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Effect of nonlinearities on actuation of a smart Cantilever beam

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ABSTRACT

Considering as an electro-mechanical continuum and obtaining the variational form for smart cantilever beam with full length ferroelectric patch brazed at the upper and lower side of host aluminum core using higher order beam bending theory. The effect of nonlinearities viz body force in the equilibrium equation and non linear term in constitutive equation due distributed body couple introduced in basic formulation because of polarization and electric field coupling is analyzed for the case of actuation. The results indicate nonlinearities show a significant influence for higher values of actuation voltages.

Keywords: Smart Cantilever, Non-linearities, Polarization, Electric field, Actuation.

1. INTRODUCTION

Advances in materials have led to the demand for portable and high-performance control design of gadgets and devices and have given birth to use of smart or intelligent materials and/or structures. The evolution of substantial and economical high-achievement building materials and systems are essential for the economic health of a nation as human infrastructure cost amounts to a considerable chunk of national capital. In order to take care of the issue of degenerating structure, research of smart materials becomes mandatory. Use of such materials will not only improve the performance of the structure, reduce the maintenance costs, ensure or confirm that the structure is sustainable in future but will as well put forth the utilization of smart materials for most favourable achievement or performance and secured infrastructure designs particularly in seismic and other naturally hazardous prone zones.

Smart is the new tech-savvy because smart materials may work completely on their own as a part of the larger smart system. In principle, these systems interconvert the fundamental energy forms viz electric, sound thermal, mechanical, magnetic etc. Smartness may range from passive to sensible depending upon its responsiveness. Smartness makes certain that under a variety of environmental conditions the system is not only giving the most favourable performance but is smart enough to get suitably actuated to tackle abnormal loads as and when required much like its biological analogue that is a human body.

1.1. Interactions in Ferroelectric Crystal

Basically three kinds of physical behaviour prevail in a Ferroelectric crystal. They are piezoelectricity, pyroelectricity and thermo-elasticity, as shown in the figure 1. An electric field in the electric domain generates piezoelectric stress in the mechanical domain, the two being related by the appropriate piezoelectric stress coefficient and is the converse, inverse or indirect piezoelectric effect. Similarly a strain in the mechanical domain generates the electric polarisation in the electrical domain, which in turn produces Electric field, the polarisation and Electric field being associated by dielectric susceptibility. In an akin manner, a temperature differential in the thermal domain and polarisation in the electrical domain are affiliated by pyroelectric constant; the arrowed path from electric field to heat indicates the electro-caloric effect (The generation of an amount of heat δQ on implementation of vector field), generally conveyed as a relationship between temperature differential and electric field. The coefficient of thermal expansion relating temperature differential ΔT with strain, the arrowed path relating stress to heat δQ indicates the thermo-elastic effect. The Constitutive equations link stress and strain in mechanical domain and specific heat relating the quantity of heat δQ and temperature differential in thermal domain. The small arrowed paths in electrical domain between polarisation P and electric field E indicating that polarisation may prevail by existence of electric field and vice versa in ferroelectric material.

The three domains driving each other basically depict the direct or basic effect. However, in every instance there is the minimum of one other way over which the process can occur, of course the coupling coefficients are assumed to be non-zero for that specific process for the crystal or material to be considered. The roundabout indirect effects are called secondary effects. A classic example will be secondary pyroelectric effect owing to piezoelectric action, which is usually many times higher than the primary effect itself. The basic pyroelectric effect is specified by the arrowed path connecting temperature differential with polarization, that is material being polarized on being subjected to temperature differential. Whereas the secondary effect pursues the way temperature differential producing strain which in turn polarizes the material. The logic for this complexity is due to the reason that all pyroelectric crystals are also piezoelectric. Thus a temperature differential to an unconstrained crystal produces a strain (deformation) and this subsequently yields the secondary polarisation superimposed on the basic or primary pyroelectric polarisation.

The foundation of thermodynamic theory of simple material was laid by Coleman (1964).A material which is considered simple is characterised as a continuous sequence wherein the stress at a specified interval of time is governed by previous strain applied. In order to fabricate the thermo-elastic problem theory, he utilized the basic principles of physics viz conservation of energy, conservation of mass, conservation of linear and angular momentum etc.

The governing equation of thermo-elasticity are derived from the conjecture of dwindling memory (deformation and temperature consummated in the remote past has minimal effect on current values of stress, entropy, heat flux and energy than deformation and temperature which happened in the latest past).

Boundary conditions were deduced by Eringen and Suhubi (1964), constitutive equations and basic field equations for a simple micro-elastic solid, taking into considerations rotations and 'micro' deformations. The duo have also taken care of inertial spin, surface tension higher order effects and stress moments in their formulation. Eringen (1966) has deducted constitutive equations, boundary conditions and equation of motion for a micro-polar fluid. These fluids are effected by micro rotational motions and spin inertia. Hence these type of fluids can support couple stresses and distributed body couples. Fluid equations have been acquired for density, velocity and micro-rotation vector. Mindlin (1974) has derived equations with two dimensions for vibrations of piezoelectric crystal at higher frequency taking into consideration, electric potential, mechanical displacement and temperature change. Field variable have been extended to thickness in terms of power series. These field variable have been incorporated in a integral equation instead of putting them in variational form of virtual work.

An energy dissipation function has also been inserted in the integral equation. Unique theory has been employed to find the unique solution of a single layer piezo plate for unknown field variable.

A microscopic theory for the dynamic response of polycrystalline ferroelectric materials have been exhibited by Chen and Peerey (1979).Their formulation comprises of the effect of the change in magnitude of electric dipoles and orientation of domains leading to energy loss. The constitutive relation contain the history of the evolution of the temperature, strain and the electric field.

Generalised thermo-elasticity theory applicable for piezoelectric materials have been revealed by Chandrasekharaiah (1988).The main thrust was on articulation for the generation of finite speed thermal signals. After Lagrangian formulation of continuum mechanics, Pak and Hermann (1986) have derived governing equations, constitutive relations and the boundary conditions for an elastic dielectric material. The interaction between electric field and polarisation in represented as a stress tensor denoted as Maxwell stress tensor. This phenomenon brings forth an equilibrium equation of a nonlinear body force term.

Tierstern (1971) applied preliminary conservation laws of continuum physics to a macroscopic model in order to deduct the non-linear governing equations of electro-thermo-elasticity. His presumption comprised of a simple material with overlapping inertia upon each other. He also deducted equilibrium equations via balancing forces, separating electronic continuum from material continuum under effect of external electric field. Applying the laws of thermodynamics to a continuum model, he derived the constitutive equations. He obtained exact boundary conditions using variational formulation, He highlighted that the objectives of interaction between electric field and material polarization is to render the stress tensor non symmetric and initiate in the equilibrium equation a non-linear body force term.

Kalpakidis and Massalas (1993) have elaborated the electro-thermo-elastic formulation of Tiersten (1971) by introducing quarter pole electric moments and the dependence of rate of change of absolute temperature with respect to time in the constitutive relations. They made their formulation on the basis of inequality in entropy production and invariable of the first law of thermodynamics, under rigid body rotation and translation. Taking the approach of Tiersten (1971), Venkatesan and Upadhyay (2002) have displayed the analysis of the smart structure using electro thermoelastic formulation. The effects of interaction between and polarisation and electric field have been reflected.

Chen and Montgomery (1980) presented a microscopic theory based on domain switching under the influence of an external electric field to model the butterfly loop and the hysteresis loop observed in ferroelectric materials. The butterfly loop is observed in the variation of strain with respect to electric field and the hysteresis loop is observed when polarization is varied with respect to electric field. A non-linear rate law relating the alignment of dipoles with the direction of electric field has been formulated. Using the constitutive relations of the stress and electric displacement along with rate law, the authors have constituted hysteresis loop have been correlated with experimental results.

Bassiouny et al (1988) exhibited a thermodynamical formulation capable of predicting electro-mechanical hysteresis effects in ferroelectric ceramics. This theory applies thermodynamic interval variables able to model both electric and plastic hysteresis effects. This is brought about by formulating evolution equation for residual electric polarisation, plastic strain and both electrical and mechanical hardening. Some of the samples have been investigated to evaluate the piezoelectric and electrostrictive couplings.

Formulation for polarisation reversal in piezoelectric materials have been presented by Zhang and Rogers (1993) A ferroelectric material comprises of millions of domains, each of domain have thousands of unidirectional dipoles which are randomly oriented with regard to each other. On the application of strong DC electric field, polarisation vectors in domains get reoriented along the external field known as domain switching. This domain switching dynamics applies to model the hysteresis effects. Combination of phenomenological part and the microscopic properties have revealed to be good way to elaborate the non-linear induced strain field behaviour and electromechanical hysteresis.

Jha et al., (2000) have devised a mathematical paradigm for hysteresis behaviour of piezoelectric materials. The very paradigm considers the effects of temperature, pressure and amplitude of the electric field on the shape of hysteresis curve.

Using Helmholtz free energy second and first law of thermodynamics an expression of entropy production rate for ferroelectric hysteresis process has been derived by Crawley, E.F.,(1994). A distribution process has been assumed for domain orientation and respective distribution parameters are selected as the internal state variables. Using this formulation they have sculptured the hysteresis effects in ferroelectric materials.

Last two decades the demand of designing and development of smart structure has been alarmingly increased, a holistic research activity have been observed in the open literature on the analysis of smart structure, broadly classified into three basic classes-studies dealing with beams, plates and shells. A detailed summary on smart technology is briefed in Chopra, I., (2000) and described implementation of smart technology to rotor system of helicopter in detail.

An overview of recent development in smart structures aimed at alleviating aero elastic response in helicopters has been elaborated by Friedman, P.P., (1997). In addition he exhibited the scaling laws associated with small scale model to full scale configuration.

2. FORMULATION

The electro-thermo-elastic interaction outcomes while modelling the performance of the ferroelectric continuum in a smart structure can be acquired about by establishing in a consistent manner the governing equations, constitutive equations and the boundary conditions. The electrical behaviour of the continuum is modelled in a quasi-static modus operandi taking into account only the laws of electro-statics as the continuums mechanical motion is by various orders smaller than the charge motion and thus enabling to neglect the electro-magnetic phenomena included with the motion of free charges and/or dipoles in insulators.

The forces and moments that act on the polarised system are obtained by applying the concepts of electrostatics of dielectrics amalgamated in the thermo-mechanical modeling of the ferroelectric continuum. The governing equations are obtained by making use of the laws of conservations to the ferroelectric medium, while applying principles of thermodynamics, first and second laws will provide the necessary constitutive relations.

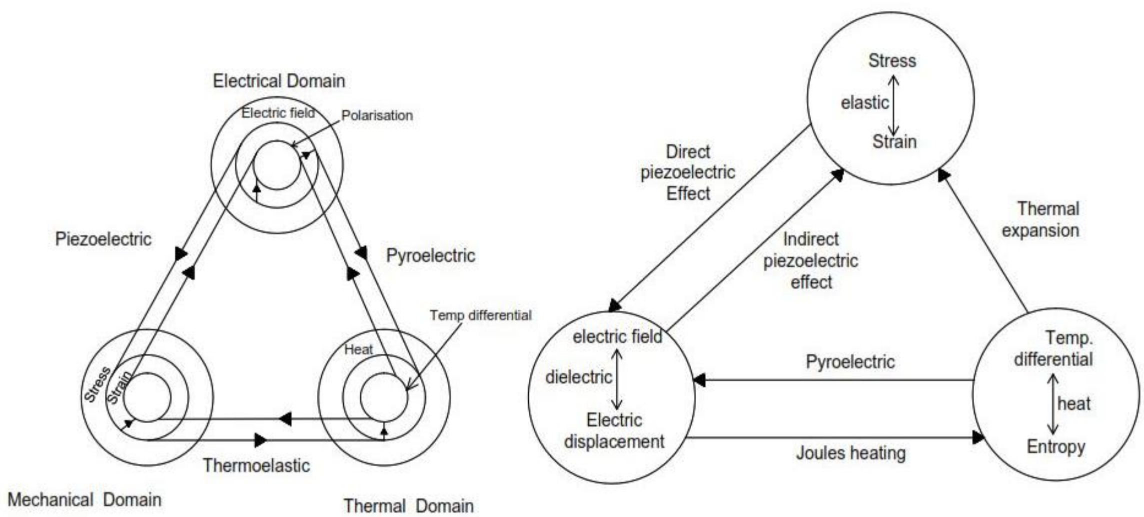


Figure 1

Interaction in Electro-Thermo-Elastic Domains

The electro-thermo-elastic problem generates thirty five equations.

One equation for conservation of Mass

One equation for conservation of Energy

Six equations (force and moment equations each three in number) of equilibrium

One equation for Gauss law of electrostatics

$$D_{i,i} = 0$$

Three equations for electric displacement in terms of electric field and polarisation vectors $D_j = \varepsilon_0 E_j + P_j$

Six equations of strain tensor in terms of displacements

$$\epsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} + \frac{\partial u_k}{\partial x_i}\frac{\partial u_k}{\partial x_j}\right)$$

Six equations for stress tensor in terms of strain tensor, polarisation vector, electric field vector and temperature

$$\sigma_{lm} = C_{lmpq}\epsilon_{pq} - P_l E_m - e_{lmj}E_j - \alpha_{lm}\theta$$

One equation for rate of entropy production equation

Three equations for electric displacement vector in terms of strain tensor, electric field vector and scalar temperature

$$D_i = e_{imn}\varepsilon_{mn} - b_{in}E_n - \eta_i\theta$$

Three equations for Electric field vector as gradient of scalar electric potential

$$E_j = -\phi_{,j}$$

One equation for entropy in terms of strain tensor, electric field vector and temperature

$$\lambda = \alpha_{mn}\varepsilon_{mn} + \eta_n E_n + C_\theta\theta$$

Three equations for heat flux in terms of temperature gradient (Fourier Law)

$$Q_i = -K_{il}\theta_{,l}$$

With corresponding nineteen variables in Mechanical domain, ten variables in Electrical and five variables in Thermal domain:

Mechanical Domain variables { one density (ρ_m) nine stress ($\boldsymbol{\sigma}_{ij}$) components, three Displacement components (axial displacement u, displacement along depth v and transverse displacement w) and six strain (ε_{ij}) components)}

Electrical Domain variables {Electric field (**E**) with three components along three directions), Electric displacement (**D** with D_X, DY & D_Z components), Polarisation (**P** with three components) and Electric potential (ϕ, one scalar)}

Thermal Domain variables {Heat flux (**Q**, three components), Temperature differential (θ, one scalar) and Entropy ($\boldsymbol{\lambda}$, one scalar)}

For the randomly chosen ferroelectric material element of volume δV, the surrounding forces and moments per unit volume experienced by it are:

Gravity force:

$$\vec{B} = B_l \vec{e}_l$$

Near action force:

$$\vec{F} = F_m \vec{e}_m$$

Force because of polarisation of the medium:

$$\vec{F}_p = (\vec{P}.\vec{\nabla})\vec{E} = P_l E_{i,l}\,\vec{e}_i$$

Force due to the existence of free charges in the polarised medium, if any:

$$(\vec{\nabla}.\vec{D})\,\vec{E} = D_i E_{j,i}\vec{e}_j$$

Moment because of polarisation of the medium:

$$-(\vec{E} \times \vec{P}) = \epsilon_{ijk}\vec{e}_i P_j E_k$$

Suppose force due to existence of free charge is zero i.e., absence of any free charges in the continuum, then linear momentum principle in Eulerian coordinate system is:

$$\iiint B_i \vec{e}_i dV + \iiint F_i \vec{e}_i\, dV + \iiint P_l E_{i,l}\, \vec{e}_i\, dV \;=\; D_t \iiint \rho_{\,m} v_i \vec{e}_i\, dV$$

Where integration is performed over an arbitrary portion of infinitesimal volume in the current deformed state of the domain Ω, let elemental volume has an absolute velocity $\vec{v}$ and $\rho_{\,m}$ is the density in the deformed state.

Using transport theorem for the time integral term & expanding time derivative term the above integral can be modified as:

$$\iiint F_i\, dV \;+ \iiint [\,B_i + P_l E_{i,l}]dV \;=\; \iiint \rho_{\,m} \frac{dv_i}{dt} dV \qquad i = 1,2,3$$

The volume integral term corresponding to the near action force can be written in terms of derivatives of stress tensor in the deformed coordinate system.

$$\iiint \{\,B_i + P_l E_{i,l} + \frac{\partial \sigma_{il}}{\partial x_l}\;\}dV \;=\; \iiint \rho_{\,m} \frac{dv_i}{dt} dV \qquad i = 1,2,3$$

As this equality is applicable to any arbitrary chosen volume, the conservation of linear momentum equation can be written as:

$$\frac{\partial \sigma_{il}}{\partial x_l} + B_i + P_l E_{i,l} = \;\rho_{\,m} \frac{dv_i}{dt}$$

It is obvious that the peculiar attribute of a polarised dielectric medium is to introduce in the force equilibrium equations a non-linear quantity $P_l E_{i,l}$ due to the coupling of polarisation and electric field. Even in the non-existence of B_i, non-linear $P_l E_{i,l}$ quantity exists. If this non-linear term is discarded then we get the equation of equilibrium for a non-polarised medium. In Vector form above equation is:

$$div\sigma^T + \vec{B} + (\vec{P}\,.\vec{\nabla})\vec{E} - \rho_{\,m} \frac{d\vec{v}}{dt} = 0$$

For uniform electric field representing coupling of polarisation and electric field vanish.

2.1. Constitutive Equations in Useful Form

As quadratic form per unit mass of Helmholtz free energy can be expressed as:

$$\chi = (2\rho_{\,m})^{-1}(\,C_{mnkl}\varepsilon_{mn}\varepsilon_{kl} - \;b_{mn}E_m E_n + \rho_{\,m}\, C_\theta \theta^2 - 2e_{mnk}E_m\varepsilon_{nk} + 2\alpha_{mn}\varepsilon_{mn}\theta + 2\eta_m E_m \theta)$$

Where different constant are C_{mnkl} elastic, b_{mn} electric susceptibility, C_θ thermal, e_{mnk} piezoelectric, $\boldsymbol{\alpha}_{mn}$ thermoelastic, $\boldsymbol{\eta}_m$ pyroelectric. Using this equation constitutive relations can be obtained as

$$\sigma_{ij} = C_{ijmn}\varepsilon_{mn} - e_{ijl}E_l - \alpha_{ij}\theta - P_i E_j$$

$$D_i = e_{imn}\varepsilon_{mn} - b_{in}E_n - \eta_i \theta$$

$$\lambda = \alpha_{mn}\varepsilon_{mn} + \;\eta_n E_n + C_\theta \theta$$

$$Q_i = -K_{il}\theta_{,l}$$

The tensor $\boldsymbol{\sigma}_{ij}$ can be divided into sum of linear and non linear

$$\sigma_{ij}^{L} \;= C_{ijmn}\varepsilon_{mn} - e_{ijl}E_l - \alpha_{ij}\theta$$

$$\sigma_{ij}^{NL} = -P_i E_j$$

Stress tensors

2.2. Variational Formulation

Consider a Ferroelectric Continuum in a domain V with boundary S. In order to obtain the variational formulation, the principle of virtual work is invoked

$$\iiint (\sigma_{il,i} + P_i E_{l,i})\, \delta u_l \, dV + \iiint D_{m,m}\, \delta\phi dV + \frac{1}{\theta_0} \iiint Q_{k,k} \delta\theta \, dV \; = 0$$

Where δu_l is the displacement variation, $\delta\phi$ is the electric potential variation and $\delta\theta$ is the temperature variation and $\boldsymbol{\theta}_0$ is the reference temperature.

In the above equation steady state heat conduction has been assumed. This decouples the heat conduction and deformation interaction of the body. However deformation and electric potential will be induced due to temperature difference. In this study temperature difference θ is not an unknown variable but is an external load, above equation can be written as:

$$\iiint \frac{\partial(\sigma_{il}\delta u_i)}{\partial x_l} dV \; - \iiint \sigma_{il} \frac{\partial \delta u_i}{\partial x_l} dV + \iiint \frac{\partial(D_m \delta\phi)}{\partial x_m} dV - \iiint D_m \frac{\partial\delta\phi}{\partial x_m} + \frac{1}{\theta_0} \iiint \frac{\partial(Q_k \delta\theta)}{\partial x_k} dV - \frac{1}{\theta_0} \iiint Q_k \frac{\partial\delta\theta}{\partial x_k} dV + \iiint P_l E_{i,l}\, \delta u_i dV = 0$$

$\iiint P_l E_{i,l}\, \delta u_i dV$

Constitute virtual work because of dispense non-linear force generated due to interaction of the vector electric field and the polarisation vector. Also, stress tensor $\boldsymbol{\sigma}_{il}$ is non-symmetric and is given as:

$$\sigma_{il} = \; \sigma_{ij}^{L} + \; \sigma_{ij}^{NL}$$

Separating the linear and nonlinear parts in equation we get:

$$\iiint \frac{\partial(\sigma_{il}\delta u_i)}{\partial x_l} dV \; - \iiint \sigma_{il}^{L} \frac{\partial \delta u_i}{\partial x_l} dV + \iiint \frac{\partial(D_m \delta\phi)}{\partial x_m} dV - \iiint D_m \frac{\partial\delta\phi}{\partial x_m} + \frac{1}{\theta_0} \iiint \frac{\partial(Q_k \delta\theta)}{\partial x_k} dV - \frac{1}{\theta_0} \iiint Q_k \frac{\partial\delta\theta}{\partial x_k} dV - \iiint P_i\, E_j \frac{\partial \delta u_i}{\partial x_j} dV \; + \iiint P_i\, E_{i,l} \delta u_i dV = 0$$

Using Gauss divergence theorem and

$$\frac{\partial\delta\phi}{\partial x_m} = -\delta E_m$$

The equation simplifies to

$$\iiint \sigma_{il}^{L}\, \delta\varepsilon_{il} dV - \iiint D_m\, \delta E_m dV + \frac{1}{\theta_0} \iiint Q_k \frac{\partial\delta\theta}{\partial x_k} dV + \iiint P_i\, E_{i,l} \delta u_i dV - \iiint P_i\, E_l \delta \frac{\partial u_i}{\partial x_l} dV = \iint (\sigma_{il}\, n_l \delta u_i + D_i n_i \delta\phi + \frac{1}{\theta_0} Q_i n_i \delta\theta) dA$$

From the right hand side it is obvious

$$\sigma_{il} n_l = T_i$$

The boundary traction applied,

$$D_l n_l = q$$

Surface charge density and

$$Q_l n_l = Q_n$$

Vector heat flux perpendicular to boundary.

$$\iiint \sigma_{il}^{L}\,\delta\varepsilon_{il}dV - \iiint D_m\,\delta E_m dV + \frac{1}{\theta_0}\iiint Q_k\frac{\partial\delta\theta}{\partial x_k}dV + \iiint P_i\,E_{i,l}\delta u_i dV - \iiint P_i\,E_l\delta\frac{\partial u_i}{\partial x_l}dV = \iint T_i\,\delta u_i dA + \iint q\,\delta\phi\,dA + \frac{1}{\theta_0}\iint Q_n\,\delta\theta dA$$

2.3. Nonlinear electro-elastic analysis of a surface mounted beam

The interaction of polarisation and electric field introduces two types of non-linearities in the smart structure, namely (i) a distributed non-linear force in equilibrium equation, this non-linearity arises due to variable electric field in piezo material and (ii) non-linear constitutive relation between stress-strain-electric field-polarisation equation this non-linearity is due to the presence of distributed body couple, Each case is presented separately in the following.

2.3.1. Non-linearity due to Distributed Force

In this Research, we assume

$$E_y = 0, \frac{\partial E_x}{\partial y} = 0, \frac{\partial E_y}{\partial y} = 0, \frac{\partial E_z}{\partial y} = 0, \frac{\partial E_y}{\partial z} = 0, \frac{\partial E_y}{\partial x} = 0\,.$$

Thus, in the variational formulation the effect of the distributed body force because of polarisation vector and electric field vector interaction, given by

$$\int (P_i E_{i,j})\delta u_i dV,$$

is to be added. This contribution can be written in an expanded form as,

$$\iota^{NL-FORCE} = \iiint P_x\frac{\partial E_x}{\partial x}\delta u_x dV + \iiint P_y\frac{\partial E_x}{\partial y}\delta u_x dV + \iiint P_z\frac{\partial E_x}{\partial z}\delta u_x dV + \iiint P_x\frac{\partial E_y}{\partial x}\delta u_y dV + \iiint P_y\frac{\partial E_y}{\partial y}\delta u_y dV + \iiint P_z\frac{\partial E_y}{\partial z}\delta u_y dV + \iiint P_x\frac{\partial E_z}{\partial x}\delta u_z dV + \iiint P_y\frac{\partial E_z}{\partial y}\delta u_z dV + \iiint P_z\frac{\partial E_z}{\partial z}\delta u_z dV$$

Revising the above contribution in terms of the electric displacement using

$$\overrightarrow{D} = \varepsilon_0\overrightarrow{E} + \overrightarrow{P}$$

$$\iota^{NL-FORCE} = \iiint \{(D_x - \epsilon_0 E_x)\frac{\partial E_x}{\partial x} + (D_z - \epsilon_0 E_z)\frac{\partial E_x}{\partial z}\}\delta u_x dV + \iiint \{(D_x - \epsilon_0 E_x)\frac{\partial E_z}{\partial x} + (D_z - \epsilon_0 E_z)\frac{\partial E_z}{\partial z}\}\delta u_z dV$$

Using the simplified constitutive relations for the electro-elastic beam, the above expression yields:

$$\iota^{NL-FORCE} = \iiint [\{(\epsilon_0 - \epsilon_1^S)\frac{\partial\phi}{\partial x} + e_{15}\gamma_{xz}\}\frac{\partial^2\phi}{\partial x^2} + \left\{(\epsilon_0 - \epsilon_1^*)\frac{\partial\phi}{\partial z} + e^*\epsilon_x\right\}\frac{\partial^2\phi}{\partial x\partial z}]\,\delta u_x dV + \iiint [\{(\epsilon_0 - \epsilon_1^*)\frac{\partial\phi}{\partial z} + e^*\epsilon_x\right\}\frac{\partial^2\phi}{\partial z^2} + \{(\epsilon_0 - \epsilon_1^S)\frac{\partial\phi}{\partial x} + e_{15}\gamma_{xz}\}\frac{\partial^2\phi}{\partial x\partial z}\delta u_z dV$$

2.3.2 Contribution due to nonlinear part of stress

It is clear that for the smart beam the nonlinear term due to stress components is

$$\iiint P_i\, E_l\delta\frac{\partial u_i}{\partial x_l}dV$$

$$\iota^{NL-\sigma} = \iiint P_x\, E_x\delta\frac{\partial u_x}{\partial x}dV + \iiint P_x\, E_y\delta\frac{\partial u_x}{\partial y}dV \iiint P_x\, E_z\delta\frac{\partial u_x}{\partial z}dV + \iiint P_y\, E_x\delta\frac{\partial u_y}{\partial x}dV + \iiint P_y\, E_y\delta\frac{\partial u_y}{\partial y}dV + \iiint P_y\, E_z\delta\frac{\partial u_y}{\partial z}dV + \iiint P_z\, E_x\delta\frac{\partial u_z}{\partial x}dV + \iiint P_z\, E_y\delta\frac{\partial u_z}{\partial y}dV + \iiint P_z\, E_z\delta\frac{\partial u_z}{\partial z}dV$$

Using the assumptions and the simplified constitutive relations can be written as:

$$\iota^{NL-\sigma} = \iiint [\{(\epsilon_1^S - \epsilon_0\,)\frac{\partial\phi}{\partial x} + e_{15}\gamma_{xz}\}\left(\frac{\partial\phi}{\partial x}\delta\frac{\partial u_x}{\partial x} + \frac{\partial\phi}{\partial z}\delta\frac{\partial u_x}{\partial z}\right)dV + \iiint \{(\epsilon_1^* - \epsilon_0)\frac{\partial\phi}{\partial z} + e^*\epsilon_x\}(\frac{\partial\phi}{\partial x}\delta\frac{\partial u_x}{\partial x} + \frac{\partial\phi}{\partial z}\delta\frac{\partial u_x}{\partial z})dV$$

Note that these nonlinear effects occur only in the piezo layers. Inserting $\iota^{NL\text{-}FORCE}$ and $\iota^{NL\text{-}STRESS}$ in the variational formulation equation for the smart electro-elastic cantilever beam with a tip load F, gives rise to:

$$\iiint (\sigma_x^L\,\delta\epsilon_x + \sigma_{xz}^L\delta\gamma_{yz})dV - \iiint (D_x\delta E_x + D_z\delta E_z)dV + \iota^{NL-FORCE} - \iota^{NL-\sigma} = F\delta w]_{x=L}$$

Where

$$\sigma_x = c^*\epsilon_x - e^*E_Z$$

$$\sigma_{xz} = -e_{15}E_x + c_{44}\gamma_{xz}.$$

3. RESULTS AND DISCUSSION

The smart beam considered for identifying the effect of nonlinearities is shown in Figures. Two piezo patches are embedded on the top and bottom faces of the aluminum core. The direction of polarization in the piezo material is along Z-direction. The interfaces between the piezo material and aluminum core are at equipotential and the free surface of the piezo patches is at zero potential. The dimensions of the beam are:

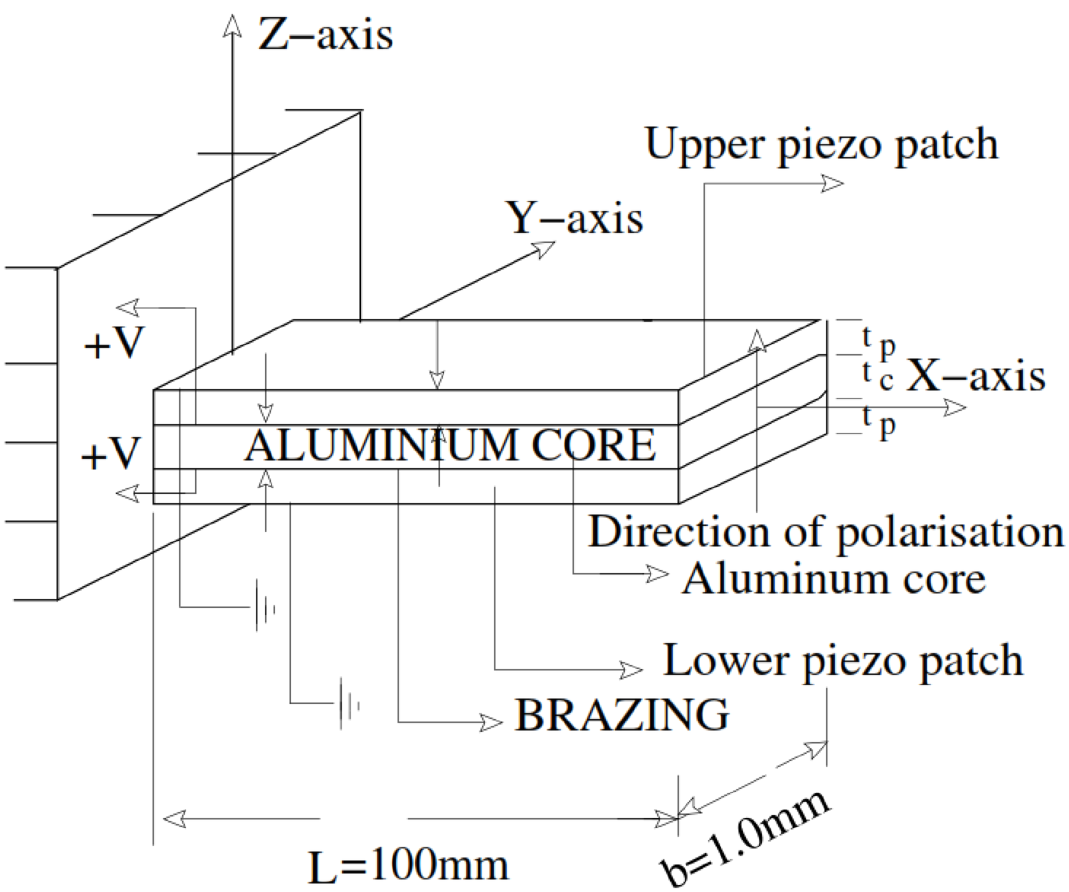
Length = 100 mm Width = 1mm
Patch thickness = 0.127mm Core thickness = 1.5875mm

The problem of beam deformation due to an actuation voltage applied symmetrically on the upper and lower piezo patches has been solved. The nonlinear effects are included as augmented stiffness matrices which are updated at every iteration and the iterations are performed till convergence is achieved. By neglecting the nonlinear stiffness terms, the results corresponding to linear actuation case are obtained.

The variation of axial deformation, axial stress and shear stress across the thickness for an actuation voltage of 40Volt, for linear and non-linear cases are shown in Figures. The introduction of non-linearity in the piezo material distorts the symmetry of the stress distribution about the reference axis. Maximum shear stress in the non-linear case has a higher value than that corresponding to linear case.

Table 1 Material properties of PZT-5H and Aluminium

PZT5H (G Pa)					PZT5H (C/m^2)			Al	
								G Pa	
C_{11}^E	C_{12}^E	C_{13}^E	C_{33}^E	C_{44}^E	e_{31}	e_{33}	e_{15}	E	ν
126	79.5	84.1	117	23	-6.5	23.3	17.0	70.3	0.345

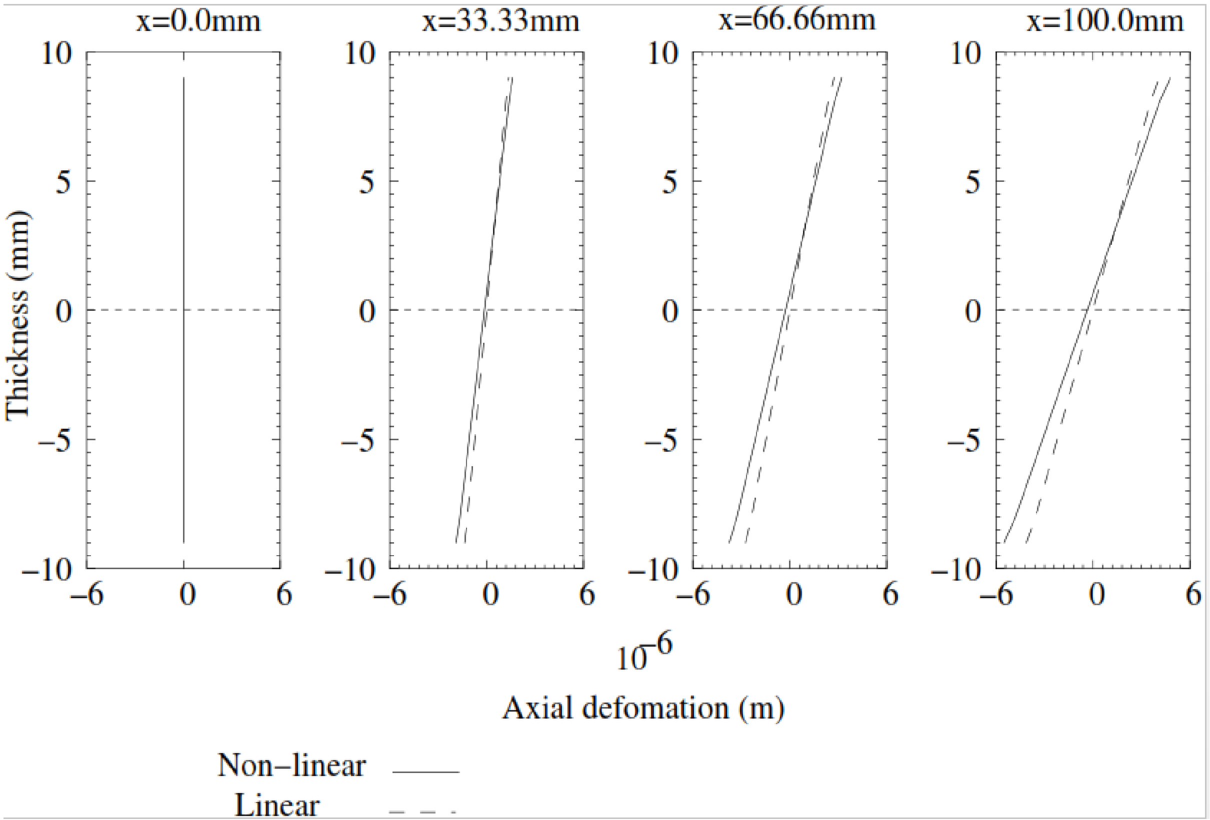


SMART BEAM

Table 2 Variation of applied voltage with tip deflection for linear and non-linear actuation

APPLIED VOLTAGE (V)	TIP DEFLECTION (m)		
	LINEAR	NON-LINEAR EFFECTS	
		DISTRIBUTED FORCE	BOTH DISTRIBUTED FORCE & TORQUE

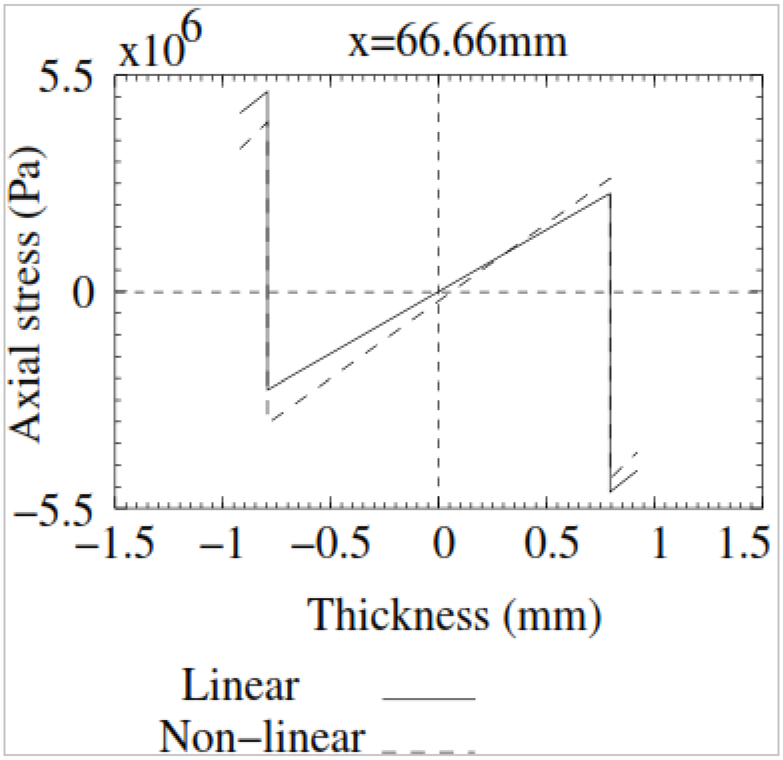
10	0.5581682×10^{-4}	0.5581683×10^{-4}	0.5637581×10^{-4}
20	1.1163366×10^{-4}	1.1163369×10^{-4}	1.164515×10^{-4}
30	1.674505×10^{-4}	1.6745058×10^{-4}	1.862935×10^{-4}
40	2.2326733×10^{-4}	2.2326755×10^{-4}	2.834189×10^{-4}



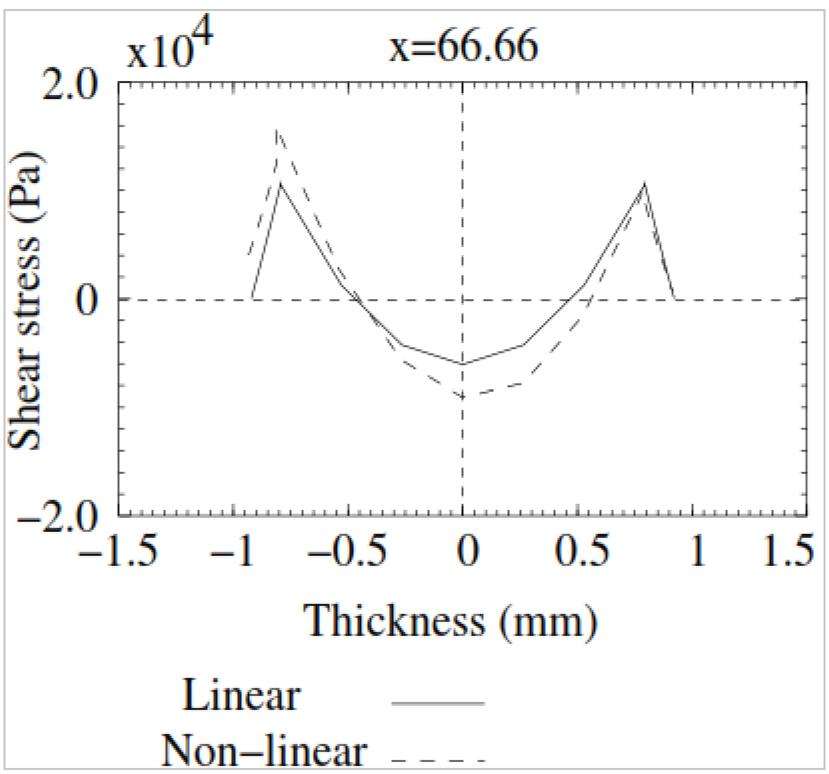
AXIAL DEFORMATION NON LINEAR

Table 3 Mid-layer voltages for linear and non-linear actuation

Applied Voltage (V)	Nodal span Location	1	2	3	4	5	6	7
		Root			Mid-Span			
10	Linear	5.029	5.029	5.029	5.029	5.029	5.029	5.030
	Non-Linear	5.029	5.029	5.029	5.029	5.029	5.029	5.030
20	Linear	10.059	10.059	10.059	10.059	10.059	10.059	10.060
	Non-Linear	10.061	10.061	10.061	10.061	10.061	10.061	10.062
30	Linear	15.088	15.088	15.088	15.088	15.089	15.088	15.091
	Non-Linear	15.099	15.099	15.099	15.098	15.098	15.096	15.098
40	Linear	20.118	20.118	20.118	20.118	20.118	20.118	20.121
	Non-Linear	20.151	20.151	20.151	20.150	20.148	20.143	20.145



NON LINEAR AXIAL STRESS



NON LINEAR SHEAR STRESS

Comparison of tip deflection of the smart beam under increasing actuation voltages are reproduced in table 2. The non-linear solutions are obtained for two cases, namely, case (i) inclusion of only non-linear distributed force terms in equilibrium equation and case (ii) inclusion of both the non-linear effects due to distributed force and moment. The results indicate that for the present problem, the non-linear effects due to distributed force alone are negligible. However introduction of non-linear effects due to distributed moments (non-linear constitutive equation) increases the effect of non-linearity. The difference in the tip deflection of the smart beam between the linear and fully non-linear cases is increase with increase in actuation voltage.

Table 3 shows the induced potential in the mid-layer of the piezo material along the span of the beam for both linear and non-linear cases. The results show that for an actuation voltage of 10Volt, there is no difference in the induced potential between linear and non-linear cases. The difference in the induced potential is increases with increase in actuation voltage. The induced voltage for the non-linear case is more than that of the linear case.

5. CONCLUSION

The effect of nonlinearities due to the interaction of electric field and polarization is analyzed by solving as example problem of actuation of a smart beam. The results indicate that the nonlinearities show a significant influence for high values of actuation voltage.

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