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## Genetic Algorithm Based Control and analysis of PMSM with PID

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### ABSTRACT

In recent years, a remarkable evolution has been achieved by control systems in different application in Robot and many other Areas. One of the significant applications of developing control systems is the Acceleration control in Permanent Magnet Synchronous Motor. Control operations are performed statically even after the proposal of diverse techniques in the literature (Doyle et al. 1992). In addition,  $H^\infty$  controllers are hardly ever utilized to accomplish this. As a result of this, delayed stability problem occurs in Permanent Magnet Synchronous Motor while controlling the acceleration or velocity. In this paper, a graphical-based acceleration stabilization technique is proposed to accomplish effective stability in Permanent Magnet Synchronous Motor controlling operations. The proposed technique is compared with PID Controller to obtain the best performance specifications such as motors with large stability margins with robust control can be effectively used in the field of robot application.

**Keywords:**  $H^\infty$  Controller, PMSM, Stability, Acceleration, Optimisation.

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### 1. INTRODUCTION

Dc motors are generally controlled by conventional proportional-integral-derivative (PID) controllers. Since they can be designed easily and can be built with low cost in expensive maintenance and effectiveness. It is necessary to know system mathematical model or to make some experiments for tuning PID parameters. However, it has been known that conventional PID controller generally do not work well for non-linear systems, and practically complex and vague systems that have no precise mathematical models to overcome these difficulties. Various types of modified conventional controllers such as auto-tuning and adaptive PID controllers can be used for this kind of problems. When compared to the conventional controller (Doyle, et al. 1982).

In this paper the combined solution we have proposed and designed a robust controller. We have used PID outer loop in controller, the gains of the  $H^\infty$  and PID are tuned on-line by use of genetic algorithm. The paper investigates the design of  $H^\infty$  controller for a permanent magnet synchronous motor drive system, the robust PID and  $H^\infty$  techniques have been applied to design the controller. This paper proposes a technique to develop a optimal robust PI and PID controller for PMSM to achieve Robustness and performance (Rosslin John Robles, et al. 2009).

### 1.1. Mathematical Modeling of a DC motor

DC motors are widely used in industrial and domestic equipment, the control of speed/ position of a motor with high accuracy in required the equivalent circuit of field and armature circuit with its rotation mechanical models as shown in Figure 1.

#### Nomenclature

The following are the physical parameters

- Ea: Input/excitation voltage (v);
- Eb: Back emf (v);
- Ra: Resistance of armature winding ( $\Omega$ );
- Ia: Armature current (A);
- La: Inductance of armature winding;
- J: M.I of the motor rotor and load  $\text{kg m}^2/\text{s}^2$ ;
- T: Motor torque (Nm);
- W: The speed of shaft and the load (angular velocity);
- $\Phi$ : shaft position (rad);
- $\beta$ : Damping ratio of mech.sys (Nm);
- Tk: Torque factor constant (Nm/A);
- Bk: The motor constant (V-S/rad);

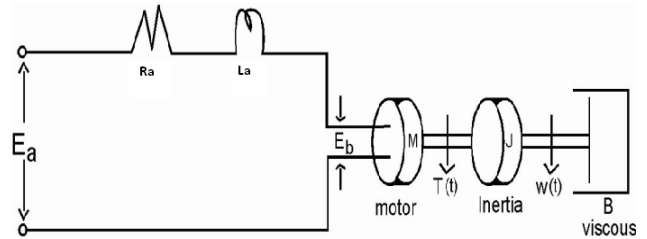


Figure 1  
Separately Excited DC Drive

A desired speed may be tracked when a desired shaft position is also required. Single controller is enough to control both position and speed of the reference signal in the form of voltage determines the desired position and speed. The controller is selected so that the error between the synchronous and reference signal eventually tends to zero. There are many DC motors depending upon the type of DC motor may be controlled by varying the input voltage while for some other motor speed can be controlled by controlling the input current. In this work the DC motor is controlled by varying input voltage. The control design for current is also same. For simplicity, a constant value as a reference signal is injected to the synchronous to obtain a desired position. However the method works successfully for any reference signal, particularly for any stepwise continuous time function. This signal may be a periodic signal or any signal to get a desired shaft position.

Equations describing the dynamics of the input circuit are expressed as follows

$$\begin{aligned}
 V_t &= I_a R_a + L_a I_a + E_a \\
 V_t &= I_a R_a + L_a \frac{dI_a}{dt} + E_a \\
 T &= J \frac{d\omega}{dt} + B\omega - T_l \\
 T &= J \frac{d}{dt} \left[ \frac{d\theta}{dt} \right] + B\omega - T_l \\
 T &= J \frac{d^2\theta}{dt^2} + B \frac{d\theta}{dt} - T_l \\
 T &= J\theta'' + B\theta' + T_l \\
 T &= K_T I_a \\
 E_a &= K_a \omega \\
 \frac{d\omega}{dt} &= \emptyset
 \end{aligned}$$

The dynamic equation in the lap less domain and open loop transfer function of DC motor[2].

$$\begin{aligned}
 s(Js + b)\theta(s) &= kI(s) & \text{---(1)} \\
 (Ls + R)I(s) &= V(s) - ks\theta(s) & \text{---(2)}
 \end{aligned}$$

$$P(s) = \frac{\theta(s)}{V(s)} = \frac{k}{(js + b)(Ls + R)} + t2 \frac{\text{rad/sec}}{\text{volt}}$$

The transfer function of the PID controller is

$$n(t) = K_p e(t) + K_i \int e(t) dt + K_d \frac{de}{dt}$$

$$c(s) = kp + \frac{ki}{s} + kds = \frac{kds^2 + kps + ki}{s}$$

All the design criteria cannot be met well PID controller as other shoot increases drastically with increase in proportional gain constant. This proportional cannot meet all the design requirements derivative or integral controller must be added to the controller.

## 2. STANDARD H-DESIGN

A standard H-infinity problem (Vishwanath, et al. 2009) which introduces a weight function to output the error is depicted in Figure 2. The closed loop transfer function can be represented as,

$$T_{zw}(s) = W(s)S(s)$$

Where, W(s) is the weight function and S(s) is the sensitivity function. In order to make the system internally stable and to minimize the H<sup>∞</sup> norm of T<sub>zw</sub>(s), an optimum feedback controller K(s) is found out by formulating an optimal H<sup>∞</sup> control problem, as follows,

$$\min_k \|T_{zw}(s)\|_{\alpha} = \gamma_0 \quad \dots(3)$$

To minimize the H<sup>∞</sup> norm, the controller parameters has to be optimized. Here, the optimization is done by determining the system parameters through GA. The GA based system parameters optimization is detailed below.

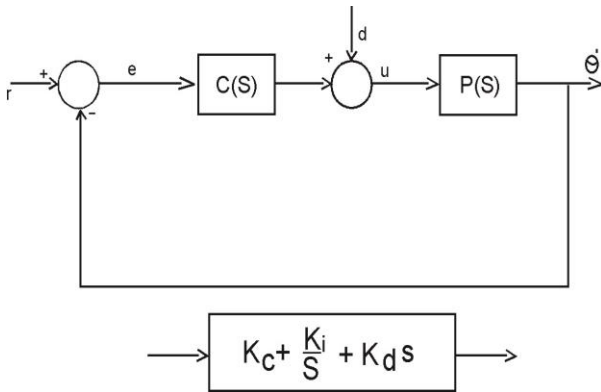


Figure 2  
Standard Control Structure

### 2.1. Parameters Optimization

The GA plays a major role in optimizing the system and controller parameters and in obtaining an optimal H<sup>∞</sup> controller. Let N<sub>T</sub> be the number of system parameters to be optimized. The parameters are considered to be multi-objective parameters. Assuming the target parameters to be the gene of the chromosomes, arbitrary chromosomes of length N<sub>T</sub> are generated. The generated chromosomes can be represented as,

$$X_i = [x_0^{(i)} x_1^{(i)} \dots x_{N_T-1}^{(i)}], \quad 0 \leq i \leq N_p - 1,$$

$$0 \leq j \leq N_T - 1$$

...(4)

Where, x<sub>j</sub><sup>(i)</sup> is the j<sup>th</sup> gene of i<sup>th</sup> chromosome, N<sub>p</sub> is the population pool and N<sub>T</sub> is the number of target parameters. Here, the target parameters considered for optimization are L<sub>q</sub>, m, R<sub>v</sub>, α and θ i.e. N<sub>T</sub>=5. Every gene of the chromosome is generated arbitrarily within their corresponding minimum and maximum intervals

$$\text{i.e. } x_0^{(i)} \in [L_q^{\min}, L_q^{\min}], x_1^{(i)} \in [R_v^{\min}, R_v^{\max}], x_2^{(i)} \in [m^{\min}, m^{\max}], x_3^{(i)} \in [\alpha^{\min}, \alpha^{\max}] \text{ and}$$

$$x_4^{(i)} \in [\theta^{\min}, \theta^{\max}]$$

The generated gene values are subjected to check whether it satisfies the controllability constraints or not. If any of the chromosomes does not satisfy the controllability constraints (Reda Ammar, et al. 2003), a new chromosome is generated repeatedly until it is satisfied. The controllability constraints are checked by generating a state space model using the gene values of the generated chromosome. For the state space model, the controllability matrix H is then determined. The constraint can be expressed as  $N_1 - H_{\text{rank}} \leq H_T$ , where,  $N_1$  is the row size of the matrix H,  $H_{\text{rank}}$  is the rank of the matrix H and  $H_T$  is the controllability threshold. Thus,  $N_p$  chromosomes are generated and the population pool is filled up. The fitness of the chromosomes that are in the population pool is determined. To determine the fitness of the chromosomes, initially, matrices of dimensions A and B are determined as follows

$$A^{(i)} = \frac{1}{x_2^{(i)} \cdot x_3^{(i)}} \begin{bmatrix} -x_1^{(i)} & \frac{3\pi\phi_p}{2\tau} & -x_2^{(i)}x_3^{(i)} \\ -\pi x_2^{(i)}x_3^{(i)}\phi_p & -R \cdot x_2^{(i)}x_3^{(i)} & 0 \\ \tau x_0^{(i)} & x_0^{(i)} & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \dots (5)$$

$$B^{(i)} = \frac{1}{x_0^{(i)}} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \quad \dots(6)$$

With the matrices of dimension and the other system parameters, the system and the process P(s) are developed as follows

$$P^{(i)}(s) = \begin{bmatrix} A^{(i)} & B^{(i)} \\ C & D \end{bmatrix} \quad \dots(7)$$

Where, D=0. Considering the system parameters and the plant function, the fitness can be determined as

$$f_i = \arg \min_{i \in [0, N_p - 1]} \frac{1}{\gamma_i} \quad \dots(8)$$

From the fitness function, the chromosomes that have maximum fitness are placed in the selection pool and the optimal solution for the  $H^\infty$  control problem can be determined as

$$\arg \min \| T_{zw}(s) \|_\infty = \gamma \quad \dots(9)$$

The fittest  $N_p/2$  chromosomes that are present in the arbitrary population pool are selected and they are subjected to the genetic operations, crossover and mutation. In the crossover operation, an exchange of genes is performed between the two parent chromosomes. The crossover is performed with a crossover rate of  $C_r$  i.e.  $C_r \cdot N_T$  genes are exchanged between two parent chromosomes. Hence, a child chromosome  $X_{\text{child}}$  is obtained for a pair of parent chromosomes. In this manner,  $N_p/2$  parent chromosomes are selected in sequence from the selection pool and the crossover operation is performed on them. Hence, new  $N_p/2$  child chromosomes are obtained from the crossover operation. After crossover operation, the chromosomes are subjected to the next genetic operation called mutation. Mutation is an operation that mutates the genes of the chromosomes to obtain new chromosomes. In our approach, an adaptive mutation is performed for fast convergence of the solution with a mutation rate of  $M_r$ . The mutation rate

decides the number of genes to be mutated. The mutation operation performed over a child chromosome is described as follows

- A fittest chromosome, say  $X^{fit}$ , is selected from the selection pool i.e. the chromosome which has the maximum fitness among all the chromosomes that are present in the selection pool

Based on the gene values of  $X^{fit}$ , the remaining chromosomes that are

- present in the selection pool are modified as follows

$$x_j^{newk} = \begin{cases} x_j^{(k)} + \frac{1}{x_j^{(k)}} & ; \text{if } x_j^{(k)} < x_j^{fit} \\ x_j^{(k)} & ; \text{if } x_j^{(k)} = x_j^{fit} \\ x_j^{(k)} - \frac{1}{x_j^{(k)}} & ; \text{if } x_j^{(k)} > x_j^{fit} \end{cases} \quad \dots(10)$$

- The genes of the obtained new chromosomes are modified such that the following criterion is satisfied.

$$x_j^{new} = \begin{cases} x_j^{min} & ; \text{if } x_j^{new} < x_j^{min} \\ x_j^{max} & ; \text{if } x_j^{new} > x_j^{max} \\ x_j^{new} & ; \text{otherwise} \end{cases} \quad \dots(11)$$

In Eq. (10),  $x_j^{fit}$ , and  $x_j^{newk}$  are the genes of children chromosomes that are obtained after performing crossover, genes of the fittest chromosome and genes of the newly obtained chromosome, respectively. Hence, all the genes present in the chromosomes are modified as per the fitness function. As this process relies on the fittest chromosome, quick convergence can be accomplished. At the end of mutation,  $N_p/2$  new chromosomes  $X_{new}$  are obtained. For the  $N_p/2$  new chromosomes, fitness is determined using the Eq.10. The entire process is repeated for  $I_{max}$  iterations. Once it reaches  $I_{max}$  iterations, the process is terminated. The chromosome with maximum fitness present in the selection pool is chosen as the best system parameters. On the basis of these parameters the system and the controller are developed. The obtained optimal  $H^\infty$  controller can work satisfactorily for all the given system parameters. Hence, the system can offer a good stabilization over the velocity, which is considered as the system output. From the obtained velocity, the acceleration can be determined as (Aboubekeur Hamdi-Cherif, et al. 2009).

$$dv/dt = (F_m - f - R_v)/m_{best} \quad \dots(11)$$

where,  $F_m$  is the output electromagnetic thrust,  $f$  is the total friction,  $R_v$  is the damper coefficient,  $v$  is the velocity output obtained from the system and  $m_{best}$  is the best mass value obtained from the proposed technique.

### 3. PID

Integral term reduces the steady state error and adding derivative term reduces the overshoot. PID controls which small  $K_D$  and  $K_i$  in this case the time required in large to go to steady state. Design PID with appropriate  $K_p$ ,  $K_D$  &  $K_i$  will give satisfactory results. PID is used to get rid of the steady state error due to disturbance. MATLAB provides tools for automatically choosing optimal PID gains which makes to trial and process.

The transfer function of the PI control

$$u = ke + \frac{1}{1+st}$$

$$u = k \left( \frac{1 + st}{st} \right) e = \left( k + \frac{k}{st} \right) e$$

The simplified dynamic model of the pm synchronous system is as shown below.

The pm synchronous motor and the +inverter need to be designed to obtain a mathematical model on which it design of the robust controller is based the inverter output current closely follow the reference current commands owing to the closed current loop control. The actual motor line currents are in d-q coordinates.  $z_q$  and  $z_d$  are thus assumed to be equal to their reference commands  $z^*_q$  and  $z^*_d$  respectively. The vector control loop places the stator current vector  $i_s$  on the q-axis and the rotor flux on the d-axis this implies that,

$$z^*_d = z_d = 0 \text{ and } \lambda_q = 0$$

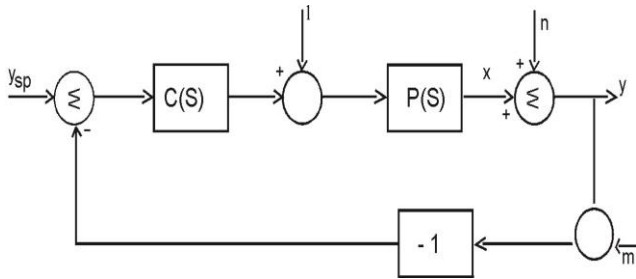


Figure 3  
Robust control scheme

The mathematical model of pm synchronous motor can be designed in the d-q synchronous rotating reference frame by the following three-state non-linear differential equation (Viswanath et al, 2011).

#### 4. ROBUST –CONTROL SCHEME

Thus the closed loop transfer function W to Z is given by (Figure 3)

$$Tzw(s) = c_d(SI - A_d)^{-1} B_d + D_d$$

The resulting closed-loop system is internally stable if and only the Eigen values of.

#### 4.1. Mathematical model of PMSM

Research has indicated that the permanent magnet motor drive, which include the PM synchronous Motor have become serious competitors to the induction motor for servo applications the PMSM has a sinusoidal back emf and require sinusoidal stator currents to produce constant torque, PMSM is very similar to the wound rotor synchronous M/c except that the PMSM that is used for servo applications tends not to have any damper windings and excitation is provided by the permanent magnet instead of a field winding. Hence the d, q model of the PMSM can be designed from the well known model of the synchronous m/c with the equations of the damper windings and field current-dynamics removed (Pragasen Pillay et al, 1989).

$$\begin{aligned} \Psi_a &= L_a Z_a + L_b Z_b + C_c Z_c + \Psi_{ra} \\ \Psi_b &= L_b Z_a + L_b Z_b + L_c Z_c + \Psi_b \\ \Psi_c &= L_c Z_a + L_c Z_b + L_c Z_c + \Psi_c \end{aligned}$$

$L_a, L_b, L_c$  are the self inductances of the stator a-b-c phases respectively. a-b-c phases respectively,

$L_{ab}, L_{bc}, L_{ac}$  Are mutual inductances, hence three phases, based on the rotor angle position the flux linkages can be expressed as,

$$\left. \begin{aligned} \Psi_a &= \psi_r \cos \theta \\ \Psi_b &= \psi_r \cos(\theta - 120^\circ) \\ \Psi_c &= \psi_r \cos(\theta + 120^\circ) \\ \Psi &= \phi_m \text{ Or max value of flux.} \end{aligned} \right\}$$

The electrical dynamic equations in terms of phase variables can be written as,

$$[V_s] = [r_s][z_s] + \frac{d}{dt}[\Delta s] \quad \text{----(12)}$$

And electromagnetic thrust is given by,

$$P_{o\omega} = F_m = \frac{3}{2} (P_n) \left( \frac{\pi}{2} \right) [\varphi_d I_q - \varphi_d Z_d]$$

If  $\omega_r = \left( \frac{P}{2} \right) \omega_{rm}$ .  $\omega_{rm}$  is the mechanical rotor speed in rad/sec. Now electromechanical torque

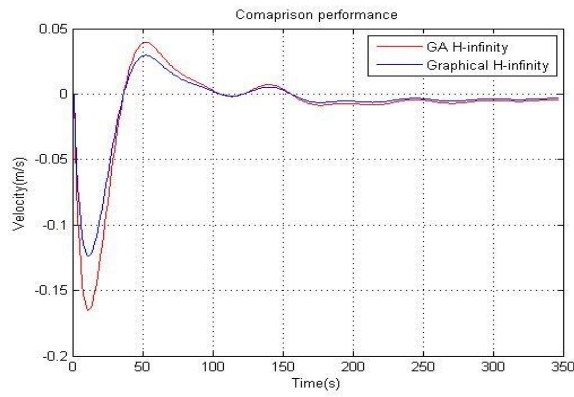


Figure 4  
Velocity response of GA H<sup>∞</sup> and graphical H<sup>∞</sup>

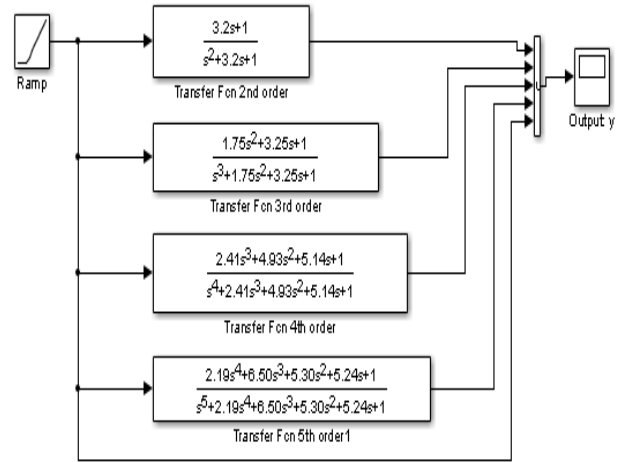


Figure 6  
Simulink model for different order controllers

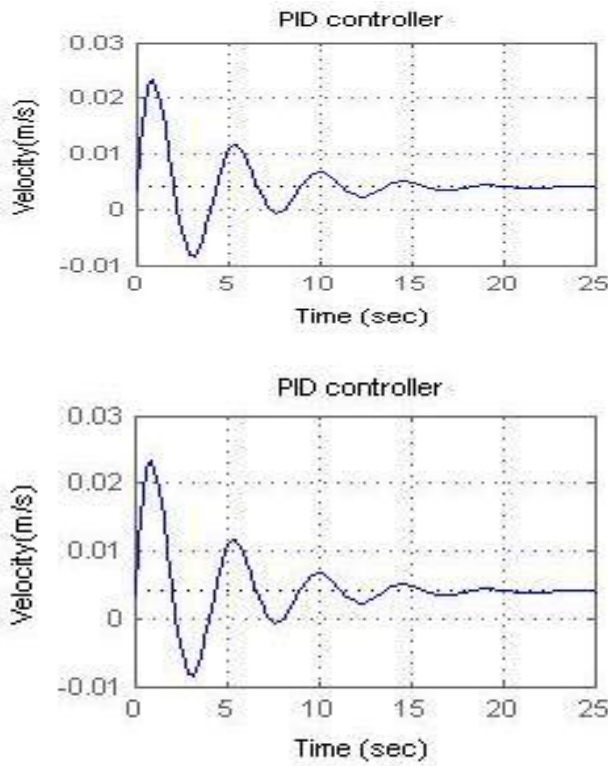


Figure 5

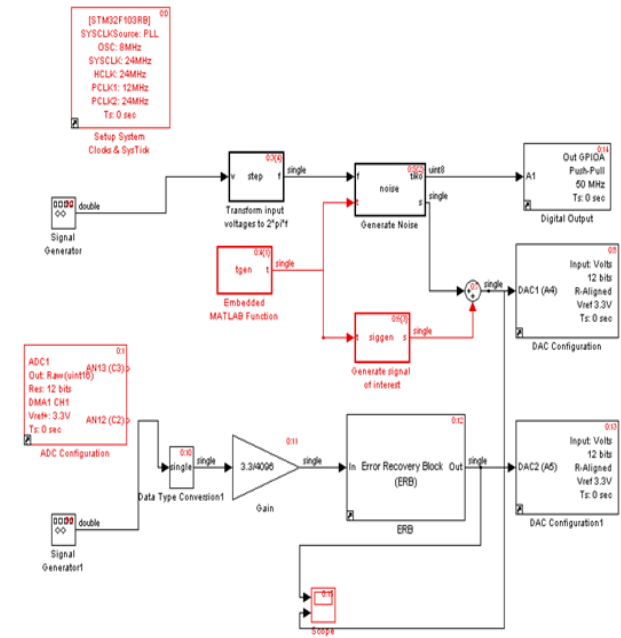


Figure 7  
Hardware implementation part of H<sup>∞</sup>

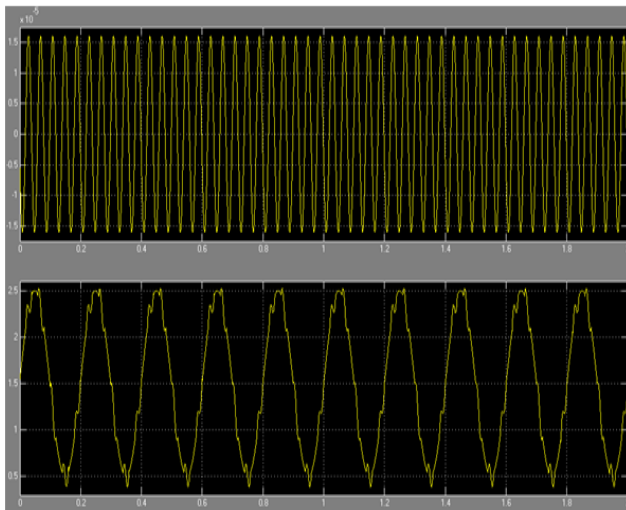
$$T_{em} = \frac{3}{2} P (\varphi_{af} L_q + (L_d - L_q) z_d z_q)$$

$$= J P \omega_r + B \omega_r + T_l$$

Where  $P = \left( \frac{d}{dt} \right)$

$\omega_r, \omega_s$  are frequency rotor speed. P is number of pole pairs.

Final the motion equation can be written as  $\frac{dv}{dt} = \frac{F_m - f - Rv}{m}$



**Figure 8**  
Stability response

The inverter o/p currents are assumed to be very close to the reference current commands. In the d-q coOrdinates, the motor line currents  $Z_q$  and  $Z_d$  are approximated to the reference commands  $Z_q^*$  and  $Z_d^*$ . Maximum torque can be obtained by means of field oriented per ampere when the stator current is properly placed on the q-axis and rotor flux on the d-axis. When  $Z_d \approx Z_d^*$  the maximum flux is constant. Hence torque is directly proportional to q-axis current. The mathematical model of PMSM is simulated and simulation. Program is written in MATLAB and used to verify the basic operation.

The mathematical model is capable of simulating the steady state as well as dynamic respectively.

$$\frac{dz_q}{dt} = (V_q - R_{zq} - \omega_s \Psi_a f) / L_q \quad \dots(13)$$

$$\frac{d\omega_r}{dt} = \frac{T_e - T_L - B\omega_r}{J}$$

$$T_e = k_t z_q \text{ Where } K_t = \frac{3P\Psi_a}{2}$$

If the PMSM is driven by const-current-source inverter the  $z_q$  is known as acts as excitation current. Here the internal motor characteristics and current feedback control play an important role to obtain the max torque/amp.

## 5. RESULTS AND DISCUSSIONS

Results are shown in Figures 4-8.

## 6. CONCLUSION

The performnce of  $H^\infty$  controller in stabilizing the accelarattion and velocity of PMSM has considerabaly improved as compared with PID controller. All the time domain specifcaitons have been met with a broader margin.

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