{-1}(C) \neq \emptyset \). By (i), there exists an open set \( V \) in \( X \) such that \( V \neq \emptyset \) and \( V \subset f^{-1}(Y \cdot C) = X \cdot f^{-1}(C) \). That is \( X \cdot f^{-1}(C) \subset D \) is a proper closed set in \( X \).

Proof: (ii) \( \Rightarrow \) (i): Let \( U \) and \( f^{-1}(U) \neq \emptyset \). Then \( Y \cdot U \) is \( r \)-closed and \( f^{-1}(Y \cdot U) = X \cdot f^{-1}(U) \neq X \). By (ii), there exists a proper closed set \( D \) such that \( D \subset f^{-1}(Y \cdot U) \). This implies that \( X \cdot D \subset f^{-1}(C) \) and \( X \cdot D = D \) is a proper closed set in \( X \).

Proof: (iii) \( \Rightarrow \) (ii): Suppose (iii) is not true. Then there exists a proper \( r \)-closed set \( C \) in \( Y \) such that \( f(M) \subset C \subset Y \). Clearly \( f^{-1}(C) \neq X \). By (ii), there exists a proper closed set \( D \) such that \( M \subset f^{-1}(C) \subset D \). This is a contradiction to the fact that \( M \) is dense in \( X \).

Proof: (ii) \( \Rightarrow \) (iii): Suppose (ii) is not true. Then there exists a \( r \)-closed set \( C \) in \( Y \) such that \( f^{-1}(C) \neq X \) but there is no proper closed set \( D \) in \( X \) such that \( f^{-1}(C) \subset D \). This means that \( f^{-1}(C) \) is dense in \( X \). But by (ii), \( f^{-1}(C) \subset C \subset X \). This is a contradiction to the fact that \( M \) is dense in \( X \).

Theorem 3.12: Let \( f \) be a function and \( X = A \cdot B \), where \( A, B \in RO(X) \). If \( f_A \) and \( f_B \) are almost somewhat continuous, then \( f \) is almost somewhat continuous.

Proof: Let \( U \in RO(X) \) such that \( f^{-1}(U) \neq \emptyset \). Then \( f_A^{-1}(U) \neq \emptyset \) or \( f_B^{-1}(U) \neq \emptyset \). Suppose \( f_A^{-1}(U) \neq \emptyset \). Since \( f_B \) is almost somewhat continuous, there exists a proper closed set \( D \) in \( Y \) such that \( V \neq \emptyset \) and \( V \subset \text{closed subset of } Y \). Since \( V \) is open in \( A \) and \( B \) is a proper closed set in \( X \), \( D \) is open in \( X \). Thus \( f \) is almost somewhat continuous.

Theorem 3.13: Let \( f(X, \tau) \rightarrow (Y, \sigma) \) be a \( r \)-almost somewhat continuous surjection and \( \tau^* \) be a topology for \( X \), which is equivalent to \( \tau \). Then \( f(X, \tau^*) \rightarrow (Y, \sigma) \) is almost somewhat continuous.

Proof: Let \( U \in RO(X) \). Since \( f \) is almost somewhat continuous, there exists a nonempty \( U \in SO(X, \tau) \). For \( \tau^* \) equivalent to \( \tau \), \( U \in SO(X, \tau^*) \). But \( U \subset f^{-1}(V) \). Hence \( f(X, \tau^*) \rightarrow (Y, \sigma) \) is almost somewhat continuous.

Theorem 3.14: Let \( f(X, \tau) \rightarrow (Y, \sigma) \) be a \( r \)-almost somewhat continuous surjection and \( \sigma^* \) is a topology for \( Y \), which is equivalent to \( \sigma \). Then \( f(X, \tau) \rightarrow (Y, \sigma^*) \) is almost somewhat continuous.

Proof: Let \( V \in RO(Y) \). Since \( \sigma^* \) is equivalent to \( \sigma \), \( V \in SO(Y, \sigma) \). Now \( f^{-1}(V) \subset f^{-1}(V) \). For \( f \) is almost somewhat continuous, \( U \in SO(X, \tau) \). Then \( U \subset f^{-1}(V) \). Hence \( f(X, \tau) \rightarrow (Y, \sigma^*) \) is almost somewhat continuous.

4. SLIGHTLY OPEN MAPPINGS, ALMOST SLIGHTLY OPEN MAPPINGS AND ALMOST SOMEWHAT OPEN FUNCTION

Definition 3.1: A function \( f : X \rightarrow Y \) is said to be

(i) \( r \)-slightly open if image of every clopen set in \( X \) is open in \( Y \)

(ii) almost \( r \)-slightly open if image of every regular-clopen set in \( X \) is open in \( Y \).

Example 4.1: Let \( X = \{a, b, c\}; \tau = (\emptyset, \{a\}, \{a, b\}, X\}; \sigma = (\emptyset, \{a\}, \{a, b\}, Y\). Let \( f : X \rightarrow Y \) be defined \( f(a) = c \), \( f(b) = b \) and \( f(c) = a \). Then \( f \) is slightly open, slightly \( r \)-open, almost \( r \)-open, and almost slightly \( r \)-open.

Example 4.2: Let \( X = \{a, b, c\}; \tau = (\emptyset, \{a\}, \{a, b\}, X\}; \sigma = (\emptyset, \{a\}, \{a, b\}, Y\). Let \( f : X \rightarrow Y \) be defined \( f(a) = c \), \( f(b) = a \) and \( f(c) = b \). Then \( f \) is not slightly open, not slightly \( r \)-open, almost \( r \)-open, and not almost \( r \)-open.

Theorem 4.1: (i) If \( f \) is \( r \)-slightly open and \( g \) is \( r \)-open then \( g \circ f \) is \( r \)-slightly open

(ii) If \( f \) is almost \( r \)-slightly open and \( g \) is \( r \)-open then \( g \circ f \) is almost \( r \)-slightly open

Proof: Let \( f \) be \( r \)-slightly open and \( g \) be \( r \)-open then \( g \circ f \) is \( r \)-slightly open.

Theorem 4.2: If \( f \) and \( g \) are \( r \)-open then \( g \circ f \) is \( r \)-open

Proof: Let \( A \) be \( r \)-open then \( f \) is \( r \)-open.

Theorem 4.3: If \( f \) is \( r \)-slightly open and \( g \) is \( r \)-open then \( g \circ f \) is \( r \)-slightly open

Proof: Follows from definitions

Theorem 4.4: If \( f \) is \( r \)-slightly open, then \( f^{-1}(A) \cap (f(A))^c = \emptyset \).
Proof: Let $A \subseteq X$ and $f$ is slightly open gives $f(A')$ is open in $Y$ and $f(A^c) \subseteq f(A)$ which in turn gives $f(A'^c) \subseteq f(A)^c$. 

Remark 2: converse is not true in general.

Theorem 4.5: If $f$ is slightly open and $A \subseteq X$ is $r$-open, then $f(A)$ is $\omega$-open in $Y$.
Proof: Let $A \subseteq X$ and $f$ is slightly open implies $f(A') \subseteq (f(A))^c$ and $(f(A)^c) \subseteq f(A)$, since $f(A) = f(A')$. But $f(A) \subseteq (f(A))^c$. Combining we get $f(A) = (f(A))^c$. Hence $f(A)$ is $\omega$-open in $Y$.

Corollary 4.1: (i) If $f$ is [almost]-slightly open, then $f(A') \subseteq (f(A))^c$.
Proof: (a) Let $A = RO(X)$, then $f(A) = (f(A))^c \subseteq f(A)$ by hypothesis. We have $f(A) \subseteq (f(A))^c$. Combining we get $f(A) = A' = r(A')$ which implies $f(A)$ is $r$-open and hence open. Thus $f$ is slightly open.

Theorem 4.6: If $A'' = r(A')$ for every $A \subseteq Y$, then the following are equivalent:
(i) $f$ is [almost]-slightly open map
(ii) $f(A) \subseteq (f(A))^c$

Proof: (a) $\Rightarrow$ (b) follows from theorem 4.4.

Remark 3: composition of two [almost]-slightly open maps is not [almost]-slightly open in general.

Theorem 4.8: Let $X$, $Y$, $Z$ be topological spaces and every open set is $r$-open in $Y$ then the composition of two [almost]-slightly open maps is [almost]-slightly open.
Proof: Let $A$ be regular clopen in $X$ then $f(A)$ is open in $Y$ (by assumption) $\Rightarrow g(f(A)) = g(f(A))$ is open in $Z$. Hence $gf$ is almost slightly open.

Theorem 4.9: If $f$ is [almost]-slightly g-open; $g$ is open-$r$-open and $Y$ is $S_{\infty}$, then $gf$ is [almost]-slightly open.
Proof: Let $A$ be regular clopen in $X$ then $f(A)$ is $g$-open and is open in $Y$ (since $Y$ is $S_{\infty}$) $\Rightarrow g(f(A)) = g(f(A))$ is open in $Z$. Hence $gf$ is almost slightly open.

Theorem 4.10: If $f$ is [almost]-slightly rgf-open; $g$ is open-$r$-open and $Y$ is $S_{\infty}$, then $gf$ is [almost]-slightly open.
Proof: Let $A$ be regular clopen in $X$ then $f(A)$ is $rg$-open and $rg$-open in $Y$ (since $Y$ is $S_{\infty}$) $\Rightarrow g(f(A)) = g(f(A))$ is open in $Z$. Hence $gf$ is almost slightly open.

Theorem 4.12: If $g$, $h$ be two mappings such that $gf$ is [almost]-slightly open then $gf$ is [almost]-slightly open.
Proof: (i) If $f$ is continuous, $r$-continuous and surjective, then $g$ is [almost]-slightly open
(ii) If $f$ is $g$-continuous, surjective and $X$ is $S_{\infty}$, then $g$ is [almost]-slightly open
(iii) If $f$ is $rg$-continuous, surjective and $X$ is $S_{\infty}$, then $g$ is [almost]-slightly open

Theorem 4.13: If $X$ is regular, $f$ is $r$-open, nearly-continuous, open surjection and $A = A$ for every open-$r$-open set in $Y$, then $Y$ is regular.

Theorem 4.14: If $f$ is [almost]-slightly open and $A$ is $r$-open set of $X$, then $f(A)(X, \tau) \rightarrow (Y, \sigma)$ is [almost]-slightly open.
Proof: Let $F$ be $r$-open set in $A$. Then $F = A \times E$ for some $r$-open set $E$ of $X$ and so $F$ is $r$-open in $X$ which implies $f(A)$ is open in $Y$. But $GF = GF$. Therefore $f(A)$ is [almost]-slightly open.

Theorem 4.15: If $f$ is [almost]-slightly open, $X$ is $S_{\infty}$ and $A$ is $g$-open set of $X$, then $f(A)(X, \tau) \rightarrow (Y, \sigma)$ is [almost]-slightly open.

Theorem 4.16: If $f, x_i : X \rightarrow Y$, be [almost]-slightly open for $i = 1, 2$. Let $f : X \times X \rightarrow Y \times Y$ be defined as $f(x_i, x_2) = (f(x_1), f(x_2))$.
Then $f$ is [almost]-slightly open.
Proof: Let $U \times U \subset X \times X$ where $U$ is $r$-open in $X$ for $i = 1, 2$. Then $f(U \times U) \subset f(U \times U) \subset f(U \times U)$. Hence $f$ is [almost]-slightly open.
Theorem 4.17: Let \( h: X \to X_i \) and \( h: X \to X_j \) be [almost]-slightly open. Let \( f: X \to X_i \) be defined as \( h(x) = (x_i, x_j) \) and \( f(x) = x \). Then \( f: X \to X_i \) is [almost]-slightly open for \( i = 1, 2 \).

\( f: \) Let \( U_i \) be \( r \)-clopen in \( X_i \), then \( U_i \times X_j \) is \( r \)-clopen in \( X \) in \( X \times X_j \) and \( h(U_i \times X_j) \) is open in \( X \). But \( h(U_i) = h(U_i \times X_j) \), therefore \( h \) is [almost]-slightly open. Similarly we can show that \( f \) is also [almost]-slightly open and thus \( f: X \to X_i \) is [almost]-slightly open for \( i = 1, 2 \).

Definition 4.2: A function \( f \) is said to be almost somewhat open provided that \( U \subseteq RO(x) \), then there exists a non-empty open set \( V \) in \( Y \) such that \( V \subseteq f(U) \).

Example 4.3: Let \( X = \{a, b, c\} \), \( \tau = \{\phi, \{a\}, \{b, c\}, X\} \) and \( \sigma = \{\phi, \{a\}, \{b\}, \{a, b\}, X\} \). The function \( f \) be defined by \( f(a) = a \), \( f(b) = c \) and \( f(c) = b \) is almost somewhat open, and somewhat open.

Example 4.4: Let \( X = \{a, b, c\} \), \( \tau = \{\phi, \{a\}, \{b, c\}, X\} \) and \( \sigma = \{\phi, \{a\}, \{b\}, \{a, b\}, X\} \). The function \( f \) be defined by \( f(a) = c \), \( f(b) = a \) and \( f(c) = b \) is almost somewhat open.

Theorem 4.18: If \( f \) is \( r \)-open and \( g \) almost somewhat open. Then \( g \circ f \) is almost somewhat open.

Theorem 4.19: For a bijective function \( f \), the following are equivalent:

(i) \( f \) is almost somewhat open.

(ii) If \( A \) is a dense subset of \( Y \), then \( f^{-1}(A) \) is a dense subset of \( X \).

(iii) If \( A \) is a dense subset of \( Y \), then \( f^{-1}(A) \) is a dense subset of \( X \).

Theorem 4.20: The following statements are equivalent:

(i) \( f \) is almost somewhat open.

(ii) If \( A \) is a dense subset of \( X \), then \( f^{-1}(A) \) is a dense subset of \( Y \).

(iii) If \( A \) is a dense subset of \( X \), then \( f^{-1}(A) \) is a dense subset of \( Y \).

Theorem 4.21: Let \( f \) be almost somewhat open and \( A \subseteq RO(X) \). Then \( f \) is almost somewhat open.

Theorem 4.22: Let \( f \) be a function and \( X = A \cup B \), where \( A, B \subseteq \sigma(X) \). If the restriction functions \( f_A \) and \( f_B \) are almost somewhat open, then \( f \) is almost somewhat open.

Example 5.1: Let \( X = \{a, b, c\} \), \( \tau = \{\phi, \{a\}, \{b, c\}, X\} \), \( \sigma = \{\phi, \{a\}, \{b\}, \{a, b\}, X\} \), \( X \). Let \( f: X \to Y \) be defined \( f(a) = c \), \( f(b) = b \) and \( f(c) = a \). Then \( f \) is almost closed, slightly \( r \)-closed, almost slightly closed and almost slightly \( r \)-closed.

Example 5.2: Let \( X = \{a, b, c\} \), \( \tau = \{\phi, \{a\}, \{b, c\}, X\} \), \( \sigma = \{\phi, \{a\}, \{b\}, \{a, b\}, X\} \). Let \( f: X \to Y \) be defined \( f(a) = c \), \( f(b) = a \) and \( f(c) = b \). Then \( f \) is not slightly closed, slightly \( r \)-closed, almost slightly closed and almost every slightly \( r \)-closed.

Theorem 5.1: Let \( f: X \to Y \) be defined \( f(a) = c \), \( f(b) = b \) and \( f(c) = a \). Then \( f \) is slightly closed, almost \( r \)-closed, almost slightly closed and almost \( r \)-closed.

Definition 5.1: A function \( f: X \to Y \) is said to be

(i) slightly closed if image of every clopen set in \( X \) is closed in \( Y \)

(ii) almost slightly closed if image of every regular-clopen set in \( X \) is closed in \( Y \)

Example 5.5: Let \( X = \{a, b, c\} \), \( \tau = \{\phi, \{a\}, \{b, c\}, X\} \), \( \sigma = \{\phi, \{a\}, \{b\}, \{a, b\}, X\} \). Let \( f: X \to Y \) be defined \( f(a) = c \), \( f(b) = b \) and \( f(c) = a \). Then \( f \) is slightly closed, slightly \( r \)-closed, almost slightly closed and almost slightly \( r \)-closed.
Theorem 5.2: If \( f \) and \( g \) are \( r \)-closed then \( g \circ f \) is [almost-]slightly closed

Proof: Let \( A \) be clopen-[r-clopen] set in \( X \Rightarrow f(A) \) is \( r \)-closed and closed and in \( Y \Rightarrow g(f(A)) \) is \( r \)-closed in \( Z \Rightarrow g(f(A)) = g \circ f(A) \) is closed in \( Z \). Hence \( g \circ f \) is [almost-]slightly closed.

Theorem 5.3: If \( f \) is almost slightly-\( r \)-closed and \( g \) is [almost-]closed then \( g \circ f \) is [almost-]slightly closed

Proof: Follows from definitions

Theorem 5.4: If \( f \) is [almost-]slightly closed, then \( cl(f(A)) = f(cl(A)) \)

Proof: Let \( A \subset X \) and \( f \) is slightly closed gives \( f(cl(A)) \) is closed in \( Y \) and \( f(A) \subset f(cl(A)) \) which in turn gives \( cl(f(A)) \subset cl(f(cl(A))) \).

Remark 5: converse is not true in general.

Theorem 5.5: If \( f \) is slightly closed and \( A \subset X \) is \( r \)-closed in \( Y \). Then \( f(A) \) is \( r \)-closed in \( Y \).

Proof: Let \( A \subset X \) and \( f \) is slightly closed implies \( cl(f(A)) \subset f(cl(A)) \) which in turn implies \( cl(f(A)) \subset f(cl(A)) \), since \( f(A) = cl(f(A)) \). But \( f(A) \subset cl(f(A)) \). Combining we get \( f(A) = cl(f(A)) \). Hence \( f(A) \) is \( r \)-closed in \( Y \).

Corollary 5.1: (i) If \( f \) is [almost-]slightly \( r \)-closed, then \( f(cl(A)) \subset cl(f(A)) \)

(ii) If \( f \) is [almost-]slightly \( r \)-closed, then \( f(A) \) is closed in \( Y \) if \( A \) is \( r \)-closed set in \( X \).

Theorem 5.6: If \( cl(A) = r(cl(A)) \) for every \( A \subset Y \), then the following are equivalent:

(i) \( f \) is [almost-]slightly closed map

(ii) \( cl(f(A)) \subset f(cl(A)) \)

Proof: (a) \( \Rightarrow \) (b) follows from theorem 5.4

(b) \( \Rightarrow \) (a) Let \( A \) be any \( r \)-closed set in \( X \), then \( f(A) = r(cl(f(A))) \) by hypothesis. We have \( f(A) \subset cl(f(A)) \). Combining we get \( f(A) = cl(f(A)) \) by given condition which implies \( f(A) \) is \( r \)-closed and hence closed. Thus \( f \) is slightly closed.

Theorem 5.7: \( f \) is [almost-]slightly closed iff for each subset \( S \) of \( Y \) and each \( r \)-clopen set \( U \) containing \( f^{-1}(S) \), there is a closed set \( V \) of \( Y \) such that \( S \subset V \) and \( f^{-1}(V) \subset U \).

Remark 6: composition of two [almost-]slightly closed maps is not [almost-]slightly closed in general

Theorem 5.8: Let \( X, Y, Z \) be topological spaces and every closed set is \( r \)-clopen in \( Y \), then the composition of two [almost-]slightly closed maps is [almost-]slightly closed.

Proof: Let \( A \) be \( r \)-clopen in \( X \Rightarrow f(A) \) is closed in \( Y \Rightarrow f(A) \) is \( r \)-clopen in \( Y \) [by assumption] \( \Rightarrow g(f(A)) = g \circ f(A) \) is closed in \( Z \). Hence \( g \circ f \) is almost slightly closed.

Theorem 5.9: If \( f \) is [almost-]slightly \( g \)-closed; \( g \) is closed-[r-closed] and \( Y \) is \( T_{
abla} \), then \( g \circ f \) is [almost-]slightly closed.

Proof: Let \( A \) be \( r \)-clopen in \( X \Rightarrow A \) be clopen in \( X \Rightarrow f(A) \) is \( g \)-closed in \( Y \Rightarrow f(A) \) is closed in \( Y \) [since \( Y \) is \( T_{
abla} \) \( \Rightarrow g(f(A)) = g \circ f(A) \) is closed in \( Z \). Hence \( g \circ f \) is almost slightly closed.

Theorem 5.10: If \( f \) is [almost-]slightly \( r \)-closed; \( g \) is closed-[r-closed] and \( Y \) is \( T_{
abla} \), then \( g \circ f \) is [almost-]slightly closed.

Proof: Let \( A \) be \( r \)-clopen in \( X \Rightarrow A \) be clopen in \( X \Rightarrow f(A) \) is \( r \)-closed and \( g \)-closed in \( Y \) [since \( Y \) is \( T_{
abla} \) \( \Rightarrow g(f(A)) = g \circ f(A) \) is closed in \( Z \). Hence \( g \circ f \) is almost slightly closed.

Theorem 5.11: If \( f \) is [almost-]slightly \( r \)-closed; \( g \) is [almost-]closed-[almost-r-closed] and \( Y \) is \( T_{
abla} \), then \( g \circ f \) is [almost-]slightly closed.

Proof: Let \( A \) be \( r \)-clopen in \( X \Rightarrow A \) be clopen in \( X \Rightarrow f(A) \) is \( r \)-closed and hence \( r \)-closed in \( Y \) [since \( Y \) is \( T_{
abla} \) \( \Rightarrow g(f(A)) = g \circ f(A) \) is closed in \( Z \). Hence \( g \circ f \) is almost slightly closed.

Theorem 5.12: If \( f, g \) be two mappings such that \( g \circ f \) is [almost-]slightly closed-[almost-] slightly \( r \)-closed. Then the following are true

(i) If \( f \) is continuous-[r-continuous] and surjective, then \( g \) is [almost-]slightly closed

(ii) If \( f \) g-continuous, surjective and \( X \) is \( T_{
abla} \), then \( g \) is [almost-]slightly closed

(iii) If \( f \) is co-continuous, surjective and \( X \) is \( T_{
abla} \), then \( g \) is [almost-]slightly closed

Proof: (i) Let \( A \) be \( r \)-clopen in \( Y \Rightarrow A \) be clopen in \( Y \Rightarrow f^{-1}(A) \) is closed in \( X \Rightarrow g(f^{-1}(A)) = g(A) \) is closed in \( Z \). Hence \( g \) is almost slightly closed.

Similarly we can prove the remaining parts and hence omitted.

Theorem 5.13: If \( X \) is regular, \( f \) is \( r \)-closed, nearly-continuous, closed surjection and \( \bar{A} = A \) for every \( r \)-closed set in \( Y \), then \( Y \) is regular.

Theorem 5.14: If \( f \) is [almost-]slightly \( r \)-closed and \( A \) is \( r \)-clopen set of \( X \), then \( f|_{A}: (X, \tau_{A}) \to (Y, \sigma) \) is [almost-]slightly closed.

Proof: For \( f|_{A} \), \( r \)-closed in \( A \). Then \( F = A \times E \) is \( r \)-closed in \( X \) for some \( r \)-closed set \( E \) of \( X \) which implies \( f(A) \) is closed in \( Y \). But \( f|_{A} = f|_{A}^{-1}(f(A)) \). Thus \( f|_{A} \) is [almost-]slightly closed.

Theorem 5.15: If \( f \) is [almost-]slightly \( r \)-closed, \( X \) is \( T_{
abla} \) and \( A \) is \( r \)-closed set of \( X \), then \( f|_{A}: (X, \tau_{A}) \to (Y, \sigma) \) is [almost-]slightly closed.

Theorem 5.16: If \( f_{x}: X_{x} \to Y_{x} \) be [almost-]slightly closed for \( i = 1, 2 \). Let \( f: X_{1} \times X_{2} \to Y_{1} \times Y_{2} \) be defined as \( f(x_{1}, x_{2}) = (f(x_{1}), f(x_{2})) \).

Then \( f(X_{1} \times X_{2}) \times Y_{1} \times Y_{2} \) is [almost-]slightly closed.
Indian Journal of Science - Analysis - Mathematics

6. COVERING AND SEPARATION PROPERTIES OF al.sl.c. and al.swt.c. FUNCTIONS

Theorem 6.1: If f is al.sl.c., then f is connected.
Proof: Let f:D→X be surjective and X is compact, then Y is compact.

Theorem 6.2: If f is al.sl.c., then f is surjective and X is compact then Y is connected.
Proof: Let {U} be a clopen cover for Y. Then each G is clopen in Y and f is al.sl.c. Thus \( f^{-1}(G) \) is open in X. Thus \( \{ f^{-1}(G) \} \) forms an open cover for X with a finite subcover, since X is compact. Since f is surjection, Y = f(X) = \( \bigcup_i G \). Therefore Y is compact.

Corollary 6.1: If f is al.sl.r.c. and X is compact then Y is connected.

Theorem 6.3: If f is al.sl.c. and X is s-closed then Y is mildly compact[ mildly lindeloff].
Proof: Let \( \{ V_i \} \) be an open cover of Y, then \( f^{-1}(V_i) \subseteq \{ U_{i,j} \} \) is open cover of X and so there is finite subset \( I \) of I such that \( f^{-1}(V_i) \subseteq \{ U_{i,j} \} \). Therefore \( f^{-1}(V_i) \subseteq \{ U_{i,j} \} \) covers Y since f is surjection. Hence Y is mildly compact.

Theorem 6.4: If f is al.sl.c., then f is surjective and X is connected, then Y is connected.
Proof: If Y is disconnected, then Y = A∪B where A and B are disjoint clopen sets in Y. Since f is al.sl.c. surjection, X = f^{-1}(Y) = f^{-1}(A)∪f^{-1}(B) are disjoint clopen sets in X, which is a contradiction for X is connected. Hence Y is connected.

Corollary 6.2: If f is al.sl.c., then X is s-closed then Y is mildly compact[mildly lindeloff].

(i) The inverse image of a disconnected space under a al.sl.c. surjection is disconnected.

Theorem 6.5: If f is al.sl.c., injection and Y is UrT, then X is T_i = 0, 1, 2.
Proof: Let \( x \neq x \in X \). Then \( f(x) \neq f(x) \in Y \) since f is injective. For Y is UrT, \( \exists U \in RO(Y) \) such that \( f(x) \subseteq U \) and \( f(x) \neq f(x) \) for j = 1, 2. By Theorem 3.1, \( f(x) \subseteq f^{-1}(U) \) for j = 1, 2 and \( f(x) \subseteq f^{-1}(U) \) for j = 1, 2. Thus X is T_2.

(ii) Let f and f be disjoint r-closed subsets of Y and X respectively. Hence \( x \in f^{-1}(U) \cap f^{-1}(V) \), \( x \neq f^{-1}(U) \cap f^{-1}(V) \), \( x \neq f^{-1}(U) \cap f^{-1}(V) \). Thus X is T_2.

Theorem 6.6: If f is al.sl.c., injection and Y is UrT, then X is T_i = 3, 4.
Proof: Let x \( \not\in X \) and f be disjoint r-closed subsets of X and Y respectively. Hence \( x \in f^{-1}(U) \cap f^{-1}(V) \), \( x \neq f^{-1}(U) \cap f^{-1}(V) \), \( x \neq f^{-1}(U) \cap f^{-1}(V) \). Thus X is T_2.

(i) Let f and f be disjoint r-closed subsets of Y and X respectively. Hence \( f(x) \neq f(x) \), \( f(x) \neq f(x) \), \( f(x) \neq f(x) \). Thus X is T_2.

(ii) Let f and f be disjoint r-closed subsets of X and Y respectively. Hence \( f(x) \neq f(x) \), \( f(x) \neq f(x) \), \( f(x) \neq f(x) \). Thus X is T_2.

(iii) Let f and f be disjoint r-closed subsets of Y and X respectively. Hence \( f(x) \neq f(x) \), \( f(x) \neq f(x) \), \( f(x) \neq f(x) \). Thus X is T_2.

Theorem 6.7: If f is al.sl.c., injection and Y is UrT, then X is T_i = 0, 1, 2.
Proof: (i) Let f be a function from X to Y such that \( f(x) \neq f(x) \) and \( f(x) \neq f(x) \). Since f is al.sl.c. such that \( x \notin X \) and \( f(x) \neq f(x) \), \( f(x) \neq f(x) \) such that \( x \notin X \) and \( f(x) \neq f(x) \), \( f(x) \neq f(x) \). Thus X is T_2.

(ii) Let f be a function from X to Y such that \( f(x) \neq f(x) \), \( f(x) \neq f(x) \), \( f(x) \neq f(x) \). Since f is al.sl.c. such that \( x \notin X \) and \( f(x) \neq f(x) \), \( f(x) \neq f(x) \), \( f(x) \neq f(x) \). Thus X is T_2.

(iii) Let f be a function from X to Y such that \( f(x) \neq f(x) \), \( f(x) \neq f(x) \), \( f(x) \neq f(x) \). Since f is al.sl.c. such that \( x \notin X \) and \( f(x) \neq f(x) \), \( f(x) \neq f(x) \), \( f(x) \neq f(x) \). Thus X is T_2.

Theorem 6.8: If f is al.sl.c., then X is UrT, then the graph G(f) is closed in X×Y.
Proof: Let \( (x, y) \in G(f) \) implies \( x \neq f(x) \) implies \( y \neq f(y) \) implies \( y \neq f(y) \) implies \( y \neq f(y) \). Since f is al.sl.c. such that \( x \in X \) and \( f(x) \neq f(x) \), \( f(x) \neq f(x) \), \( f(x) \neq f(x) \). Hence G(f) is closed in X×Y.

Theorem 6.9: If f is al.sl.c. and Y is UrT, then A = \{ (x, y) \in X×Y \} is closed in X×X.
Proof: Let \( (x, y) \in X×X \), then \( f(x) \neq f(x) \) implies \( y \neq f(y) \) implies \( y \neq f(y) \) implies \( y \neq f(y) \). Since f is al.sl.c. such that \( f(x) \neq f(x) \), \( f(x) \neq f(x) \), \( f(x) \neq f(x) \). Thus \( f(x) \neq f(x) \), \( f(x) \neq f(x) \), \( f(x) \neq f(x) \). Hence A is closed.

Theorem 6.10: If f is al.sl.c., then \( f \subseteq X×Y \) is al.sl.c. and Y is UrT, then \( f \subseteq X×X \) is closed in X×X.
Proof: Let \( (x, y) \in X×X \), then \( f(x) \neq f(x) \) implies \( y \neq f(y) \) implies \( y \neq f(y) \) implies \( y \neq f(y) \). Since f is al.sl.c. such that \( f(x) \neq f(x) \), \( f(x) \neq f(x) \), \( f(x) \neq f(x) \). Hence A is closed.

Theorem 6.11: If f is al.sl.c., then \( f \subseteq X×Y \) is compact, then Y is compact.

Theorem 6.12: If f is al.sl.c., then X is compact, then Y is compact.

Corollary 6.3: (i) If f is al.sl.r.c. and X is compact, then Y is compact.

(ii) If f is al.sl.r.c. and X is compact, then Y is compact.
(iii) If \( f \) is al.swt.c.[resp: al.swt.r.c] surjection and \( X \) is locally compact[resp: Lindeloff; locally Lindeloff], then \( Y \) is locally compact[resp: Lindeloff; locally mildly compact; locally mildly Lindeloff].

**Theorem 6.13:** If \( f \) is al.swt.c., surjection and \( X \) is s-closed then \( Y \) is mildly compact[ mildly Lindeloff].

**Theorem 6.14:** If \( f \) is al.swt.c.[resp: al.swt.r.c ] surjection and \( X \) is connected, then \( Y \) is connected.

**Corollary 6.4:** (i) If \( f \) is al.swt.c[resp: al.swt.r.c ] surjection and \( X \) is s-closed then \( Y \) is mildly compact[mildly Lindeloff].

(ii) The inverse image of a disconnected space under an al.swt.c.[resp: al.swt.r.c ] surjection is disconnected.

**Theorem 6.15:** (i) If \( f \) is al.swt.c.[al.swt.r.c ], injection and \( Y \) is UrT, then \( X \) is \( T_i = 0, 1, 2 \).

(ii) If \( f \) is al.swt.c.[al.swt.r.c ] injection; \( r \)-closed and \( Y \) is UrT, then \( X \) is \( T_i = 3, 4 \).

**Theorem 6.16:** If \( f \) is al.swt.c.[resp: al.swt.r.c ], injection and \( Y \) is \( UrC[resp: UrD] \) then \( X \) is \( C[resp: D]_i = 0, 1, 2 \).

(iii) \( Y \) is \( UrR, \) then \( X \) is \( R, i = 0, 1 \).

**Theorem 6.17:** If \( f \) is al.swt.c.[resp: al.swt.r.c ] and \( Y \) is \( UrT_2 \), then

(i) the graph \( G(f) \) is closed in \( X \times Y \).

(ii) \( A = \{(x_1, x_2) \mid f(x_1) = f(x_2)\} \) is closed in \( X \times X \).

**Theorem 6.18:** If \( f \) is al.swt.r.c[resp: al.swt.c]; \( g: X \rightarrow Y \) is al.swt.c[resp: al.swt.r.c]; and \( Y \) is \( UrT_2 \), then \( E = \{x \in X : f(x) = g(x)\} \) is closed in \( X \).

### 7. CONCLUSION

In this paper we introduced the concept of almost slightly-continuous functions, almost somewhat continuous functions, almost somewhat open mappings, slightly open mappings, almost slightly open mappings, almost closed mappings, almost slightly closed mappings, almost closed mappings, studied their basic properties and the interrelationship between other such maps.

### REFERENCE


