Indian journal of
Engineering

Development of an Empirical Correction Model for Improving Single-Degree-of-Freedom Predictions of Beam Response Under Impact Loading

Ali M. Lafta[1]*, Salih Meri Al-Absi[2,3], Ahmed Raad Al-Adhadh[4]

To Cite:
Lafta AM, Al-Absi SM, Al-Adhadh AR. Development of an Empirical Correction Model for Improving Single-Degree-of-Freedom Predictions of Beam Response Under Impact Loading. *Indian Journal of Engineering*, 2026, 23, e8ije1716
doi: https://doi.org/10.54905/disssi.v23i59.e8ije1716

Author Affiliation:
[1]Civil Engineering Department, College of Engineering, Al-Muthanna University, Al-Muthanna, Iraq. E-mail: ali.majd@mu.edu.iq, Orcid: https://orcid.org/0000-0002-0925-0714.
[2]Department of Refrigeration and Air-Conditioning, College of Technical Engineering, Sawa University, Al-Muthanna, Iraq.
[3]Iraqi Cement State Company, Ministry of Industry and Minerals, Baghdad, Iraq. E-mail: saleh.m@sawauniversity.edu.iq
[4]Civil Engineering Department, College of Engineering, Al-Muthanna University, Al-Muthanna, Iraq. E-mail: ahmad_al_iraqi2000@mu.edu.iq

Corresponding author:
Ali M. Lafta,
Civil Engineering Department, College of Engineering, Al-Muthanna University, Al-Muthanna, Iraq. E-mail: ali.majd@mu.edu.iq

Peer-Review History
Received: 29 September 2025
Reviewed & Revised: 18/October/2025 to 01/May/2026
Accepted: 09 May 2026
Published: 18 May 2026

Peer-Review Model
External peer-review was done through double-blind method.

Indian Journal of Engineering
pISSN 2319–7757; eISSN 2319–7765

DISCOVERY
SCIENTIFIC SOCIETY

ABSTRACT

This study investigated the response of a reinforced concrete beam subjected to drop-weight impact using both a single-degree-of-freedom (SDOF) model and a multi-degree-of-freedom (MDOF) model. The study examined the displacement response in both models and then developed a correction factor to bring the SDOF displacement results closer the corresponding MDOF values. Both models were analyzed under various realistic conditions relevant to structural applications. The main advantages of the SDOF model ARE its simplicity, speed, and low computational cost; however, its accuracy under impact conditions remains questionable, whereas the MDOF model does not suffer from this limitation. We created and analyzed a simply supported beam and built the MDOF model using the finite element method (FEM). The study focused on four main variables: drop mass, drop height, span length, and section depth. The accuracy of the results was also verified by comparing these results with published experiments and Abaqus simulations, and the agreement was very close. Additionally, a parametric investigation determined the conversion factor. The results showed that the SDOF model gave higher maximum mid-span displacement values than the MDOF model. Where the difference between the two models are 44.6% for effect of span length and 43% for effect of section depth. After applying the correction factor, the results showed a high correlation with MDOF values (R2 = 0.9866) with a maximum error of less than 2% in most simulated cases. Therefore, the proposed conversion factor provides a practical and computationally efficient tool for preliminary design and structural evaluation.

Keywords: Single-degree-of-freedom (SDOF) model; multi-degree-of-freedom (MDOF) model; Finite element method (FEM); Impact loading; Displacement prediction; Empirical correction model.

1. INTRODUCTION

Previous studies on impact loads on beams can be divided into three directions. The research in the first direction aimed to study the behavior of the reinforced concrete (RC) beams under impact loads, and to understand the failure modes, and the factors that affect the maximum response. Fujikake et al., (2009) investigated the behavior or RC beams that subjected to a falling weight and provided an analytical evaluation. Kishi et al., (2002) and Saatci & Vecchio, (2009)) studied the behavior of the beams that showed a shear failure under the impact loads. Zhong et al., (2023) studied the dynamic response and the failure modes of the beams subjected to the impact loads. They proved that the maximum and residual displacement of an RC beam under low velocity impact increases with the improvement of impact velocity. These studies contributed to understanding the impact response, but they still cover a limited number of cases of impact methods and the beam characteristics.

The second direction depended on high-accuracy numerical studies whose analysis was based on multi-degree-of-freedom systems using the finite element method. This method gives realistic results of the damage that occurs to the structural element along the time of impact and rebound, as well as its ease in representing the variables of drop mass, drop height, span length, and the rigidity. In this context, the studies conducted by (Zhao et al., 2018; Li et al., 2019; Li et al., 2021; Samadzad et al., 2025; Saini & Shafei, 2019) addressed important issues related to impact-force representation, the effect of drop-weight shape, and the ability of these models to predict the response, damage, and residual capacity. And provide the fundamental references in structural dynamics and finite element analysis, such as the works of (Bathe, 2014; Zienkiewicz & Taylor, 2015; Clough & Penzien, 2015; Cook, 2007). This method has contributed to producing highly reliable studies, but at the same time, this method needs advanced programming skills that cannot be available to all users. It also requires detailed calibration of the structural element's material properties, and experimental results for the same structural element whose remaining characteristics are to be studied which may also be unavailable to many.

The researchers in the third direction focused on developing simplified models, especially the single-degree-of-freedom (SDOF) models, which give rapid results with fair accuracy. This method is accurate when the vibration behavior of the wave after impact is controlled by the fundamental vibration mode (Meyers,1994; Paz & Leigh, 2012; Newmark, 1959). Throughout the years, the (SDOF) method has improved by using two-degree-of-freedom models like the studies presented by Jin et al., (2026). The 2DOF method contributed to improve the results of the simplified method. However, the literature still shows notable variation in the accuracy of these models when the beam geometry or the impact method and conditions change (Cui et al., 2025; Jin et al., 2026). The major problem with the SDOF model is that it converts the entire beam into just a single moving point, which makes it fail to capture important details, such as how the beam vibrates in more than one direction, which means it may not capture the higher vibration modes, or how the inertia and curvature are actually distributed, or even how the deformation develops at the impact location and the rest of the beam.

Many previous studies have been done before, but something is still missing. Most research used real experiments, complicated and slow computer programs, or simple formulas. The simple SDOF model has not been compared directly and carefully to the detailed MDOF model over a large range of conditions.

How accurately does the SDOF model predict the deflection or displacement of a concrete beam when impacted?

Based on the research problem explained above, this study first aims to compare the SDOF and MDOF models for beams under impact loading. Then, it develops an empirical correction factor that brings the maximum displacement values from the SDOF model closer to those from the MDOF model.

To investigate this, the researchers developed two independent numerical models in MATLAB. They used equivalent system parameters for the SDOF model. They used the finite element method for the MDOF model, dividing the beam into eighty elements. Then, the researchers compared the models' results with published experimental results of previous studies to determine their accuracy. The results were also compared with Abaqus simulations. The four major variables studied were the mass of the falling object, the drop height, the span length, and the section depth. This was done to get an idea of what factors have the greatest impact on the differences between the two models. This led to the development of an empirical correction factor which proved a strong capacity to reduce the typical error of conventional SDOF, predictions.

Thus, it provides a practical and reliable tool to estimate the maximum deflection of reinforced concrete beams under impact loading without the need for complex MDOF analysis or detailed finite element modeling for each case. We must mention on the studies, that reported about the effect of the thermal stresses on the results of the members that may be subjected to an impact load (Ahmed et al., 2026; Ali et al., 2023).

2. METHODOLOGY

Theoretical Background and Formulation

Single Degree of Freedom (SDOF) Model

In SDOF models, the structural element is replaced by a simple equivalent system consist of one point as shown in Fig. 1. This system is consisting of equivalent of mass, damping, external force, and stiffness. in this study (Pham et al., 2017; Luo et al., 2025; Sobol, 2001). the damping ratio was taken equal to 5% because it was given a best result when our model verification with the compared previous results studies, and it is a common approximate value use in many design codes like the ASCE 7-22 (ASDE 7-22,22) standard recommended as 5% damping ratio for RC structures in seismic design.

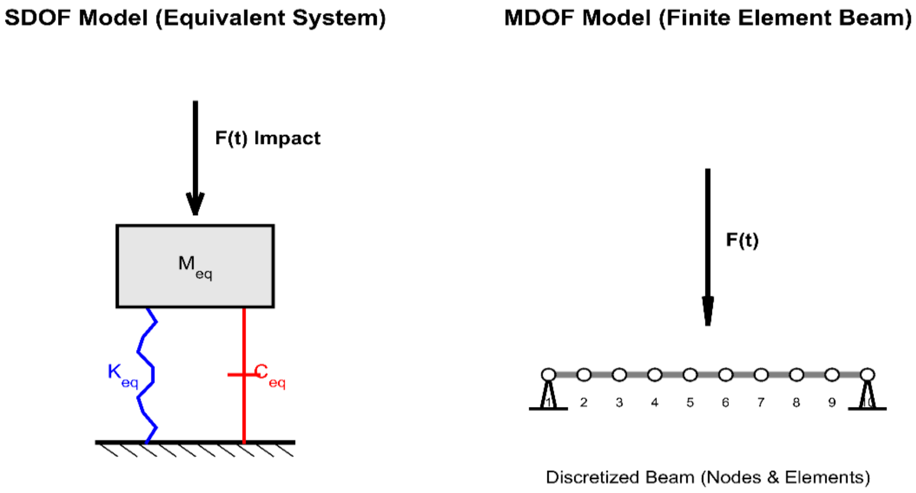

Figure 1. Schematic representation comparing the simplified SDOF model (equivalent system) and the discretized MDOF model

In the SDOF idealization, a distributed structural system is replaced by an equivalent system with a single generalized coordinate, as shown in Fig. 1. The equivalent system is defined by an equivalent mass, an equivalent stiffness, an equivalent damping ratio, and an equivalent external force. In the present study, the damping ratio was taken as 5% (Pham et al., 2017; Luo et al., 2025). This idealization provides a simple representation of the dominant global response (Sobol, 2001).

The 5% damping ratio was used as an equivalent damping value for the simplified dynamic model. This value represents the general energy loss in the beam, but it does not describe all local effects, such as cracking, impact contact, and local damage.

The SDOF model is a simple method to evaluate the maximum displacement that occurs in the first vibration mode, and it is not a complete model that characterizes the complex nonlinearity occurs under high-speed impact that lead to severe localized concrete damage and the effect of shear stress is substantial on the beam. Thus, we can write the equation of motion as shown in Eq. (1) (Paz & Leigh, 2012; Gisladottir et al., 2025).

$$M_{eq}\ddot{u}(t) + \mathrm{C_{eq}}\dot{u}(t) + K_{eq}u(t) = F(t) \quad (1)$$

$\dot{u}(0) = v_o = \sqrt{2gh}$ Where M_{eq}, C_{eq}, and k_{eq} are the equivalent mass, damping, and stiffness, respectively. In the MATLAB model, the impact was applied through the initial velocity, while the initial displacement was taken as zero.

The stiffness is the resistance of a beam to the deformation or bending when subjected to external loads. When a very stiff beam is subjected to a load, the maximum displacement is small, but its vibration is very rapid. While if the stiffness is low or the applied external load is very high, the beam will bend significantly, and nonlinear cracks may appear.

For this reason, the equivalent stiffness used in the SDOF model is important for estimating the displacement response and the maximum mid-span deflection. In the case of a simply supported beam loaded at its mid-span, the equivalent stiffness can be calculated from Eq. (2):

$$K_{eq} = (48\, EI)/L^3 \tag{2}$$

Where k_eq is the equivalent stiffness, although the SDOF model is simple and gives quick results, it cannot describe the full vibration pattern or the complete deflection shape along the beam (Newmark, 1959). For this reason, the limitation of the first-mode assumption was later reduced by comparing the SDOF results with the MDOF model and by using the correction factor to account indirectly for higher-mode effects and distributed inertia. Fig. 1 illustrates the fundamental difference between the idealized SDOF system and the discretized MDOF beam model.

Regarding nonlinearity, the present SDOF model is a simplified equivalent elastic model. It does not explicitly simulate concrete cracking, reinforcement yielding, or plastic damage. Instead, the effect of these nonlinear phenomena is included indirectly through the equivalent dynamic properties and through the correction factor obtained by comparing the SDOF results with the MDOF response. The MDOF model used in MATLAB also focuses on the global displacement response and does not explicitly include a detailed concrete damage model. Therefore, the proposed model should be used mainly for estimating the maximum displacement response and not for predicting crack patterns, local damage, or reinforcement plasticity. The influence of cracking and nonlinear behavior was considered indirectly by validating the results against experimental data and Abaqus simulations (Abaqus, 2024).

Multi-Degree of Freedom (MDOF) Model

To model the structural behavior more realistically, the mass, stiffness, damping, and applied forces are often distributed at many discrete points. All the nodes have the ability to move or rotate. The deformations of the structure under different vibration modes can be analyzed, including higher-order modes. The degrees of freedom of the system are brought into clearer perspective, and provide a better idea of the internal stresses involved. Equation (3) gives the FEM solution for this system (Muho & Kalapodis, 2024; Cook & Malkus, 2025).

$$[M]\{\ddot{u}\} + [C]\{\dot{u}\} + [K]\{u\} = \{F(t)\} \tag{3}$$

The standard Euler-Bernoulli beam formulation is used to build the global stiffness matrix [K]. The local stiffness matrix [k_e] for a single element connecting two nodes with two degrees of freedom each (vertical translation and rotation) with length le, is given by Equation 4:

$$[\mathrm{k_e}] = \frac{\boldsymbol{EI}}{\boldsymbol{l^3}} \begin{bmatrix} 12 & 6\,\mathrm{l_e} & -12 & 6\,\mathrm{l_e} \\ 6\,\mathrm{l_e} & 4\mathrm{l_e^2} & -6\,\mathrm{l_e} & 2\mathrm{l_e^2} \\ -12 & -6\,\mathrm{l_e} & 12 & -6\,\mathrm{l_e} \\ 6\,\mathrm{l_e} & 2\mathrm{l_e^2} & -6\,\mathrm{l_e} & 4\mathrm{l_e^2} \end{bmatrix} \tag{4}$$

After the element matrices are formed, they are assembled to obtain the global MDOF system. This model can describe the beam response in more detail than the SDOF model because it includes the distribution of mass and stiffness along the beam. Therefore, it can represent more complex deformation patterns. In this study, the beam was divided into eighty elements.

Numerical Implementation and calibration

The equations of motion of the SDOF and MDOF models were solved numerically using a MATLAB R2020a code prepared for this study. The main steps of the numerical procedure are shown in Fig. 2.

At the beginning, we entered the geometrical and engineering details of the beam. For MDOF model, the beam was divided into eighty elements, and the global stiffness and mass matrices were built. For impact case, the initial velocity was calculated from the drop height (H) through $v_o = \sqrt{2gH}$. The initial displacement value of the beam was considered equal to zero.

The purpose of this study is to compute the maximum displacement at mid-span of the beam after impact. The impact load was represented through the previous equation of initial velocity (v_o). The model does not simulate the detailed contact force between the beam and the impactor and does not study the propagation of stress waves along the entire beam.

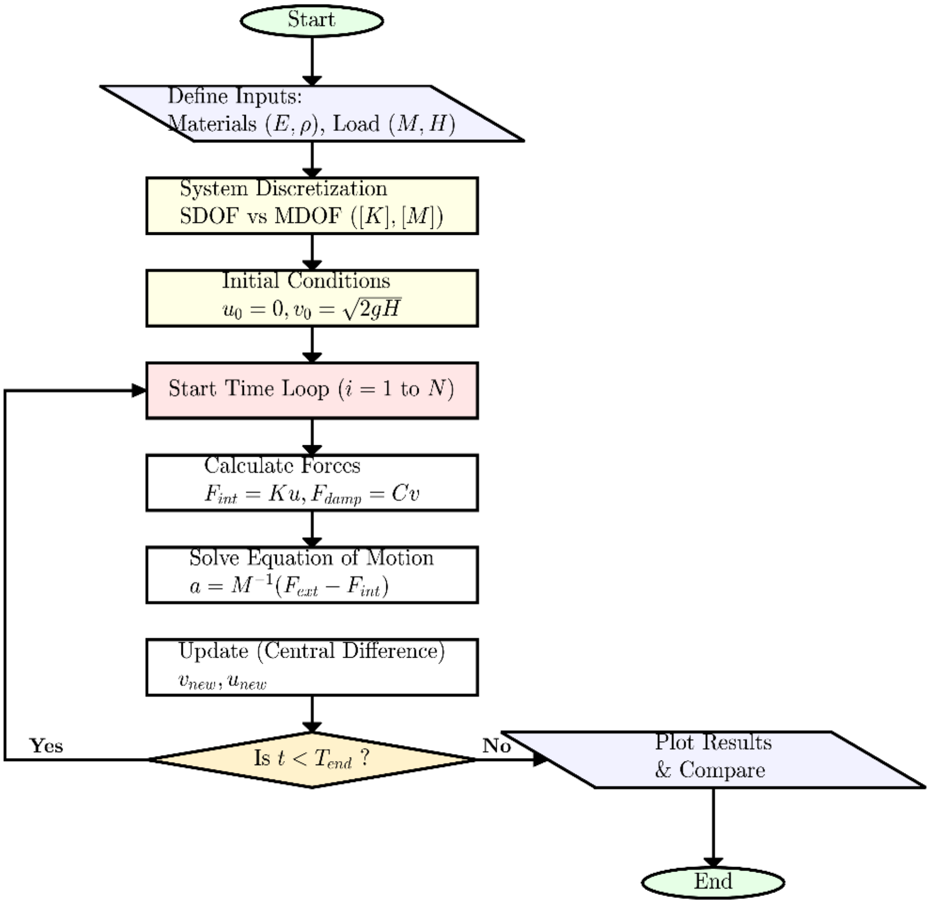

Figure 2. Flowchart of the numerical algorithm developed in MATLAB

The MDOF model provides a better representation of the distributed dynamic response than the SDOF model. At the same time the proposed correction factor further reduces the difference between the simplified and distributed responses. The solution was then carried out using the Central Difference Method as an explicit time-integration method. At each time step (dt), the algorithm calculates internal elastic and damping forces, solves for nodal accelerations, and updates the kinematic state for the next step. The process repeats until the simulation ends.

Model Calibration

To verify the accuracy of the developed code, a preliminary comparison was conducted between the experimental results reported by (Fujikake et al., 2009) and the analytical SDOF solution and the FEM-based MDOF model. The program computes the time-history response and visualizes the instantaneous deflection profile. To increase the reliability of our developed code, a comparison with the Abaqus finite element program was performed. The details of the experimental specimens adopted from (Fujikake et al., 2009; Satadru et al., 2015) are presented in Table 1, and the failure mode is illustrated in Fig. 3. while the Abaqus finite element model is illustrated in Fig. 4.

Table 1. Experimental specimens' details from Fujikake et al (Fujikake et al., 2009*(1); Satadru et al., 2015*(2))

Specimen ID	Reinforcement Type (Top/Bottom)	Yield Strength fy (MPa)	Drop Height H (m)	impactor mass (kg)	Fc' MPa
S1616-015 (1)	2-D16 / 2-D16	426 / 426	0.15	400	42
S1616-030 (1)	2-D16 / 2-D16	426 / 426	0.30	400	42
S1616-060 (1)	2-D16 / 2-D16	426 / 426	0.60	400	42
S1616-120 (1)	2-D16 / 2-D16	426 / 426	1.20	400	42
S1322-030 (1)	2-D13 / 2-D22	397 / 418	0.30	400	42
S1322-060 (1)	2-D13 / 2-D22	397 / 418	0.60	400	42
S1322-120(1)	2-D13 / 2-D22	397 / 418	1.20	400	42
S1322-240 (1)	2-D13 / 2-D22	397 / 418	2.40	400	42
S2222-030 (1)	2-D22 / 2-D22	418 / 418	0.30	400	42
S2222-060 (1)	2-D22 / 2-D22	418 / 418	0.60	400	42
S2222-120 (1)	2-D22 / 2-D22	418 / 418	1.20	400	42
S2222-240 (1)	2-D22 / 2-D22	418 / 418	2.40	400	42
DR3.8_0.8_0.15 (2)	2T13 / 2T13	520	0.6	300	40
DR3.8_0.8_0.15(2)	2T13 / 2T13	520	0.9	300	40
DR3.8_0.8_0.15(2)	2T13 / 2T13	520	1.2	300	40

Figure 3. Failure modes: DR3.8_0.8_0.15 [Satadru et al., 2015].

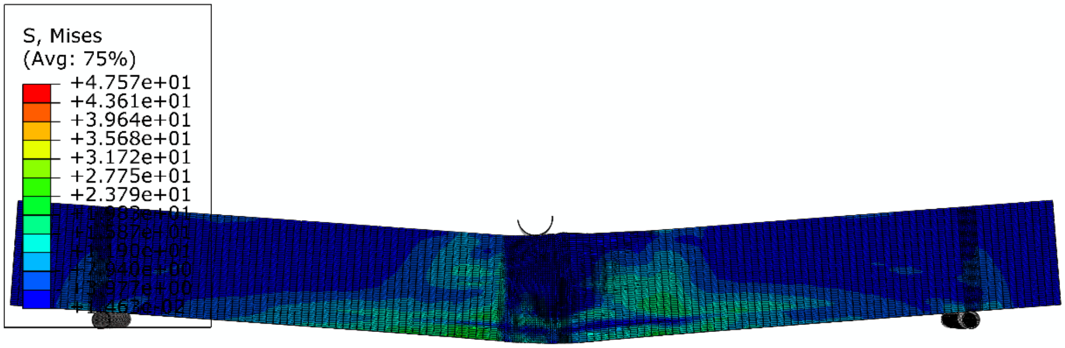

Figure 4. Finite element model of the reinforced concrete beam developed in Abaqus.

A comparison of mid-span displacement among the developed model, the experimental results, and Abaqus is listed in Table 2. Table 2 demonstrated that there is close agreement between the results of the developed code and the laboratory results and the Abaqus model, which confirms that the code is accurate and able to predict the beam behavior correctly.

Table 2. Verification of the developed model against experimental and Abaqus mid-span displacement results.

Specimen ID	Experimental displacement (mm)	Abaqus displacement (mm)	Developed SDOF prediction (mm)	Developed MDOF prediction (mm)
S1616-015	5.8	6.5	7.1	6.3
S1616-030	10.2	11.3	12.5	10.7
S1616-060	18.8	20	22.0	19.5
S1616-120	36	39	42.5	37.3
S1322-030	6.3	6.0	8.2	7.0
S1322-060	11.3	11.4	13.7	11.9
S1322-120	21.3	22.1	24.9	21.9
S1322-240	41.2	40.8	46.6	41.6
S2222-030	6.3	6.9	7.7	6.6
S2222-060	11.2	11.5	13.6	12.1
S2222-120	20.7	21.3	24.2	22.0
S2222-240	39.6	39	42.1	40.1
DR3.8_0.8_0.15	19.6	21.2	23.3	20.1
DR3.8_0.8_0.15	28.8	27.9	30.4	27.7
DR3.8_0.8_0.15	39.2	36.9	41.2	28.5

Model Verification

To verify the accuracy of the developed code, a preliminary comparison was conducted between the analytical SDOF solution and the FEM-based MDOF model. The program computes the time-history response and visualizes the instantaneous deflection profile. Fig. 5 displays an example of the output from our MATLAB code.

The results showed that the code captured the beam deformation in both models of the beam deformation in both models. The results also showed that the SDOF method overestimated the maximum displacement in the mid-span of the beam (16.03 mm) compared to the MDOF method, which was 11.21 mm, i.e., a difference of approximately 42.98%. This confirms the necessity of using the MDOF method for beam analysis to represent the realistic stiffness and boundary conditions that are neglected by the simplified SDOF method.

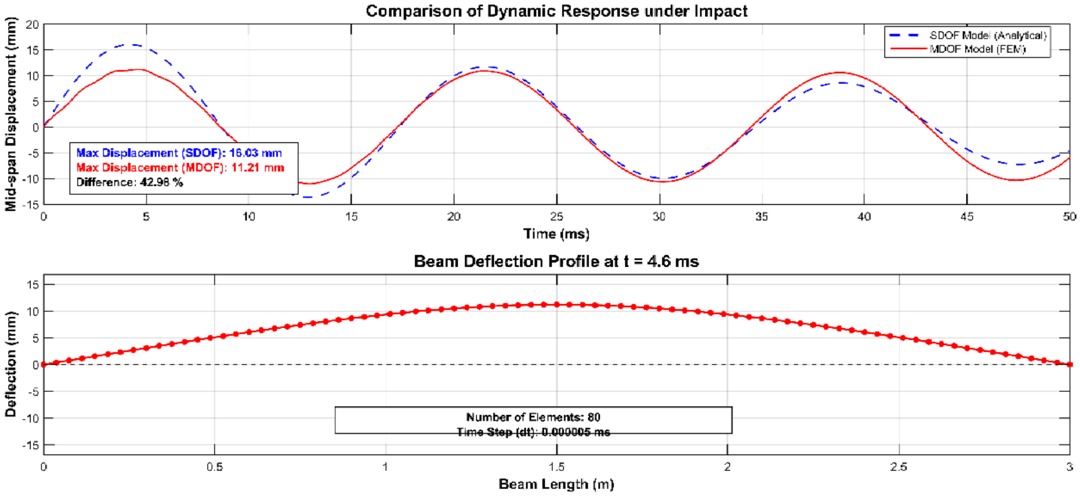

Figure 5. Comparison of dynamic response time-histories computed using the developed MATLAB codes for SDOF and MDOF models.

Mesh Convergence Study

A convergence study was carried out on the developed MDOF model to check mesh sensitivity and ensure numerical stability. This was achieved by varying the mesh density from 10 elements to 90 elements. The main goal was to find the optimal discretization level required to obtain a grid-independent solution.

Table 3 shows the maximum mid-span displacement and the critical time step (Δt) for different mesh refinements. As shown in the results, the coarse mesh with ten elements was unstable and failed to produce a valid solution. However, increasing the number of elements from 40 to 90 caused a negligible variation of less than 0.7% (decreasing slightly from 35.59 mm to 35.34 mm). This confirms that grid independence was achieved. Therefore, the 80-element high-fidelity model was selected as the converged benchmark for this study.

Table 3. Mesh convergence results for the MDOF model

Mesh Density	Time step (Δt, s)	Max Displacement (mm)	% Difference
10 Elements	1×10^{-6}	(Unstable)	-
20 Elements	0.25×10^{-6}	37.09	-
40 Elements	0.06×10^{-6}	35.59	4.0%
80 Elements	0.08×10^{-6}	35.34	0.7%
90 Elements	0.08×10^{-6}	35.33	0.7%

Case Study Parameters

The dynamic response of a simply supported beam under impact load was evaluated using a numerical model, as shown in Fig. 6, (other boundary conditions, such as fixed or continuous supports involve different dynamic mechanisms, and require a separate study), with the geometric and material properties listed in Table 4. These values act as the reference sample employed in the numerical simulation. In the parametric study, the impactor mass and drop height were adjusted to evaluate their effect on the dynamic response, while the beam dimensions and material properties remained constant.

The present study represents the simply supported beam only; other boundary conditions, such as fixed or continuous supports, involve different dynamic mechanisms and need to be addressed in a separate study to further generalize the proposed correction model.

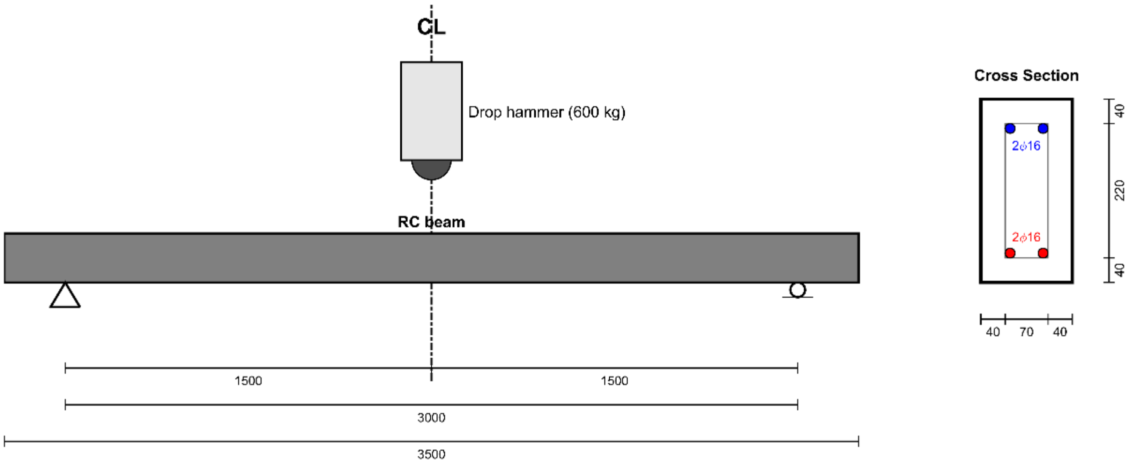

Figure 6. Schematic representation of the reinforced concrete beam under drop-weight impact.

Table 4. Geometric and material properties of the beam used in the simulation.

Lo (m)	b (m)	h (m)	E(Pa)	Density (kg/m³)	Mo_drop(kg)	Ho_drop(m)	Initial velocity, V0 (m/s)
3	0.15	0.3	2.57E+10	2500	600	2	6.264184

3. RESULTS & DISCUSSIONS

The accuracy of the SDOF model was checked against the MDOF results by changing four key parameters: impact mass, drop height, span length, and section depth.

Effect of Impact Mass

The effect of the impact mass was studied by changing the impactor mass from 600 kg to 1400 kg, while keeping the drop height and beam geometry constant. Fig. 7 illustrates the relationship between the impact mass and the maximum mid-span displacement for both SDOF and MDOF models and the percentage error between the two methods.

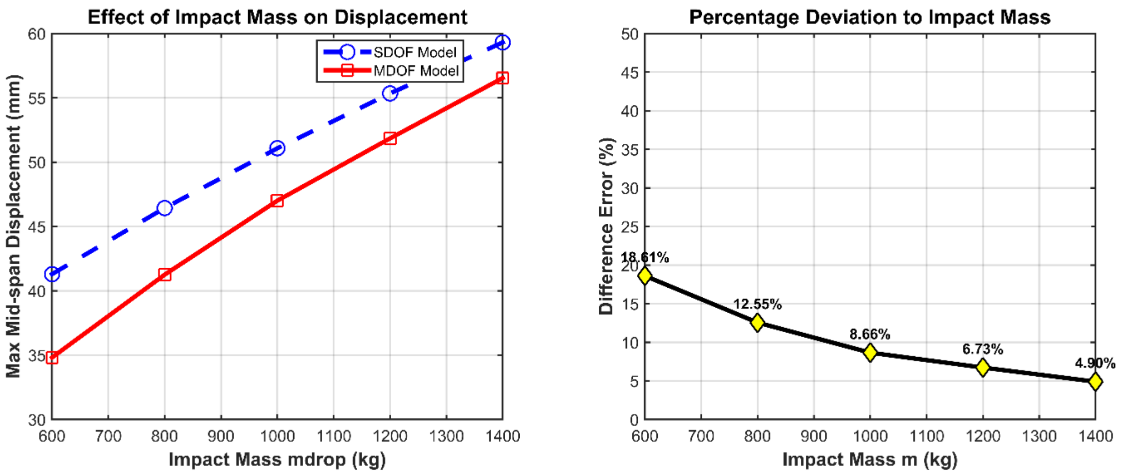

Figure 7. Influence of impact mass on maximum mid-span displacement and the percentage discrepancy between SDOF and MDOF models

The results show that, in every simulated case, the SDOF model yielded larger displacements than the MDOF model, and the SDOF predictions became much closer to the MDOF predictions as the mass increased. At a lower impact mass (600 kg), the SDOF model overestimates the displacement by 18.61%.

An important result of the analysis is that when the impactor mass increases, the discrepancy percentage between the two models decreases, as the accuracy of SDOF becomes very close to that of MDOF. It decreased to 4.9% when the impactor mass increased to 1400 kg. This can be explained physically by the fact that the increase in the impactor mass increases the inertia affecting the system, forcing the beam to react as a single mass and bend in the fundamental mode without allowing for the appearance of complex oscillations. This behavior is fully consistent with the SDOF assumption. In contrast, lighter impactor masses lighter impactor masses are more likely to excite higher modes and subtle undulations in the beam's body, details that the SDOF model is unable to detect, which explains the large differences at lower impactor masses.

Effect of Drop Height (Impact Velocity)

For this parameter, different impact velocities (represented by the change in drop height) were used to show the response of the two models and measure the percentage difference between them. We used a drop height extending from 0.5 m to 3.0 m, while keeping the beam specifications and other parameters fixed for all stages of analysis. The results of the analysis, as shown in Fig. 8, showed that the increase in the drop height led to a proportional increase in the maximum displacement in the mid-span of the beam for both models. It was also observed that the SDOF model gave greater displacements at the same drop height values.

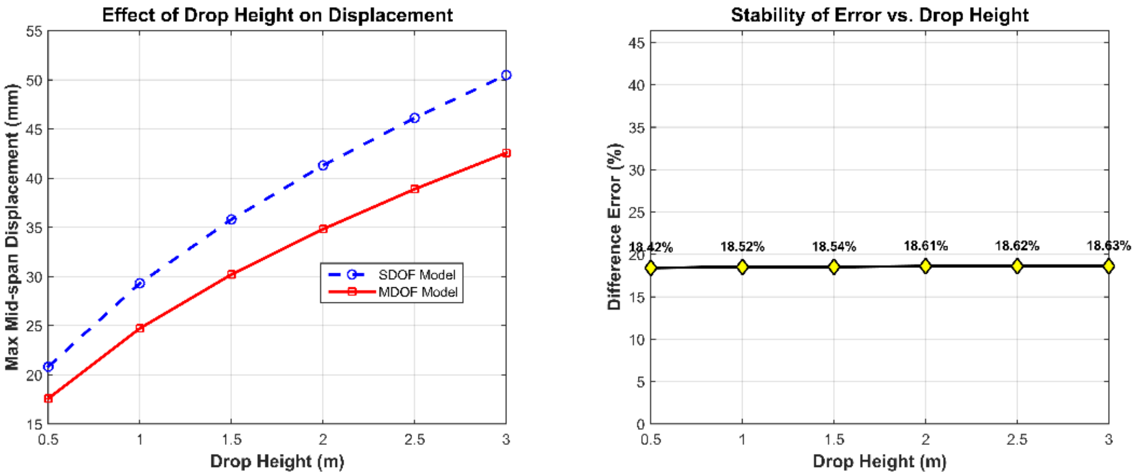

Figure 8. Influence of Drop Height on maximum mid-span displacement and the percentage discrepancy between SDOF and MDOF.

Fig. 8 also shows that the percentage difference between the two models remains almost constant across all simulated cases (approximately 18%). This indicates that both models respond nearly linearly to the same energy transfer mechanism, and that the discrepancy between them is due to the structural representation method rather than the intensity of the loading.

Effect of Span Length

The second parameter examined was the effective beam span. The analysis varied span lengths from 3 m to 6 m while keeping the other parameters constant. Fig. 9 indicates that the percentage difference between the two models became significantly higher from 18.6% at 3 m to 44.6% at 6 m. Moreover, the SDOF model showed greater displacements than the MDOF model in all the simulated cases.

The large increase in the difference is due to the flexibility of the beam. Physically, the longer the beam, the less stiff it becomes (according to the relationship $K = \frac{48EI}{L^3}$)Therefore, the beam does not move in a single simple shape. Rather it exhibits complex vibrations. Given that the SDOF model only represents a single simple mode shape, the model cannot accurately predict the displacements since it is unable to represent these intricate vibrations.

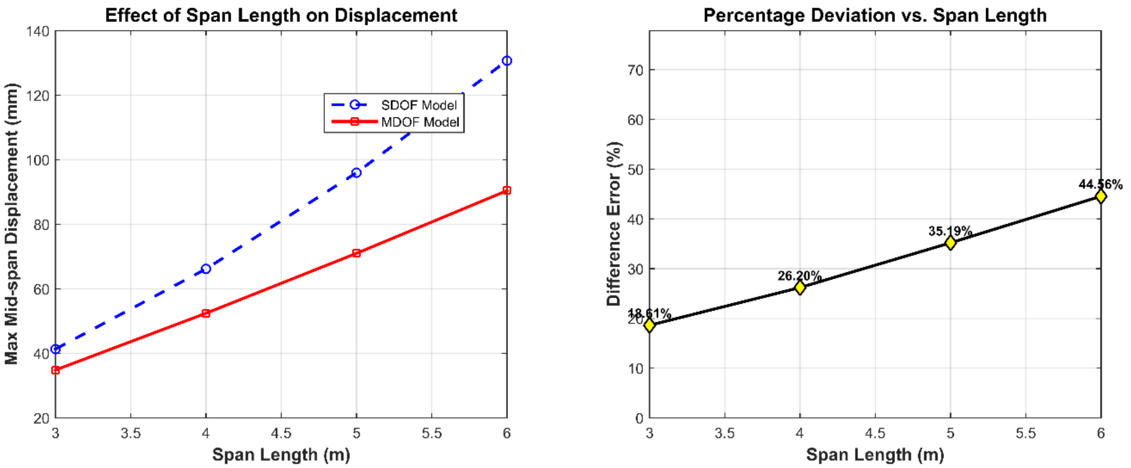

Figure 9. Influence of Span Length on maximum mid-span displacement and the percentage discrepancy between SDOF and MDOF

Effect of Section Depth

This study also investigated the beam section depth (h) by changing the depth between 0.2 m and 0.6 m, while keeping all other parameters stable, including the span length, mass of the impactor, and height of drop in all the simulated cases. The percentage change in mid-span displacement of the two models as illustrated in Fig. 10 indicates that the difference between the two models grew by a small value of 10.71% with the thin beam (h = 0.2 m) and 43.0% with the deep beam (h = 0.6 m). Moreover, the SDOF model always gave higher values of displacement as compared to the MDOF model.

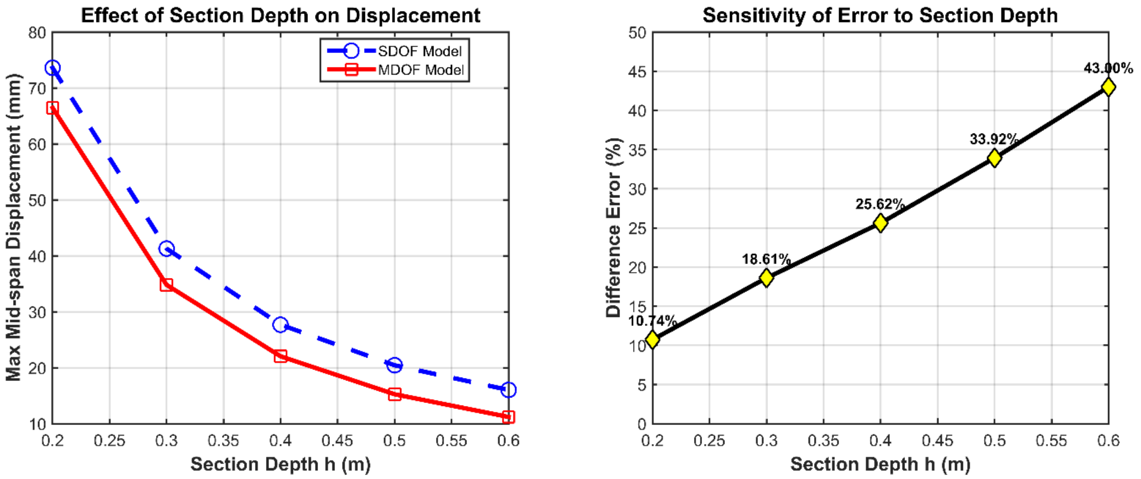

Figure 10. Influence of Section Depth on maximum mid-span displacement and the percentage discrepancy between SDOF and MDOF

The principal cause of the noted difference between the two techniques upon increasing the beam depth is that the governing hypotheses of the two models are different. The SDOF system follows the Euler-Bernoulli beam theory, according to which the cross-sections remain plane and perpendicular to the neutral axis; therefore, this model disregards shear deformations.

In contrast, the MDOF model (Muho and Kalapodis, 2024) follows Timoshenko beam theory, which is more precise and detailed because it takes into consideration the rotation and deformation of the section due to shear stresses.

The following comparison clearly shows this difference:

At a depth (h = 0.2 m), the analysis considered the beam a slender beam, where bending action dominates, and shear action is insignificant; hence, the two methods gave extremely similar results of the displacement.

At a depth (h = 0.6m), the behavior of the structure changed to a deep beam, where shear distortion became a significant and inherent factor that the analysis could not ignore. This is why the difference in the results is large and the SDOF model cannot represent this multifaceted response.

Development of a Correction Model for SDOF Predictions

To improve the accuracy of the simplified SDOF model, this study introduces a correction factor derived from the findings of the parametric study. This factor aims to reduce the discrepancy observed between the SDOF and MDOF predictions of the maximum mid-span displacement.

The correction factor, α, represents the ratio of the displacement predicted by the MDOF model to that predicted by the SDOF model:

$$\alpha = \frac{\delta_{MDOF}}{\delta_{SDOF}} \tag{5}$$

Accordingly, the following expression gives the corrected SDOF displacement:

$$\delta(SDOF)_{\ corrected} = \alpha.\delta_{SDOF}$$

The calculations showed that, as the (α) value approaches (1), the maximum displacement values between the SDOF and MDOF models converge and become nearly equal. Consequently, the values of (α) were far from value (1) in the case of beams which have high slenderness (large span to depth ratio), because increasing slenderness leads to an increase in beam flexibility, which makes the effect of higher vibration modes significant. For this reason, the difference of displacement between the two models was large and the results of SDOF become inaccurate compared to the equivalent MDOF. In terms of studying the effect of increasing the impactor mass, the calculations showed that, the value of (α) approaches to (1) as the impactor mass was increases, the reason is that the dynamic response is governed by the fundamental mode of vibration.

To provide a clearer theoretical basis for the proposed correction factor, the empirical relation was supported by dimensional analysis. Since (α) is defined as the ratio between the MDOF displacement and the SDOF displacement, it is dimensionless. Therefore, the governing variables were rewritten in normalized form using dimensionless groups inspired by the Buckingham a theorem.

The main dimensionless groups used in this study are h/L, L/ L0, m/m0, and H/HO, where L0 = 3 m, m0= 600 kg, and HO = 2 m are the reference values adopted in the parametric study. These groups represent the geometric slenderness effect, the relative span length, the relative impact mass, and the relative drop height, respectively.

According to the previous, this study introduces the following empirical relationship to estimate a correction factor as a function of the effective dimensionless parameters:

$$\alpha = 0.4879 \left(\frac{h}{L}\right)^{-0.237} \frac{L}{3}^{-0.503} \left(\frac{m}{600}\right)^{0.156} \frac{H}{2}^{-0.002} \tag{6}$$

where L is the span length of the beam, h is the section depth, m is impactor mass, and H is the drop height.

The very small exponent of (H/H0) (-0.002) indicates that the drop height has only a minor influence on the correction factor compared with span length, section depth, and impact mass. This is consistent with the parametric results, where increasing the drop height mainly changed the displacement magnitude but did not strongly change the discrepancy between the SDOF and MDOF models.

This form improves the theoretical justification of the proposed model because the correction factor is not expressed only as a curve-fitting relation, but also in terms of dimensionless groups with clear mechanical meanings.

This provides an easy and effective approach that can enhance SDOF prediction accuracy without relying on the MDOF or explicit finite-element-based models. Results from the corrected SDOF, compared to the MDOF results, further emphasize this ability of overcoming prediction error with this technique. Fig. 11 compares the predicted correction factor with the one from the numerical simulations.

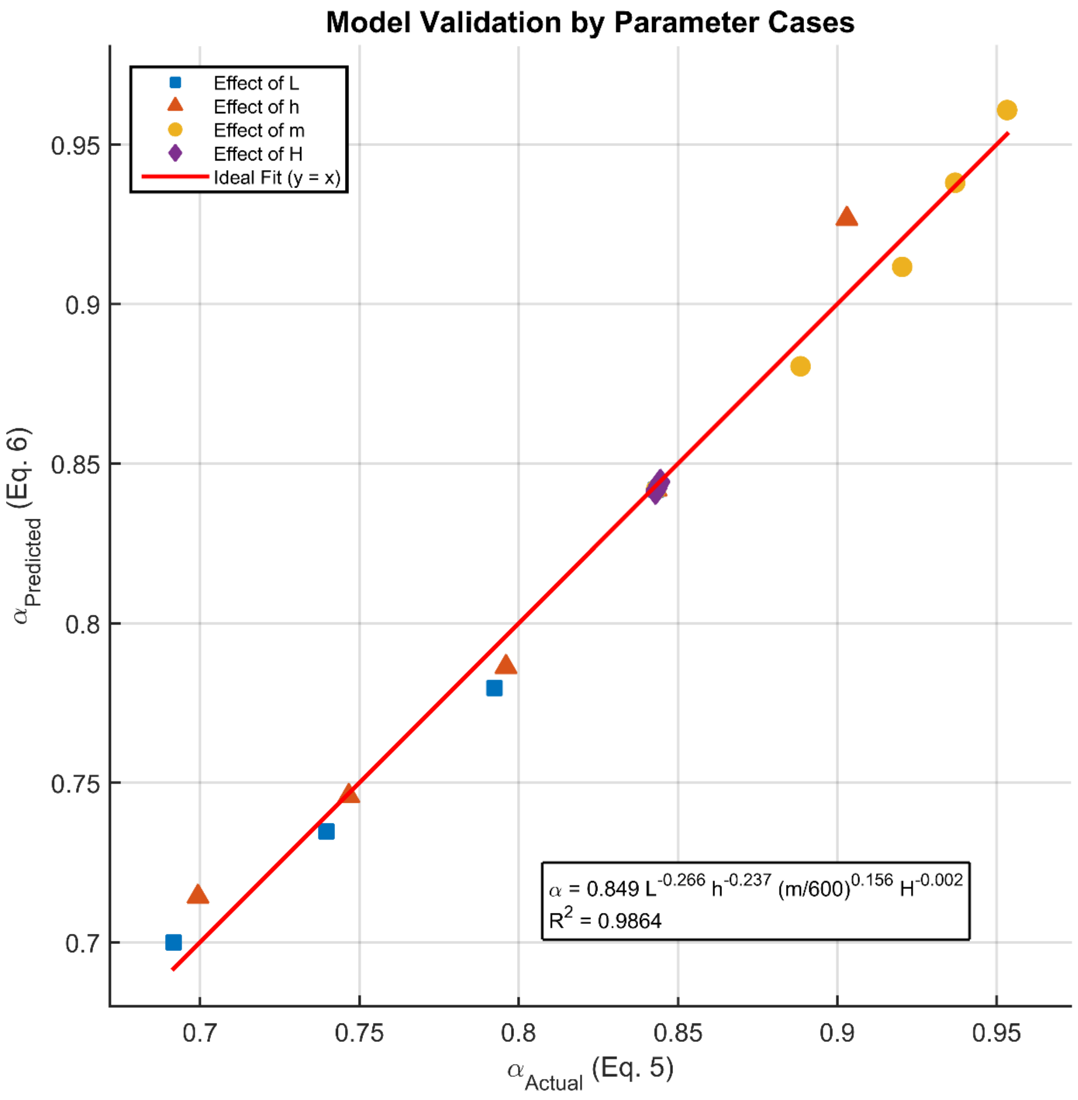

Figure 11. Comparison between predicted (Eq. 6) and actual (Eq. 5) values of the correction factor (α)

Figure 11 shows a strong agreement between the values calculated from the correction coefficient according to Equation (6) and the actual values calculated according to Equation (5), as most of the data points representing the numerical results (blue marks) lie near the diagonal line (red line), which represents the ideal case, where (y=x), and indicates an almost complete agreement between the calculations of the proposed equation and the numerical results. This indicates that the proposed empirical model is able to represent the relationship between the governing variables and the correction coefficient with good accuracy. The high value of the coefficient of determination ($R^2 = 0.9866$) confirms the validity and reliability of the proposed formulation in this convergence.

Overall, this model significantly reduces the error of the SDOF model's calculated results and provides a reliable correction factor that users can apply directly to improve the corrected results. Table 5 and Table 6 summarizes the corrected SDOF and MDOF results, Mean Square Error (RMSE), Mean Absolute Error (MAE). Figure 12 display the residual error in the displacement values at mid-span of the beams tested.

Table 5. Comparison between corrected SDOF and MDOF results

Length (L)	section depth (h)	impact mass (m)	drop height (H)	d_{SDOF}	α (Eq. 6)	Corrected SDOF displacement	d_{MDOF}	% error
3	0.3	600	2	41.3	0.84200	34.775	34.82	0.13
4	0.3	600	2	66.14	0.77997	51.587	52.41	1.57
5	0.3	600	2	95.96	0.73502	70.533	70.98	0.63
6	0.3	600	2	130.7	0.70023	91.520	90.41	1.23
3	0.2	600	2	73.61	0.92693	68.231	68.47	0.35
3	0.3	600	2	41.3	0.84200	34.775	34.82	0.13
3	0.4	600	2	27.7	0.78651	21.786	22.05	1.20
3	0.5	600	2	20.46	0.74599	15.263	15.27	0.05
3	0.6	600	2	16.03	0.71445	11.453	11.21	2.16
3	0.3	600	2	41.3	0.84200	34.775	34.82	0.13
3	0.3	800	2	46.44	0.88065	40.897	41.26	0.88
3	0.3	1000	2	51.08	0.91185	46.577	47.01	0.92
3	0.3	1200	2	55.34	0.93815	51.917	51.85	0.13
3	0.3	1400	2	59.31	0.96099	56.996	56.54	0.81
3	0.3	600	0.5	20.83	0.84434	17.588	17.59	0.01
3	0.3	600	1	29.31	0.84317	24.713	24.73	0.07
3	0.3	600	1.5	35.81	0.84249	30.169	30.21	0.13
3	0.3	600	2	41.3	0.84200	34.775	34.82	0.13
3	0.3	600	2.5	46.13	0.84163	38.824	38.89	0.17
3	0.3	600	3	50.5	0.84132	42.487	42.57	0.20

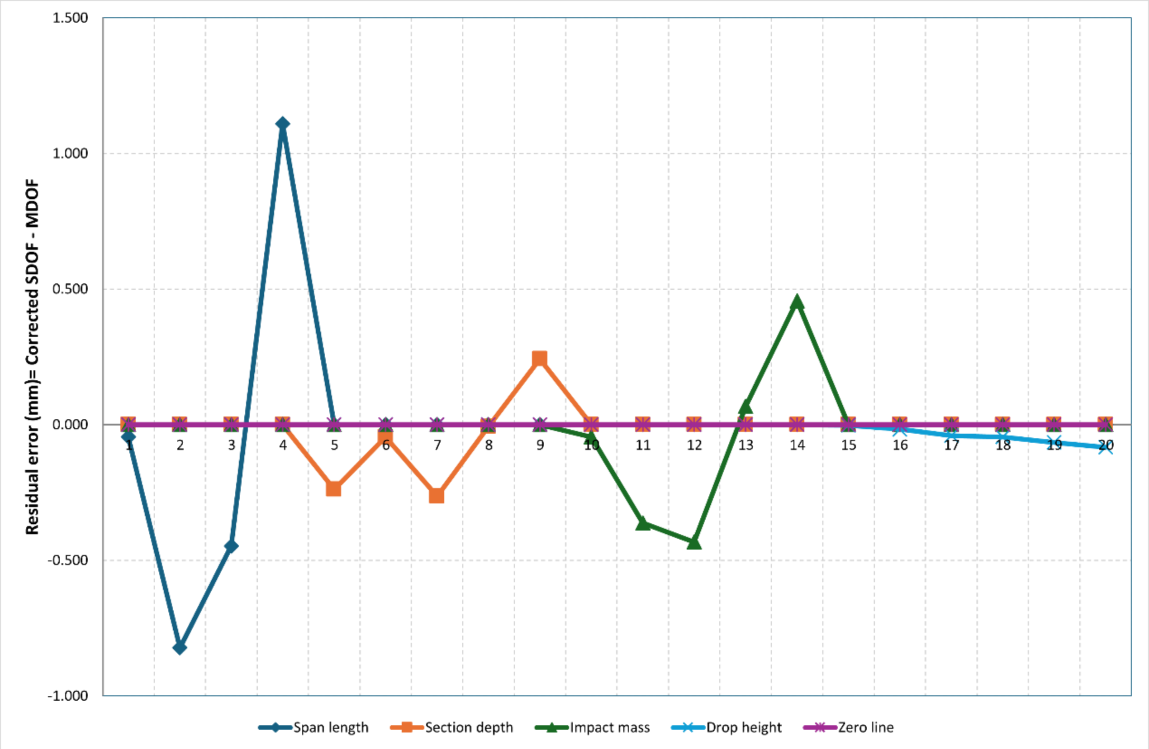

Figure 12: Comparison between d(SDOF)$_{corrected}$ and d_{MDOF} results and their relative percentage errors under varying conditions.

Table 6. Calculate Root Mean Square Error (RMSE), Mean Absolute Error (MAE)

Case	L (m)	Residual (mm)	Abs. error (mm)	Squared error	% error	RMSE (mm)	MAE (mm)
L=3m	3.00	-0.045	0.045	0.002	0.13	0.3773	0.2420
L=4m	4.00	-0.823	0.823	0.677	1.57		
L=5m	5.00	-0.447	0.447	0.200	0.63		
L=6m	6.00	1.110	1.110	1.232	1.23		
h=0.2mm	3.00	-0.239	0.239	0.057	0.35		
h=0.3mm	3.00	-0.045	0.045	0.002	0.13		
h=0.4mm	3.00	-0.264	0.264	0.070	1.20		
h=0.5mm	3.00	-0.007	0.007	0.000	0.05		
h=0.6mm	3.00	0.243	0.243	0.059	2.16		
m=600kg	3.00	-0.045	0.045	0.002	0.13		
m=800kg	3.00	-0.363	0.363	0.131	0.88		
m=1000kg	3.00	-0.433	0.433	0.187	0.92		
m=1200kg	3.00	0.067	0.067	0.005	0.13		
m=1400kg	3.00	0.456	0.456	0.208	0.81		
H=0.5m	3.00	-0.002	0.002	0.000	0.01		
H=1m	3.00	-0.017	0.017	0.000	0.07		
H=1.5m	3.00	-0.041	0.041	0.002	0.13		
H=2m	3.00	-0.045	0.045	0.002	0.13		
H=2.5m	3.00	-0.066	0.066	0.004	0.17		
H=3m	3.00	-0.083	0.083	0.007	0.20		

The residual error is given by the difference between the predictions of the corrected SDOF model and the reference results of the MDOF model. Most of the data points fluctuate close to the zero-error line, indicating high predictive accuracy. While the 'span length' showed extreme sensitivity in some cases (e.g., Case No. 4), other parameters such as 'drop height' showed almost perfect agreement. This convergence confirms that the proposed correction model effectively reduces the inherent errors of traditional SDOF analysis, with the relative error being less than 2% in most cases and not exceeding 2.2% in worst-case scenarios. This large error minimization indicates that the suggested correction factor is effective in reducing the error that is present in the SDOF results. The results affirm that the proposed model is capable of providing credible displacement estimates while retaining the same level of computational simplicity as the SDOF approach. Engineers can also use it as a practical substitute for the more computationally intensive MDOF or finite element analysis for preliminary design and evaluation.

4. CONCLUSION

This paper examined the dynamic behavior of impact-loaded beams in simplified Single-Degree-of-Freedom (SDOF) and Multi-Degree-of-Freedom (MDOF) models. This study conducted a detailed parametric study to test the impact of the most important governing parameters, as well as to create a correction model that enhances the predictive performance of the simplified SDOF method. According to the obtained results, it is possible to draw the following conclusions:

Effect of Drop Height

The findings suggest that changes in the drop height mainly scale the input impact energy without drastically changing the deformation mechanism of the beam. As a result, both SDOF and MDOF models show proportional changes in the displacement with the drop height, and the relative difference between the two models is relatively stable.

Effect of Impact Mass

The simplified SDOF model is more accurate with an increase in the impact mass. Increasing the impactor masses results in higher inertia of the system and leads to the fundamental vibration mode taking control of the structural response. In this case, the dynamic behavior will be closer to the assumptions of the SDOF formulation. Conversely, the lighter masses of impact are more likely to excite the higher modes of vibration, resulting in multifaceted patterns of deformation that the simplified SDOF model does not sufficiently resolve.

The span length also affected the dynamic response

A significant factor in the dynamic behavior of the SDOF system under impact loads is the span length of the beam where, longer span length results in lower stiffness and lower natural frequency. The decrease in stiffness leads to an increase in the flexibility of the beam and in turn, leads to a significant increase in maximum displacement, which leads to less accurate results, which were clearly confirmed by the results, as they showed a significant increase in the discrepancy between the two systems, increasing from 18.6% in the short spans to 44.6% in the long spans. In this case, it is important to consider the effect of the section depth.

A cubic increase in the depth of the beam section results in a cubic increase in the moment of inertia and results in an increase in the stiffness of the beam and a decrease in the displacement. The shear deformations are higher with larger beam depth. The SDOF system does not consider the influence of shear deformations as the MDOF system does; therefore, the results revealed that the difference between the two approaches grows with the section depth where the difference is 10.74% at a depth of 0.2 m and 43% at a depth of 0.6 m. Therefore, this study recommends avoiding the SDOF system in deep beams.

Correction Model Development

To overcome the points where the SDOF model cannot give correct results or requires a more detailed model, this study suggested a correction factor depending on the findings of the numerical parametric study. The empirical model developed considers the influence of the span length, section depth and impact mass and drop height. The proposed equation showed a close agreement with the MDOF results, with ($R^2 = 0.9866$) and prediction errors mostly less than 2%. The SDOF results with corrections also indicated that there was a close agreement with the MDOF predictions in a large range of simulated conditions.

Practical Implications

The proposed correction model makes the simplified SDOF model much more accurate while maintaining the simplicity and speed of the model. Thus, engineers can use the revised version of this model as a viable and effective method to estimate the beam displacement during impact loading in the pre-design phase and the initial assessment of the structure without involving sophisticated computations based on MDOF or the finite element approach.

Acknowledgement

We would like to express my sincere gratitude to all those who supported me throughout the course of this research.

Author Contributions

Ali M. Lafta contributed to the study conception and design, material preparation, data collection, numerical modeling, analysis, and drafting of the manuscript. Salih Meri Al-Absi and Ahmed Raad Al-Adhadh contributed to the interpretation of the results, critical revision of the manuscript, and supervision of the work. All authors read and approved the final manuscript.

Ethical issues

Not applicable. This study does not involve any experiments on humans or animals. Hence, ethical approval was not required.

Informed consent

Not applicable.

Funding

This research did not receive any external funding like specific grant from funding agencies in the public, commercial, or nonprofit sectors.

Conflict of Interest

The authors declare that they have no conflicts of interest, competing financial interests or personal relationships that could have influenced the work reported in this paper.

Data and materials availability

The data that support the findings of this study are available from the corresponding author upon reasonable request.

REFERENCES

1. Abaqus. Analysis user's guide. Dassault Systemes Simulia Corp; 2024.
2. Ahmed SK, Lafta AM, Hama SM. Methods of the uniformity of thermal load applied to elements in standard fire resistance tests: a contemporary review. Innov Infrastruct Solut 2026;11: 122. doi: 10.1007/s41062-026-02506-3.
3. Ali M. Lafta, Hussein Kareem Sultan, Ameer J. Abdulkareem and Ali A. Hassan. Effect of the heat from fire on the strength of the concrete. AIP Conf Proc 2023;2806(1):040018. doi: 10.1063/5.0163239.
4. Bathe KJ. Finite element procedures. 2nd ed. Watertown: Klaus-Jurgen Bathe; 2014.
5. Clough RW, Penzien J. Dynamics of structures. 4th ed. Berkeley: Computers & Structures; 2015.
6. Cook RD, Malkus DS. Concepts and applications of finite element analysis. 5th ed. New York: John Wiley & Sons; 2025.
7. Cook RD. Concepts and applications of finite element analysis. New York: John Wiley & Sons; 2007.
8. Cui L, Zhang X, Hao H. Prediction of dynamic shear and maximum displacement of clamped reinforced concrete beams subjected to impact loading. Int J Impact Eng 2025;195: 105131. doi: 10.1016/j.ijimpeng.2024.105131.
9. Fujikake K, Li B, Soeun S. Impact response of reinforced concrete beam and its analytical evaluation. J Struct Eng 2009;135(8):938-950. doi: 10.1061/(ASCE)ST.1943-541X.0000039
10. Gisladottir LM, Castellino M, Dermentzoglou D, Hendriks MAN, de Girolamo P, van Gent MRA, Antonini A. Curved concrete crownwalls on vertical breakwaters under impulsive wave load: finite element analysis. Coastal Eng 2025;201: 104791. doi: 10.1016/j.coastaleng.2025.104791.
11. Jin L, Wu S, Zhang R, Li J, Du X. Impact velocity and mass effects on the impact force of geometrically scaled reinforced concrete beams: simulation, mechanisms, and prediction. J Build Eng 2026;117:114794. doi: 10.1016/j.jobe.2025.114794.
12. Kishi N, Mikami H, Matsuoka KG, Ando T. Impact behavior of shear-failure-type RC beams without shear rebar. Int J Impact Eng 2002;27(9):955-968. doi: 10.1016/S0734-743X(01)00149-X.
13. Li H, Chen W, Hao H. Influence of drop weight geometry and interlayer on impact behavior of RC beams. Int J Impact Eng 2019;131:222-237. doi: 10.1016/j.ijimpeng.2019.04.028.
14. Li H, Chen W, Pham TM, Hao H. Analytical and numerical studies on impact force profile of RC beam under drop weight impact. Int J Impact Eng 2021;147:103743. doi: 10.1016/j.ijimpeng.2020.103743.
15. Luo H, Liu Z, Fan C, Li Y. Numerical simulation and experimental study of high-speed impact penetrator for lunar subsurface. Acta Astronaut 2025;233:250-265. doi: 10.1016/j.actaastro.2025.04.019.
16. Meyers MA. Dynamic behavior of materials. New York: John Wiley & Sons; 1994.
17. Muho EV, Kalapodis NA. The MDOF equivalent linear system and its applications in seismic analysis and design of framed structures. Resilient Cities Struct 2024;3(4):107-125. doi: 10.1016/j.rcns.2024.11.003.
18. Newmark NM. A method of computation for structural dynamics. J Eng Mech Div 1959;85(3):67-94. doi: 10.1061/JMCEA3.0000098.
19. Paz M, Leigh W. Structural dynamics: theory and computation. New York: Springer; 2012.
20. Pham AT, Tan KH, Yu J. Numerical investigations on static and dynamic responses of reinforced concrete sub-assemblages under progressive collapse. Eng Struct 2017;149: 2-20. doi: 10.1016/j.engstruct.2016.07.042.
21. Saatci S, Vecchio FJ. Effects of shear mechanisms on impact behavior of reinforced concrete beams. ACI Struct J 2009;106(1):78-86.
22. Saini D, Shafei B. Concrete constitutive models for low velocity impact simulations. Int J Impact Eng 2019;132:103329. doi: 10.1016/j.ijimpeng.2019.103329.
23. Samadzad A, Whelan M, Cathey S, Braxtan N, Chen S. Investigation of concrete constitutive models for predicting the response, damage, and residual capacity of reinforced concrete beams subject to low velocity impact. Int J Impact Eng 2025;202:105310. doi: 10.1016/j.ijimpeng.2025.105310.
24. Satadru Das Adhikary, Bing Li and Kazunori Fujikake. Low Velocity Impact Response of Reinforced Concrete Beams:

Experimental and Numerical Investigation. Int J Protect Struct 2015; 6:1

25. Sobol IM. Global sensitivity indices for nonlinear mathematical models. Math Comput Simul 2001;55:271-280. doi: 10.1016/S0378-4754(00)00270-6.
26. Zhao DB, Yi WJ, Kunnath SK. Numerical simulation and shear resistance of reinforced concrete beams under impact. Eng Struct 2018;166:387-401. doi: 10.1016/j.engstruct.2018.03.072.
27. Zhong J, Song C, Xu J, Cheng Y, Liu F. Experimental and numerical simulation study on failure mode transformation law of reinforced concrete beam under impact load. Int J Impact Eng 2023;179:104645. doi: 10.1016/j.ijimpeng.2023.104645.
28. Zienkiewicz OC, Taylor RL. The finite element method for solid and structural mechanics. Amsterdam: Elsevier; 2015.