On the action of mobile loads on an uninterrupted cylindrical tunnel

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ABSTRACT
The stationary transport load acts on the surface of the cavity or on the inner surface of the shell reinforcing cavity. The speed of the load movement is assumed to be subsonic, which corresponds to the modern transport speeds in the underground facilities under study. To describe the motion of a half-space and thick-walled shells, dynamic equations of the theory of elasticity in Lamé potentials are used, and for thin-walled shells, the classical equations of the theory of thin shells are used. Equations are recorded in the moving coordinate system associated with the load.

Key words: thick-walled shell, stationary load, cavity, mobile coordinate system, Lame potentials.

1. INTRODUCTION

In the theoretical aspect, the solution was based on the works [1,2,3]. In [4], the first and second boundary-value problems of the theory of elasticity for a half-plane with a point source of stationary waves concentrated within it, the potential of which is represented in terms of cylindrical functions, are solved by the method of expanding potentials on plane waves. And in [5], using this approach, the problem of the stationary load on the contour of a circular hole in a half-space was solved. Using the idea of these papers on the superposition of solutions and the re-expansion of plane waves into series in cylindrical functions, in [6], in contrast to the exact analytical solution for the subsonic case, when the velocity of the moving load is less than the velocity of the Rayleigh waves.

Statement of the problem for a circular tunnel

Using the model approach for research, we will represent the tunnel as an infinitely long circular cylindrical cavity with a radius $r = R$, located in a linear viscoelastic, homogeneous and isotropic half-space $x \leq h$ (Figure 1) parallel to its horizontal boundary (the earth's surface). We define the reaction of a half-space on a moving with a constant subsonic velocity c along the cavity surface in the direction of the load axis Z P.
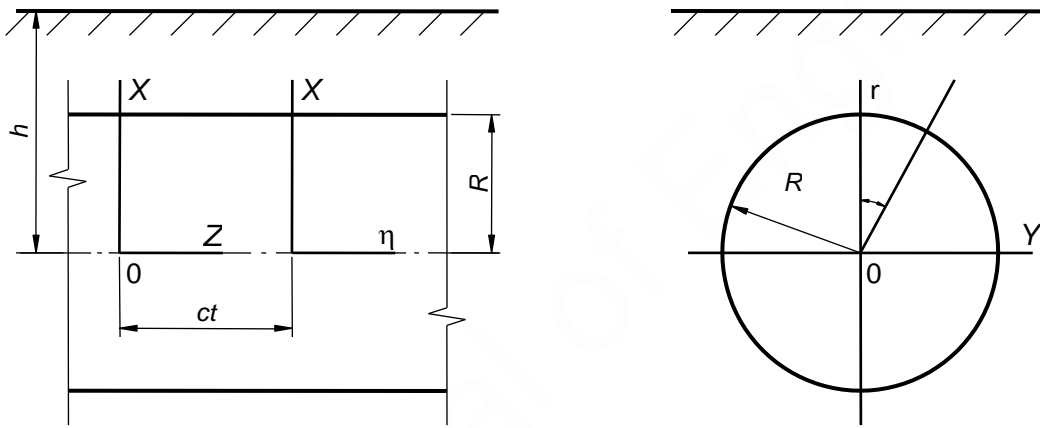

Figure 1 The calculated scheme of a reinforced tunnel

For this, we use the equations of motion of an elastic medium in vector form [9, 10]

$$\tilde{\mu}\nabla^2\vec{u} + (\tilde{\lambda} + \tilde{\mu})\,graddi\vartheta\,\vec{u} = \rho\frac{\partial^2\vec{u}}{\partial t^2}, \quad (1)$$

Here $\vec{u}(u_x, u_y, u_z)$ - vector of displacement of points of the medium; ρ - material density; u – displacement components;

ν_j - Poisson's ratio;

$$\tilde{\lambda}_j = \frac{v_j\tilde{E}_j}{(1+v_j)(1-2v_j)}; \quad \tilde{\mu}_j = \frac{v_j\tilde{E}_j}{2(1+v_j)}, \text{где}$$

$\tilde{E}$ – Operational modulus of elasticity, which have the form [11,12]:

$$\tilde{E}\varphi(t) = E_{01}\left[\varphi(t) - \int_0^t R_E(t-\tau)\varphi(t)d\tau\right] \quad (2)$$

$\varphi(t)$– arbitrary time function; $R_E(t-\tau)$– relaxation core; E_{01} – instantaneous modulus of elasticity; We assume the integral terms in (5) to be small, then the functions $\varphi(t) = \psi(t)e^{-i\omega_R t}$, where $\psi(t)$- a slowly varying function of time, ω_R - real constant. Further, applying the freezing procedure [13], we note relations (2) as approximations of the form

$$\overline{E}\varphi = E\left[1 - \Gamma^C(\omega_R) - i\Gamma^S(\omega_R)\right]\varphi,$$

where $\Gamma^C(\omega_R) = \int\limits_0^\infty R(\tau)\cos\omega_R\tau\, d\tau$, $\Gamma^S(\omega_R) = \int\limits_0^\infty R(\tau)\sin\omega_R\tau\, d\tau$, respectively, the cosine and sine Fourier images of the relaxation core of the material. As an example of a viscoelastic material, we take three parametric relaxation nuclei $R(t) = Ae^{-\beta t}/t^{1-\alpha}$. On the influence function $R(t-\tau)$ the usual requirements of inerrability, continuity (except for $t = \tau$), sign of uncertainty and monotony:

$$R > 0, \qquad \frac{dR(t)}{dt} \le 0, \ \ 0 < \int\limits_0^\infty R(t)dt < 1.$$

$\vec{u}$ - the vector of displacements of the environment.

Since the steady-state process is considered, the strain pattern is stationary with respect to the moving load. Therefore, it is convenient to move to a moving coordinate system η = z − ct, connected with the load P.
Then equation (1) can be rewritten in the form

$$\left(\frac{1}{M_p^2} - \frac{1}{M_s^2}\right)\text{grad div}\,\mathbf{u} + \frac{1}{M_s^2}\nabla^2\mathbf{u} = \frac{\partial^2\mathbf{u}}{\partial\eta^2}. \quad (3)$$

2. TASKS OF THE ACTION OF MOBILE LOADS ON AN UNREINFORCED TUNNEL

In the theoretical aspect, the solution was based on the papers [15, 16]. In [14], the first and second boundary-value problems of the theory of elasticity for a half-plane with a point source of stationary waves concentrated within it, the potential of which is represented in terms of cylindrical functions, are solved by the method of expanding potentials on plane waves. And in [15], using this approach, the problem of the stationary load on the contour of a circular hole in a half-space was solved. Using the idea of these papers on the superposition of solutions and the re-expansion of plane waves into series in cylindrical functions, in [16], in contrast to the exact analytical solution for the subsonic case, when the velocity of a moving load is less than the velocity of the Rayleigh waves.

3. STATEMENT OF THE PROBLEM FOR A CIRCULAR TUNNEL

Using the model approach for research, we will represent the tunnel as an infinitely long circular cylindrical cavity with a radius $r = R$, located in a linear viscoelastic, homogeneous and isotropic half-space x ≤ h (Figure 1) parallel to its horizontal boundary (the earth's surface). We define the reaction of a half-space on a moving with a constant subsonic velocity c along the surface of the cavity in the direction of the Z-axis of the load P.

Since the steady-state process is considered, the strain pattern is stationary with respect to the moving load. Therefore, it is convenient to move to the mobile coordinate system η = z − ct, connected with the load P.

Then equation (1) can be rewritten in the form

$$\left(\frac{1}{M_p^2} - \frac{1}{M_s^2}\right)\text{grad div }\mathbf{u} + \frac{1}{M_s^2}\nabla^2\mathbf{u} = \frac{\partial^2\mathbf{u}}{\partial\eta^2}. \quad (4)$$

Here $M_p = c/c_p$, $M_s = c/c_s$ – Mach numbers; $c_p = \sqrt{(\overline{\lambda} + 2\overline{\mu})/\rho}$, $c_s = \sqrt{\overline{\mu}/\rho}$ – complex propagation velocities of expansion waves - compression and shear in a medium.

When the load acts on the cavity surface, we have

$$\sigma_{rj}\big|_{r=R} = P_j(\theta, \eta), \quad j = r, \theta, \eta, \quad (5)$$

where σ_{rj} – components of the stress tensor in a medium, $P_j(\theta,\eta)$ – components of the intensity of the mobile load $P(\theta,\eta)$. Since the boundary of the half-space is free from loads, $x = h$

$$\sigma_{xx} = \sigma_{xy} = \sigma_{x\eta} = 0. \quad (6)$$

We transform equation (1) by expressing the displacement vector of an elastic medium through Lame potentials

$$\mathbf{u} = \text{grad }\varphi_1 + \text{rot }\psi. \quad (7)$$

Potential ψ can be represented in the form [7]

$$\psi = \varphi_2\mathbf{e}_\eta + \text{rot}(\varphi_3\mathbf{e}_\eta), \quad (8)$$

where $\mathbf{e}_\eta$ – ort axis η.

With this in mind, (5) takes the form

$$\mathbf{u} = \text{grad div }\varphi_1 + \text{rot}(\varphi_2\mathbf{e}_\eta) + \text{rot rot}(\varphi_3\mathbf{e}_\eta). \quad (9)$$

It follows from (3) and (8) that the potentials φ_j satisfy the modified wave equations

$$\nabla^2\varphi_j = M_j^2\frac{\partial^2\varphi_j}{\partial\eta^2}, \quad j = 1, 2, 3. \quad (10)$$

Here $M_1 = M_p$, $M_2 = M_3 = M_s$.

We express the components of the stress-strain state (VAT) of the medium through the potentials φ_j.

The components of the vector u (7) in cylindrical (8) and Cartesian (9) coordinate systems [17]:

$$u_r = \frac{\partial \varphi_1}{\partial r} + \frac{1}{r}\frac{\partial \varphi_2}{\partial \theta} + \frac{\partial^2 \varphi_3}{\partial \eta \partial r},$$

$$u_\theta = \frac{1}{r}\frac{\partial \varphi_1}{\partial \theta} - \frac{\partial \varphi_2}{\partial r} + \frac{1}{r}\frac{\partial^2 \varphi_3}{\partial \eta \partial \theta}, \tag{11}$$

$$u_\eta = \frac{\partial \varphi_1}{\partial \eta} + m_s^2 \frac{\partial^2 \varphi_3}{\partial \eta^2};$$

$$u_x = \frac{\partial \varphi_1}{\partial x} + \frac{\partial \varphi_2}{\partial y} + \frac{\partial^2 \varphi_3}{\partial x \partial \eta},$$

$$u_y = \frac{\partial \varphi_1}{\partial y} - \frac{\partial \varphi_2}{\partial x} + \frac{\partial^2 \varphi_3}{\partial y \partial \eta},$$

$$u_\eta = \frac{\partial \varphi_1}{\partial \eta} + m_s^2 \frac{\partial^2 \varphi_3}{\partial \eta^2},$$

where $m_s^2 = 1 - M_s^2$.

Volumetric strain

$$\varepsilon = \operatorname{div} \mathbf{u} = \nabla^2 \varphi_1. \tag{12}$$

Using Hooke's law, taking into account (9), (11), we find expressions for the stress tensor components in cylindrical and Cartesian coordinates

$$\sigma_{\eta\eta} = (2\mu + \lambda M_p^2)\frac{\partial^2 \varphi_1}{\partial \eta^2} + 2\mu m_s^2 \frac{\partial^3 \varphi_3}{\partial \eta^3},$$

$$\sigma_{\theta\theta} = \lambda M_p^2 \frac{\partial^2 \varphi_1}{\partial \eta^2} + \frac{2\mu}{r}\left(\frac{1}{r}\frac{\partial^2 \varphi_1}{\partial \theta^2} + \frac{\partial \varphi_1}{\partial r} + \frac{1}{r}\frac{\partial \varphi_2}{\partial \theta} - \frac{\partial^2 \varphi_2}{\partial r \partial \theta} + \frac{1}{r}\frac{\partial^3 \varphi_3}{\partial \theta^2 \partial \eta} + \frac{\partial^2 \varphi_3}{\partial r \partial \eta}\right),$$

$$\sigma_{rr} = \lambda M_p^2 \frac{\partial^2 \varphi_1}{\partial \eta^2} + 2\mu\left(\frac{\partial^2 \varphi_1}{\partial r^2} + \frac{1}{r}\frac{\partial^2 \varphi_2}{\partial \theta \partial r} - \frac{1}{r^2}\frac{\partial \varphi_2}{\partial \theta} + \frac{\partial^3 \varphi_3}{\partial r^2 \partial \eta}\right),$$

$$\sigma_{r\eta} = \mu\left(2\frac{\partial^2 \varphi_1}{\partial \eta \partial r} + \frac{1}{r}\frac{\partial^2 \varphi_2}{\partial \theta \partial \eta} + (1+m_s^2)\frac{\partial^3 \varphi_3}{\partial \eta^2 \partial r}\right), \tag{13}$$

$$\sigma_{\eta\theta} = \mu\left(\frac{2}{r}\frac{\partial^2 \varphi_1}{\partial \theta \partial \eta} - \frac{\partial^2 \varphi_2}{\partial r \partial \eta} + \frac{(1+m_s^2)}{r}\frac{\partial^3 \varphi_3}{\partial \theta \partial \eta^2}\right),$$

$$\sigma_{r\theta} = 2\mu\left(\frac{1}{r}\frac{\partial^2 \varphi_1}{\partial \theta \partial r} - \frac{1}{r^2}\frac{\partial \varphi_1}{\partial \theta} - \frac{\partial^2 \varphi_2}{\partial r^2} - \frac{m_s^2}{2}\frac{\partial^2 \varphi_2}{\partial \eta^2} + \frac{1}{r}\frac{\partial^3 \varphi_3}{\partial r \partial \eta \partial \theta} - \frac{1}{r^2}\frac{\partial^2 \varphi_3}{\partial \eta \partial \theta}\right);$$

$$\sigma_{\eta\eta} = (2\mu + \lambda M_p^2)\frac{\partial^2 \varphi_1}{\partial \eta^2} + 2\mu m_s^2 \frac{\partial^3 \varphi_3}{\partial \eta^3},$$

$$\sigma_{yy} = \lambda M_p^2 \frac{\partial^2 \varphi_1}{\partial \eta^2} + 2\mu\left(\frac{\partial^2 \varphi_1}{\partial y^2} - \frac{\partial^2 \varphi_2}{\partial x \partial y} + \frac{\partial^3 \varphi_3}{\partial y^2 \partial \eta}\right),$$

$$\sigma_{xx} = \lambda M_p^2 \frac{\partial^2 \varphi_1}{\partial \eta^2} + 2\mu\left(\frac{\partial^2 \varphi_1}{\partial x^2} + \frac{\partial^2 \varphi_2}{\partial x \partial y} + \frac{\partial^3 \varphi_3}{\partial x^2 \partial \eta}\right),$$

$$\sigma_{x\eta} = \mu\left(2\frac{\partial^2 \varphi_1}{\partial \eta \partial x} + \frac{\partial^2 \varphi_2}{\partial y \partial \eta} + (1+m_s^2)\frac{\partial^3 \varphi_3}{\partial \eta^2 \partial x}\right),$$

$$\sigma_{\eta y} = \mu\left(2\frac{\partial^2 \varphi_1}{\partial y \partial \eta} - \frac{\partial^2 \varphi_2}{\partial x \partial \eta} + (1+m_s^2)\frac{\partial^3 \varphi_3}{\partial y \partial \eta^2}\right),$$

$$\sigma_{xy} = 2\mu\left(\frac{\partial^2 \varphi_1}{\partial x \partial y} - \frac{\partial^2 \varphi_2}{\partial x^2} - \frac{m_s^2}{2}\frac{\partial^2 \varphi_2}{\partial \eta^2} + \frac{\partial^3 \varphi_3}{\partial x \partial y \partial \eta}\right).$$

Thus, to determine the components of the stress-strain state of the medium, it is necessary to solve equations (9) together with the boundary conditions.

In the moving coordinate system, we apply to the equations of motion and the boundary conditions a complex Fourier transform of the form [16]

$$\overline{\varphi}(\xi) = \int_{-\infty}^{\infty} \varphi(\eta)e^{-i\xi\eta}d\eta \quad , \tag{14}$$

$$\varphi(\eta) = -\frac{1}{2\pi}\int_{-\infty}^{\infty} \overline{\varphi}(\xi)e^{i\xi\eta}d\xi \quad .$$

Writing general solutions of the transformed equations of motion of the tunnel in the form (4) - (14), we find the following system of algebraic equations for determining the dimensionless trans formants of displacements of an intermediate surface

$$-\xi^2 U_0 + i\xi G_1 w_0 = -\zeta^2 \frac{1-G_1}{3}G_0^2 U_0; \tag{15}$$

$$i\xi G_1 U_0 - \frac{1-G_1}{3}G_0^2\zeta^2 w_0 + \left(1 + \frac{k^2\zeta^4}{12}\right)w_0 -$$

$$-\frac{1-G_1}{2k}\frac{\zeta G_{11}}{\gamma}w_0 = C_2 P_{10};$$

where $\quad G_1 = G_2/G_1;\ k = h/a;\ P_{10} = P_0 a/Eh$;

$$\{U_0,\ W_0\} = \frac{1}{h}\{U_1,\ W_1\};\ C_0^2 = C\left(\frac{3}{2}\frac{\rho_1}{G_1}\right);$$

The stress at the boundary of the soft layer and elastic among (r = b) in the dimensionless form has the form:

$$\sigma_{rr}^* = \int \frac{4}{\pi}\left\{-\frac{(1-\eta)H_1^{(1)}(\overline{\alpha}a)}{\delta_1}\sin\theta + \sum_{}^{\infty}\frac{i^{n+1}H_n(\overline{a}\ a)}{\Delta n}\sin\ n\theta\right\}e^{i\xi\eta}\,d\xi$$

$$\sigma_{r\theta}^* = \int \frac{2}{\pi}\left\{-\frac{i\ \overline{\beta}a^2}{\overline{\beta}^3 a^3 H_1^{(1)}(\overline{\beta}a) + 8\eta\left(\frac{\overline{\beta}^2 a^2}{2}H^{(1)}{}_0(\beta a) - \overline{\beta}aH_1(\beta a)\right)} - \right.$$

$$-\frac{2}{\delta_1}\left[(1+\eta)H_1(\overline{\alpha}\ a) - \overline{\alpha}\ aH_0(\overline{\alpha}\ a)cos\theta\right]-$$

$$\left.-2\sum_{n=2}^{\infty}\frac{i^{n+1}\left[-nH_n^{(1)}(\overline{\alpha}\ a) + (\overline{\alpha}\ a)H_{n-11}(\overline{\alpha}\ a)\right]}{\Delta n}cosn\theta\right\}e^{i\xi\eta}$$

$$\text{где}\quad \delta_1 = -4\eta H_1^{(1)}(\overline{\alpha}\ a)H_1(\overline{\beta}\ a) + (1+\eta)\overline{\alpha}\ aH_0(\overline{\alpha}\ a) + H_0(\overline{\beta}\ a),$$

$$\Delta n = n\overline{\alpha}\ aH_{n-1}(\overline{\alpha}\ a)H_{n1}(\overline{\beta}\ a) + n(\overline{\beta}\ a)H_{n-1}(\overline{\beta}\ a)H_n(\overline{\alpha}\ a) -$$

$$-\overline{\alpha}\overline{\beta}a^2 H_{n-1}^{(1)}(\overline{\alpha}\ a)H_{n-1}^{(1)}(\overline{\beta}\ a)$$

Here $\delta = \rho/\rho_B$ is the ratio of the density of the environment to the density of the soft layer; $\overline{\alpha}, \overline{\beta}$ - are functions of ξ and η. We find the following expression for the load transformer, which is transferred to the shell from the side of the soft layer

$$\overline{q}_{rc} = -G_1\frac{\xi}{q}C_1 w_0 - C_2 P_0(\xi);$$

$$C_1 = \sum_{j=1}^{4}\frac{A_{4j}\big|_{k_{c1}=0}B_{3j}}{\det\|A_{ke}\|};\ C_2 = \sum_{j=1}^{4}\frac{(-1)^j A_{4i}\big|_{k_{c1}=0}B_{3j}}{\det\|A_{ke}\|}.$$

Elements of the determinant $\det\|A_{ke}\|$ is computed then formula $A_{11} = -2M_1;\ A_{12} = -a_{11};\ A_{13} = nM_{12};$

$A_{14} = -A_{13};\ A_{21} = -S_1 A_{11};\ A_{22} = A_{12} * k_0(z_1)/k_1(z_1);$

$$A_{23} = A_{13} * I_0(z_2)/I_2(z_2); \quad A_{24} = A_{13} * k_0(z_1)/k_1(z_2); \quad A_{31} = \frac{1}{2}A_{11}; \quad A_{32} = -\frac{1}{2}A_{11};$$

$$A_{41} = n_1 k_0(z_3)/k_1(z_4) - 2A_{21}/(z_3/M_2); \quad A_{31} = A_{13}/n_1; \quad A_{34} = -A_{13}/n_1;$$

$$A_{42} = n_1 I_0(z_3)/I_1(z_4) - 2M_1 S_1(z_3)/I_1(z_4);$$

$$A_{43} = -2M_{12}^2 (k_0(z_5)/k_1(z_6) + I_1(z_5)/I_1(z_6)/(z_6/M_2));$$

$$A_{44} = -2M_{12}^2 (I_0(z_5)/k_1(z_6) + I_1(z_6)/k_1(z_6)/(z_6/M_2));$$

$$\text{где} \quad m_1 = \sqrt{1 - M_p^2}; \quad m_{12} = \sqrt{1 - M_S^2};$$

$$z_1 = M_1 \eta; \quad z_2 = M_{121} \eta; \quad z_3 = M_1 \eta;$$

$$z_4 = m_1 \eta (1 + k_{11}); \quad z_5 = m_1 \eta; \quad k_{11} = (b-a)/a;$$

k_{1_0} k_1 - modified Neumann functions; I_{1_0} I_1 - modified Bessel functions; The general solution of the equations of the motion of the environment has the form $(C_f < C_S < C_p)$

$$\varphi(r, \xi) = A_n(\xi) k_n(m_1 \xi r) + B_n(\xi) I_4(m_1 \xi r)$$

$$\psi(r, \xi) = C_n(\xi) k_n(m_{i2} \xi r) + D_n(\xi) I_4(m_{12} \xi r) \tag{16}$$

The expression for the trans formant of normal displacement has the form

$$w_0 = -\frac{1-v}{m} \sum_{i=o}^{\infty} \left\{ \int_{-\infty}^{\infty} \frac{\Delta_1 [a \cos(\zeta \eta) - \zeta \lim(\zeta \eta)] d\xi}{[a^2 + \zeta^2] \det|A_{ke}|} \right\} \tag{17}$$

Define Δ_j $(j = 1.2.......5)$ is obtained from $\det\|A_{ke}\|$ by replacing j = 20 by the column C with the elements {0; 0; 1; 0; 0}. After this function $A(\zeta).....D(\zeta)$ from (23) can be calculated from formulas

$$\{A, B, C, D\} = \frac{a^2}{\xi^2 \det\|A_{ke}\|} \left\{ \frac{A_1^1}{k_1(m_1 \xi)}; \quad -\frac{A_2^1}{I_1(m_2 \xi)}; \quad -i\frac{a A_3^1}{\zeta k_1(m_{12} \xi)}; \quad i\frac{a A_4^1}{\zeta I_1(m_{12} \xi)} \right\}$$

$$A_j^1 = \frac{\xi}{a} M_{3k} w_0 + P_0 |m_{4k}| G_1 \left(\kappa = 1,.......4 \right)$$

m_{ie} — minors of the element $A_{je.}$ For a specific value of the load velocity C, the denominators under the integral expressions in formulas (14) are transcendental functions with respect to ζ C real coefficients depending on C, as well as on the mechanical parameters of the shell and the layer. Analysis of the integrals of treatment must begin with consideration of cases $D(\xi, C_0) = 0,$ which is equivalent to the construction of the dispersion relation in the corresponding problem of propagation of free waves and the determination of the denominator from the dispersion curves of the roots for the chosen velocity of the load C. at C< C_5 are possible for cases. Figure 2 shows the change in the movement of the filler, depending on the thickness of the bodies for different values of the rigidity of the aggregate. As can be seen from the drawing (γ = 100, 50,10,2), that for a sufficiently rigid layer (γ = 100), the deflections of the shell essentially decrease.
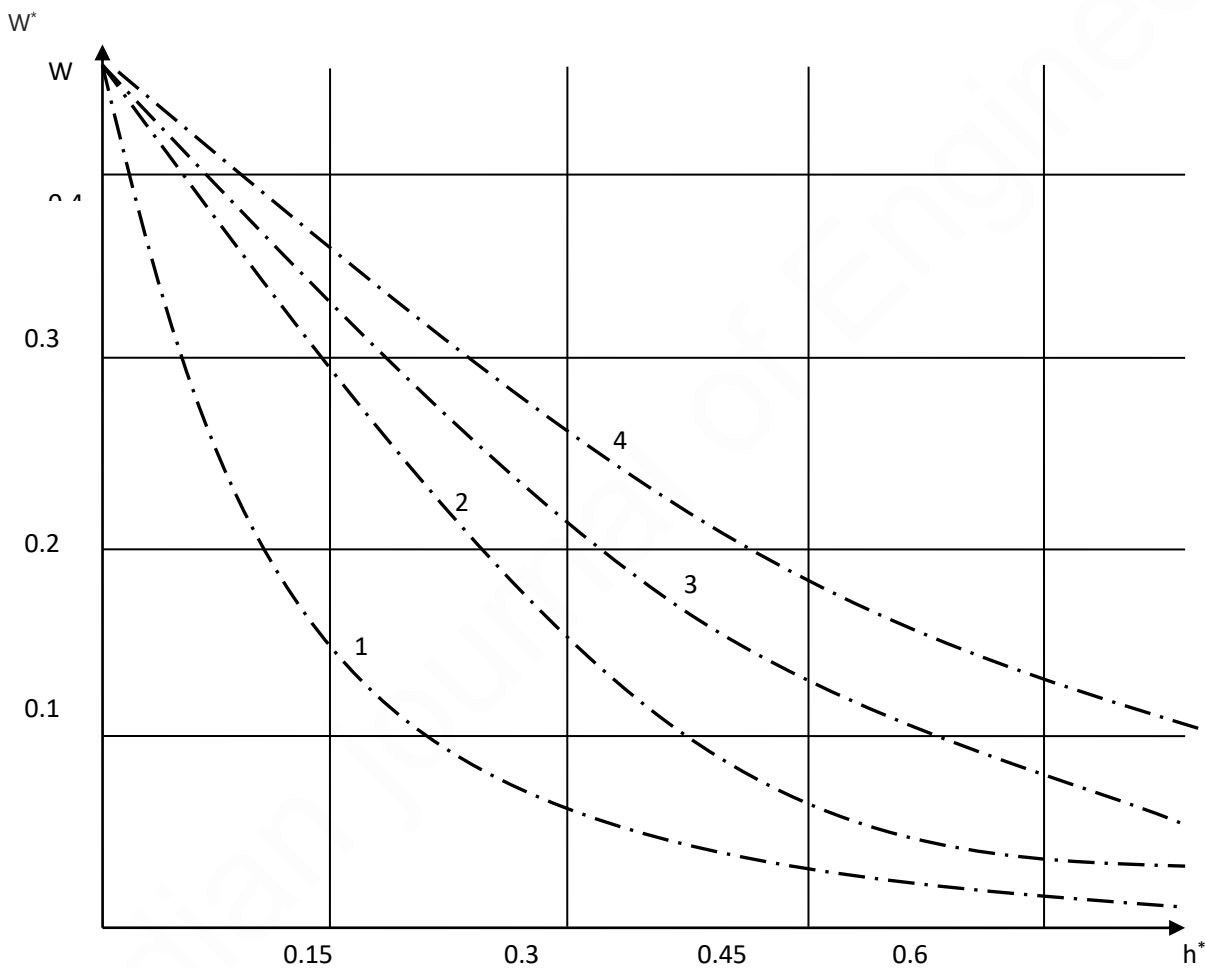

Figure 2 Shell deflections as a function of thickness

1. For a given speed C, there are one or two different denominator roots.
2. For some values of C, the denominator has a double root. This case corresponds to a minimum of the corresponding dispersion curve in Fig. Such a velocity is called resonance and is denoted by C^x. A resonance effect appears, or which deflections and contact pressures tend to infinity.
3. For a given value of C, the denominator has no roots on the real axis, as seen in Figure 2, this will be either, C<C_ϕ (up to resonance mode). At this speed of motion, the inversion integrals are not special and can be found by effective numerical methods.

Dividing the integral (21) into two terms

$$w_0 = \frac{1}{\pi}\int_0^{\infty} x_1(\Omega)d\Omega \quad \text{or} \quad w_0 = \frac{1}{\pi}\int_{\omega_1}^{\omega_2} x_1(\Omega)d\Omega \quad (18)$$

The value of the integral (18) was found numerically, using the Rmberg method [2]. When the integral is calculated by the Romberg method, it is necessary to repeatedly calculate the integrand function. The inverse Fourier transform (29) was numerically fulfilled. It is shown that at an integration step of 1.01, the error of the procedure does not exceed 0.3-0.5%.

4. CONCLUSION

1. To describe the behavior of viscoelastic materials with unstable properties that do not obey the principle of temperature-time analogy, the non-singular singular kernel of heredity is proposed.
2. A universal algorithm for solving the problems is proposed.

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