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sg-separation axioms

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ABSTRACT

In this paper we discuss new separation axioms using sg-open sets.

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1. INTRODUCTION

Norman Levine introduced generalized closed sets in 1970. After him various Authors [1-18; 20-29] studied different versions of generalized sets and related topological properties. Recently V.K. Sharma studied separation axioms for g-open. Following V.K. Sharma we are going to study further properties of sg-separation axioms.Throughout the paper a space X means a topological space (X,τ). For any subset A of X its complement, interior, closure, sg-interior, sg-closure are denoted respectively by the symbols A^c, A^o, cl(A), sg-int(A) and sg-cl(A).

1.1. Definition 1.1

$A\subset X$ is called

(i) g-closed[resp: sg-closed] if $cl(A) \subseteq U$[resp: $scl(A) \subseteq U$] whenever $A \subseteq U$ and U is open[resp: semi-open] in X.

(ii) g-open[resp: sg-open] if its complement is (i) g-closed[resp: sg-closed].

Note 1: The class of regular open sets, open sets, g-open sets and sg-open sets are denoted by RO(X), $\tau(X)$, GO(X) and SGO(X) respectively. Clearly $RO(X)\subset\tau(X)\subset GO(X)\subset SGO(X)$.

Note 2: $A\in SGO(X, x)$ means A is a semipro generalized-open neighborhood of X containing x.

1.2. Definition 1.2

$A\subset X$ is called clopen[resp: nearly-clopen; semi-clopen; g-clopen; sg-clopen] if it is both open[resp: regular-open; semi-open; g-open; sg-open] and closed[resp: regular-closed; semi-closed; g- closed; sg-closed]

1.3. Definition 1.3

A function $f: X \to Y$ is said to be

(i) Continuous [resp: nearly continuous, semi-continuous] if inverse image of open set is open[resp: regular-open, semi-open]

(ii) g-continuous [resp: sg-continuous] if inverse image of closed set is g-closed [resp: sg-closed]

(iii) irresolute [resp: nearly irresolute, sg-irresolute] if inverse image of semi-open [resp: regular-open, sg-open] set is semi-open [resp: regular-open, sg-open]

(iv) g-irresolute [resp: sg-irresolute; sg-irresolute] if inverse image of g-closed [resp: sg-closed, sg-closed] set is g-closed [resp: sg-closed; sg-closed]

(v) open [resp: nearly open, semi-open] if the image of open set is open [resp: regular-open, semi-open]

(vi) g-open [resp: sg-open] if the image of open set is g-open [resp: sg-open]

(vii)homeomorphism [resp: nearly homeomorphism, semi-homeomorphism] if *f* is bijective, continuous [resp: nearly-continuous, semi-continuous] and f^{-1} is continuous[resp: nearly-continuous, semi-continuous]

(viii)rc-homeomorphism [resp: sc-homeomorphism] if *f* is bijective r-irresolute [resp: irresolute] and f^{-1} is r-irresolute [resp: irresolute]

(ix) g-homeomorphism [resp: sg-homeomorphism] if *f* is bijective g-continuous [resp: sg-continuous] and f^{-1} is g-continuous [resp: sg-continuous]

(x) gc-homeomorphism [resp: sgc-homeomorphism] if *f* is bijective g-irresolute [resp: sg-irresolute] and f^{-1} is g-irresolute[resp: sg-irresolute]

1.4. Definition 1.4

X is said to be

(i) compact [nearly compact, semi-compact, g-compact, sg-compact] if every open[regular-open, semi-open, g-open, sg-open] cover has a finite sub cover.

(ii) T_0 [rT_0, sT_0, g_0] space if for each $x \neq y\in X$ $\exists$ $U\in\tau(X)$[RO(X); SO(X); GO(X)] containing either x or y.

(iii) T_1 [rT_1, sT_1, g_1] space if for each $x \neq y\in X$ $\exists$ U, $V\in\tau(X)$[RO(X); SO(X); GO(X)] such that $x\in U-V$ and $y\in V - U$.

(iv) T_2[rT_2, sT_2, g_2] space if for each $x \neq y\in X$ $\exists$ U, $V\in\tau(X)$[RO(X); SO(X); GO(X)] such that $x\in U$; $y\in V$ and $U\cap V = \phi$.

(v) $T_{1/2}$ [$rT_{1/2}$, $pT_{1/2}$] if every g-closed[rg-closed, pg-closed] set is closed [r-closed, pre-closed]

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2. SG-CONTINUITY AND PRODUCT SPACES

2.1. Theorem 2.1

Let Y and $\{X_\alpha:\alpha\in I\}$ be Topological Spaces. Let f: Y$\to \Pi X_\alpha$ be a function. If f is sg-continuous, then $\pi_\alpha\bullet f$: Y$\to X_\alpha$ is sg-continuous.

Proof: Suppose f is sg-continuous and $\pi_\alpha:\Pi X_\beta\to X_\alpha$ is continuous for each $\alpha\in I$, $\pi_\alpha\bullet f$ is sg-continuous.

Converse of the above theorem is not true in general.

Example 2.1: Let X = {p, q, r, s}; τ_X = {ϕ, {q}, {p, q}, {q, r}, {p, q, r}, X}, $Y_1 = Y_2$ = {a, b}; τ_{Y1} = {ϕ, {a}, Y_1}; τ_{Y2} = {ϕ, {a}, Y_2}; Y = $Y_1\times Y_2$ = {(a, a), (a, b), (b, a), (b, b)} and τ_Y = {ϕ, {(a, a)}, {(a, a), (a, b)}, {(a, a), (b, a)}, {(a, a), (a, b), (b, a)}, $Y_1\times Y_2$}. Define f by f(p) = (a, a), f(q) = (b, b), f(r) = (a, b), f(s) = (b, a). It is easy to see that $\pi_1\bullet f$ and $\pi_2\bullet f$ are sg-continuous. However {(b, b)} is closed in Y but f^{-1}({(b, b)}) = {q} is not sg-closed in X. Therefore f is not sg-continuous.

2.2. Theorem 2.2

If Y is $sT_{1/2}$ and $\{X_\alpha:\alpha\in I\}$ be Topological Spaces. Let f: Y$\to \Pi X_\alpha$ be a function, then f is sg-continuous iff $\pi_\alpha\bullet f$: Y$\to X_\alpha$ is sg-continuous.

2.3. Corollary 2.3

Let f_α: $X_\alpha\to Y_\alpha$ be a function and let f: $\Pi X_\alpha\to \Pi Y_\alpha$ be defined by $f(x_\alpha)_{\alpha\in I} = (f_\alpha(x_\alpha))_{\alpha\in I}$. If f is sg-continuous then each f_α is sg-continuous.

2.4. Corollary 2.4

For each α, let X_α be $sT_{1/2}$ and let f_α: $X_\alpha\to Y_\alpha$ be a function and let f: $\Pi X_\alpha\to \Pi Y_\alpha$ be defined by $f(x_\alpha)_{\alpha\in I} = (f_\alpha(x_\alpha))_{\alpha\in I}$, then f is sg-continuous iff each f_α is sg-continuous.

3. SG_I SPACES i = 0, 1, 2

3.1. Definition 3.1

X is said to be

(i) a sg_0 space if for each pair of distinct points x, y of X, there exists a sg-open set G containing either of the point x or y.

(ii) a sg_1 space if for each pair of distinct points x, y of X there exists a sg-open set G containing x but not y and a sg-open set H containing y but not x.

(iii)a sg_2 space if for each pair of distinct points x, y of X there exists disjoint sg-open sets G and H such that G containing x but not y and H containing y but not x.

Note 2: X is $sg_2\to$ X is $sg_1\to$ X is sg_0.

Example 3.1: Let X = {a, b, c} and τ = {ϕ, {a, c}, X} then X is sg_0 but not rT_0 and T_0, i = 0, 1, 2.

(ii) τ = {ϕ, {a}, {a, c}, X} then X is not sg_i for i = 0, 1, 2.

Example 3.2: Let X = {a, b, c, d} and

(i) τ = {ϕ, {a}, {c}, {a, c}, X} then X is sg_i ; i = 0, 1, 2.

(ii) τ = {ϕ, {a}, {a, b}, {c, d}, {a, c, d}, X} then X is not sg_i for i = 0, 1, 2.

Remark 3.1: If X is $sT_{1/2}$ then sT_i and sg_i are one and the same for i = 0,1,2.

3.2. Theorem 3.1

(i) Every [resp: regular open] open subspace of sg_i space is sg_i for i = 0, 1, 2.

(ii) [25]The product of sg_i spaces is again sg_i for i = 0, 1, 2.

(iii) sg-continuous image of T_i[resp: rT_i] spaces is sg_i for i = 0, 1, 2.

3.3. Theorem 3.2

(i) X is sg_0 iff $\forall$ x$\in$ X, $\exists$ U$\in$ SGO(X) containing x such that the subspace U is sg_0.

(ii)X is sg_0 iff distinct points of X have disjoint sg-closures.

3.4. Theorem 3.3

The following are equivalent:

(i) X is sg_1.

(ii) Each one point set is sg-closed.

(iii)Each subset of X is the intersection of all sg-open sets containing it.

(iv) For any x$\in$ X, the intersection of all sg-open sets containing the point is the set {x}.

3.5. Theorem 3.4

If X is sg_1 then distinct points of X have disjoint sg-closures.

3.6. Theorem 3.5

Suppose x is a sg-limit point of a subset of A of a sg_1 space X. Then every neighborhood of x contains infinitely many distinct points of A.

3.7. Theorem 3.6

X is sg_2 iff the intersection of all sg-closed, sg-neighborhoods of each point of the space is reduced to that point.

Proof: Let X be sg_2 and x$\in$X, for each y $\neq$ x in X, $\exists$ U, V$\in$SGO(X) such that x$\in$U, y$\in$V and U$\cap$V = ϕ. Since x$\in$U-V, hence X-V is a sg-closed, sg-neighborhood of x to which y does not belong. Consequently, the intersection of all sg-closed, sg-neighborhoods of x is reduced to {x}.

Conversely let y $\neq$ x in X, by hypothesis $\exists$ a sg-closed, sg-neighborhood U of x such that y$\notin$U. Now $\exists$ G$\in$SGO(X) such that x$\in$G$\subset$U. Thus G and X-U are disjoint sg-open sets containing x and y respectively. Hence X is sg_2.

3.8. Theorem 3.7

If to each x$\in$X, there exist a sg-closed, sg-open subset of X containing x which is also a sg_2 subspace of X, then X is sg_2.

Proof: Let x$\in$X, U a sg-closed, sg-open subset of X containing x and which is also a sg_2 subspace of X, then the intersection of all sg-closed, sg-neighborhoods of x in U is reduced to {x}. U being sg-closed, sg-open, these are sg-closed, sg-neighborhoods of x in X. Thus the intersection of all sg-closed, sg-neighborhoods of x is reduced to {x}. Hence by Theorem 3.6, X is sg_2.

3.9. Theorem 3.9

If X is sg_2 then the diagonal Δ in X$\times$X is sg-closed.

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Proof: Let $(x, y)\in X\times X-\Delta$, then $x \neq y$. Since X is sg_2 $\exists$ U; $V\in SGO(X)$ s.t. $x\in U$; $y\in V$ and $U\cap V = \phi$, implies $(U\times V)\cap\Delta = \phi$ and therefore $(x, y)\in(U\times V)\subset X\times X-\Delta\in SGO(X\times X)$. Hence Δ is sg-closed.

3.10. Theorem 3.9

In sg_2-space, sg-limits of sequences, if exists, are unique.

3.11. Theorem 3.10

In a sg_2 space, a point and disjoint sg-compact subspace can be separated by disjoint sg-open sets.
Proof: Let X be a sg_2 space, $x\in X$ and C a sg-compact subspace of X not containing x. Let $y\in C$ then for $x \neq y$ in X, $\exists$ disjoint sg-open nbds G_x and H_y. Allowing this for each y in C, we obtain a class $\{H_y\}$ whose union covers C; and since C is sg-compact, some finite subclass $\{H_i, i = 1 \text{ to } n\}$ covers C. If G_i is sg-nbd of x corresponding to H_i, we put $G = \cup_{i=1-n}G_i$ and $H = \cap_{i=1-n}H_i$, satisfying the required properties.

3.12. Corollary 3.1

(i) In a T_1 [resp: rT_1; g_1] space, each singleton set is sg-closed.
(ii) If X is T_1 [resp: rT_1; g_1] then distinct points of X have disjoint sg-closures.
(iii)If X is T_2 [resp: rT_2; g_2] then the diagonal Δin $X\times X$ is sg-closed.
(iv) Show that in a T_2 [resp: rT_2; g_2] space, a point and disjoint compact[resp: nearly-compact; g-compact] subspace can be separated by disjoint sg-open sets

3.13. Theorem 3.11

Every sg-compact subspace of a sg_2 space is sg-closed.
Proof: Let C be sg-compact subspace of a sg_2 space. If $x\in C^c$, by above Theorem x has a sg-nbd G s.t $x\in G \subset C^c$. Thus C^c is the union of sg-open sets and therefore C^c is sg-open. Thus C is sg-closed.

3.14. Corollary 3.2

Every compact [resp: nearly-compact; g-compact] subspace of a T_2 [resp: rT_2; g_2] space is sg-closed.

3.15. Theorem 3.12

If $f: X\to Y$ is injective, sg-irresolute and Y is sg_i then X is sg_i, i = 0, 1, 2.
Proof: Let $x \neq y\in X$, then $\exists$ a sg-open set $V_x\subset Y$ such that $f(x)\in V_x$ and $f(y)\notin V_x$ and $\exists$ a sg-open set $V_y\subset Y$ such that $f(y)\in V_y$ and $f(x)\notin V_y$ with $f(x) \neq f(y)$. By sg-irresoluteness of f, $f^{-1}(V_x)$ is sg-open in X such that $x\in f^{-1}(V_x)$; $y\notin f^{-1}(V_x)$ and $f^{-1}(V_y)$ is sg-open in X such that $y\in f^{-1}(V_y)$; $x\notin f^{-1}(V_y)$. Hence X is sg_2

Similarly one can prove the remaining part of the Theorem.

3.16. Corollary 3.3

(i) If $f: X\to Y$ is injective, sg-continuous and Y is T_i then X is sg_i, i = 0, 1, 2.
(ii) If $f: X\to Y$ is injective, r-irresolute[r-continuous] and Y is rT_i then X is sg_i, i = 0, 1, 2.
(iii)The property of being a space is sg_0 is a sg-Topological property.
(iv) Let $f: X \to Y$ is a sgc-homeomorphism, then X is sg_i if Y is sg_i, i = 0, 1, 2.

3.17. Theorem 3.13

Let X be T_1 and $f: X \to Y$ be sg-closed surjection. Then X is sg_1.

3.18. Theorem 3.14

Every sg-irresolute map from a sg-compact space into a sg_2 space is sg-closed.
Proof: If $f: X \to Y$ is sg-irresolute where X is sg-compact and Y is sg_2. Let $C\subset X$ be closed, then $C\subset X$ is sg-closed and hence C is sg-compact and so $f(C)$ is sg-compact. But then $f(C)$ is sg-closed in Y. Hence the image of any sg-closed set in X is sg-closed set in Y. Thus f is sg-closed.

3.19. Theorem 3.15

Any sg-irresolute bijection from a sg-compact space onto a sg_2 space is a sgc-homeomorphism.
Proof: Let f be a sg-irresolute bijection from a sg-compact space onto a sg_2 space. Let $G\in SGO(X)$. Then $X-G\in SGC(X)$ and hence $f(X-G)\in SGC(Y)$. Since f is bijective $f(X-G) = Y-f(G)$ and therefore $f(G)\in SGO(Y)$. Hence f is M-sg-open. Thus f is sgc-homeomorphism.

3.20. Corollary 3.4

Any sg-continuous bijection from a sg-compact space onto a sg_2 space is a sg-homeomorphism.

3.21. Theorem 3.16

The following are equivalent:
(i) X is sg_2.
(ii) For each pair $x \neq y\in X$ $\exists$ a sg-open, sg-closed set V such that $x\in V$ and $y\notin V$, and
(iii)For each pair $x \neq y\in X$ $\exists$ $f: X\to [0, 1]$ such that $f(x) = 0$ and $f(y) = 1$ and f is sg-continuous.

3.22. Theorem 3.17

If $f: X\to Y$ is sg-irresolute and Y is sg_2 then
(i) the set $A = \{(x_1, x_2): f(x_1) = f(x_2)\}$ is sg-closed in $X\times X$.
(ii)$G(f)$, Graph of f, is sg-closed in $X\times Y$.
Proof: (i) Let $A = \{(x_1, x_2): f(x_1) = f(x_2)\}$. If $(x_1, x_2)\in X\times X-A$, then $f(x_1) \neq f(x_2) \Rightarrow \exists$ disjoint V_1 and $V_2\in SGO(Y)$ such that $f(x_1)\in V_1$ and $f(x_2)\in V_2$, then by sg-irresoluteness of f, $f^{-1}(V_j)\in SGO(X, x_j)$ for each j. Thus $(x_1, x_2)\in f^{-1}(V_1)\times f^{-1}(V_2) \subset X\times X-A \in SGO(X\times X)$. Hence $A\in SGC(X\times X)$.
(ii) Let $(x, y)\notin G(f) \Rightarrow y \neq f(x) \Rightarrow \exists$ disjoint V; $W\in SGO(Y)$ such that $f(x)\in V$ and $y\in W$. Since f is sg-irresolute, $\exists$ $U\in SGO(X)$ such that $x\in U$ and $f(U)\subset W$. Therefore we obtain $(x, y)\in U\times V\subset X\times Y-G(f)$. Hence $X\times Y-G(f)\in SGO(X\times Y)$. Hence $G(f)\in SGC(X\times Y)$.

3.23. Theorem 3.18

If f: X→ Y is sg-open and A = {(x_1, x_2): $f(x_1) = f(x_2)$} is closed in X×X. Then Y is sg_2.

3.24. Theorem 3.19

Let Y and {X_α:α∈ I} be Topological Spaces. If f: Y→ Π X_α be a sg-continuous function and Y is $rT_{1/2}$, then Π X_α and each X_α are sg_i, i = 0,1,2.

3.25. Theorem 3.20

Let X be an arbitrary space, R an equivalence relation in X and p: X → X/R the identification map. If R⊂ X×X is sg-closed in X×X and p is sg-open map, then X/R is sg_2.

Proof: Let p(X), ≠ p(y)∈X/R. Since x and y are not related, R⊂X×X is sg-closed in X×X. There are sg-open sets U and V such that x∈U, y∈V and U×V⊂ R^c. Thus {p(U), p(V)} are disjoint and also sg-open in X/R since p is sg-open.

3.26. Theorem 3.21

The following four properties are equivalent:

(i) X is sg_2

(ii) Let x∈ X. For each y ≠ x, ∃ U∈SGO(X) such that x∈U and y∉sgcl(U)

(iii)For each x∈ X, ∩{sgcl(U)/U∈SGO(X) and x∈ U} = {x}.

(iv) The diagonal Δ = {(x, x)/x∈X} is sg-closed in X× X.

Proof: (i) ⇒ (ii) Let x∈ X and y ≠ x. Then there are disjoint sg-open sets U and V such that x∈ U and y∈ V. Clearly V^c is sg-closed, sgcl(U) ⊂ V^c, y∉V^c and therefore y∉ sgcl(U).

(ii) ⇒ (iii) If y ≠ x, then ∃ U∈SGO(X, x) and y∉sgcl(U). So y∉∩{sgcl(U)/U∈SGO(X) and x∈U}.

(iii) ⇒ (iv) We prove Δ^c is sg-open. Let (x, y) ∉Δ. Then y ≠ x and ∩{sgcl(U)/U∈SGO(X) and x∈U} = {x} there is some U∈SGO(X) with x∈U and y∉ sgcl(U). Since U∩$(sgcl(U))^c$ = ϕ, U×$(sgcl(U))^c$ is a sg-open set such that (x, y)∈U×$(sgcl(U))^c$⊂Δ^c.

(iv) ⇒ (i) y ≠ x, then (x, y)∉Δ and thus there exist sg-open sets U and V such that (x, y)∈U×V and (U×V)∩Δ = ϕ. Clearly, for the sg-open sets U and V we have; x∈U, y∈V and U∩V = ϕ.

4. SGG_3 AND SGG_4 SPACES

4.1. Definition 4.1

X is said to be

(i) a sg_3 space if for every sg-closed sets F and a point x∉F ∃ disjoint U, V∈SPO(X)such that F⊆U; x∈V

(ii) a sgg_3 space if for every sg-closed sets F and x∉ F ∃ disjoint U, V∈SGO(X)such that F⊆U; x∈V

(iii)a sg_4 space if for each pair of disjoint F; H∈SGC(X), ∃ disjoint U, V∈SPO(X) s.t. F⊆U; H⊆V

(iv) a sgg_4 space if for each pair of disjoint F; H∈SGC(X), ∃ disjoint U, V∈SGO(X) s.t. F⊆U; H⊆V

Note: rT_i → sg_i → sgg_i , i = 3, 4. but the converse is not true in general.

Example 4.1: Let X = {a, b, c} and

(i) τ = {ϕ, {a}, {b, c}, X} then X is sgg_i.

(ii) τ = {ϕ, {a}, X} then X is not sgg_i, sg_i and rT_i for i = 3, 4.

4.2. Lemma 4.1

X is sg-regular iff X is nearly-regular and $rT_{1/2}$.

Proof: X is sg-regular, then obviously X is nearly-regular. Let A⊆X be sg-closed. For each x∉A ∃ V_x∈ SGO(X, x) such that V_x∩A = ϕ. If V = ∪{V_x:x∉ A}, then V is sg-open and V = X-A. Hence A is sg-closed implies X is $rT_{1/2}$.

4.3. Theorem 4.1

If X is sg_3. Then for each x∈X and each U∈SGO(X, x) ∃ a sg-neighborhood V of x such that sgcl(A)⊂U.

Proof: Let x∈X and U a sg-neighborhood of x. Let B = X - U, then B is sg-closed and by sg-regularity of X, ∃ disjoint V, W∈SGO(X) such that x∈V and B⊆W. Then sgcl(V)∩B = ϕ ⇒ sgcl(V)⊆X - B.

4.4. Theorem 4.2

The following are equivalent:

(i) X is sg_3.

(ii) For every point x∈X and for every G∈SGO(X, x), ∃ U∈SGO(X) such that x∈U⊆ sgcl(U) ⊆ G.

(iii)For every sg-closed set F, the intersection of al sg-closed sg-neighborhoods of F is exactly F.

(iv) For every set A and B∈SGO(X) such that A∩B ≠ ϕ, ∃ G∈SGO(X)such that A∩G ≠ ϕ and sgcl(G)⊆B.

(v) For every A ≠ ϕ and B∈SGC(X) with A∩B = ϕ, ∃ disjoint G; H∈SGO(X) such that A⊆G and B⊆H.

4.5. Theorem 4.3

If X is sgg_3. Then for each x∈X and each U∈SGO(X, x), ∃ V∈SGO(X, x) such that sgcl(A)⊂U.

Proof: Let x∈ X and U a sg-nbd of x. Let B = X - U, then B is sg-closed and by sgg-regularity of X, ∃ disjoint V, W∈SGO(X) such that x∈V and B⊆W. Then sgcl(V)∩B = ϕ ⇒ sgcl(V)⊆X - B.

4.6. Corollary 4.1

If X is T_3 [resp: rT_3; g_3]. Then for each x∈ X and each sg-open neighborhood U of x there exists a sg-open neighborhood V of x such that sgcl(A)⊂U.

4.7. Theorem 4.4

If f: X→ Y is sg-closed, sg-irresolute bijection. Then X is sgg_3 iff Y is sgg_3.

Proof: Let F be closed set in X and x∉F, then f(x)∉f(F) and f(F) is sg-closed in Y. By sgg_3 of Y, ∃ V; W∈ SGO(y) such that f(X)∈V and f(F)⊆W. Hence x∈f^{-1}(V) and F⊆f^{-1}(W), where f^{-1}(V) and f^{-1}(W) are disjoint sg-open sets in X (by sg-irresoluteness of f). Hence X is sgg_3.

Conversely, X be sgg_3 and K any sg-closed in Y with y∉K, then f^{-1}(K) is sg-closed in X such that f^{-1}(y) ∉ f^{-1}(K). By sgg_3 of X, ∃ disjoint V, W∈SGO(X) such that f^{-1}(y)∈V and f^{-1}(K)⊆W. Hence y∈ f(V) and K⊆ f(W) such that f(V) and f(W) are disjoint sg-open sets in X. Thus Y is sgg_3

4.8. Theorem 4.5

X is sg-normal iff for every sg-closed set F and a sg-open set G containing A, there exists a sg-open set V such that F⊆ V⊆ sgcl(V) ⊆ G

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4.9. Theorem 4.6

X is sg-normal iff for every pair of disjoint sg-closed sets A and B, there exist disjoint sg-open sets U and V such that $A \subseteq U$ and $B \subseteq V$.

Proof: Necessity: Follows from the fact that every sg-open set is sg-open.

Sufficiency: Let A, B be are disjoint sg-closed sets and U, V are disjoint sg-open sets such that $A \subseteq U$ and $B \subseteq V$. Since U and V are sg-open sets, $A \subseteq U$ and $B \subseteq V \Rightarrow A \subseteq sg(U)^o$ and $B \subseteq sg(V)^o$. Hence $sg(U)^o$ and $sg(V)^o$ are disjoint sg-open sets satisfying the axiom of sg-normality.

4.10. Theorem 4.7

The following are equivalent:

(i) X is sg-normal

(ii) For any pair of disjoint closed sets A and B, $\exists$ disjoint U; $V \in SGO(X)$ such that $A \subseteq U$ and $B \subseteq V$

(iii)For every closed set A and an open B containing A, $\exists$ $U \in SGO(X)$ such that $A \subseteq U \subseteq sgcl(U) \subseteq B$

(iv) For every closed set A and a sg-open B containing A, $\exists$ $U \in SGO(X)$ such that $A \subseteq U \subseteq sgcl(U) \subseteq (B)^o$

(v) For every $A \in SGC(X)$, and every $B \in \tau(X,A)$, $\exists$ $U \in SGO(X)$ such that $A \subseteq sgcl(A) \subseteq U \subseteq sgcl(U) \subseteq B$.

4.11. Theorem 4.8

The following are equivalent:

(i) X is sg-normal

(ii) For every $A \in SGC(X)$ and every sg-open set containing A, $\exists$ a sg-clopen set V such that $A \subseteq V \subseteq U$.

4.12. Theorem 4.9

Let X be an almost normal space and $F \cap A = \phi$ where F is regularly closed and A is sg-closed, then $\exists$ disjoint U; $V \in \tau$ such that $F \subseteq U$; $B \subseteq V$.

4.13. Theorem 4.10

X is almost normal iff for every disjoint sets F and A where F is regular closed and A is closed, $\exists$ disjoint sg-open sets in X such that $F \subseteq U$; $B \subseteq V$.

Proof: Necessity: Follows from the fact that every open set is sg-open.

Sufficiency: Let F, A be disjoint s.t. $F \in RC(X)$ and A is closed, $\exists$ disjoint U; $V \in SGO(X)$ s.t. $F \subseteq U$; $B \subseteq V$. Hence $F \subseteq U^o$; $B \subseteq V^o$, where U^o and V^o are disjoint open sets. Hence X is almost regular.

5. SG-R_i SPACES; i = 0,1:.

5.1. Definition 5.1

Let $x \in X$. Then

(i) sg-kernel of x is defined and denoted by $Ker_{\{sg\}}\{x\} = \cap\{U : U \in SGO(X) \text{ and } x \in U\}$

(ii)$Ker_{\{sg\}}F = \cap\{U : U \in SGO(X) \text{ and } F \subset U\}$

5.2. Lemma 5.1

Let $A \subset X$, then $Ker_{\{sg\}}\{A\} = \{x \in X: sgcl\{x\} \cap A \neq \phi.\}$

5.3. Lemma 5.2

Let $x \in X$. Then $y \in Ker_{\{sg\}}\{x\}$ iff $x \in sgcl\{y\}$.

Proof: Suppose that $y \notin Ker_{\{sg\}}\{x\}$. Then $\exists$ $V \in SGO(X)$ containing x such that $y \notin V$. Therefore we have $x \notin sgcl\{y\}$. The proof of converse part can be done similarly.

5.4. Lemma 5.3

For any points $x \neq y \in X$, the following are equivalent:

(i) $Ker_{\{sg\}}\{x\} \neq Ker_{\{sg\}}\{y\}$; (ii) $sgcl\{x\} \neq sgcl\{y\}$.

Proof: (i) $\Rightarrow$ (ii): Let $Ker_{\{sg\}}\{x\} \neq Ker_{\{sg\}}\{y\}$, then $\exists$ $z \in X$ such that $z \in Ker_{\{sg\}}\{x\}$ and $z \notin Ker_{\{sg\}}\{y\}$. From $z \in Ker_{\{sg\}}\{x\}$ it follows that $\{x\} \cap sgcl\{z\} \neq \phi \Rightarrow x \in sgcl\{z\}$. By $z \notin Ker_{\{sg\}}\{y\}$, we have $\{y\} \cap sgcl\{z\} = \phi$. Since $x \in sgcl\{z\}$, $sgcl\{x\} \subset sgcl\{z\}$ and $\{y\} \cap sgcl\{x\} = \phi$. Therefore $sgcl\{x\} \neq sgcl\{y\}$. Now $Ker_{\{sg\}}\{x\} \neq Ker_{\{sg\}}\{y\} \Rightarrow sgcl\{x\} \neq sgcl\{y\}$.

(ii) $\Rightarrow$ (i): If $sgcl\{x\} \neq sgcl\{y\}$. Then $\exists$ $z \in X$ such that $z \in sgcl\{x\}$ and $z \notin sgcl\{y\}$. Then $\exists$ a sg-open set containing z and therefore containing x but not y, namely, $y \notin Ker_{\{sg\}}\{x\}$. Hence $Ker_{\{sg\}}\{x\} \neq Ker_{\{sg\}}\{y\}$.

5.5. Definition 5.2

X is said to be

(i) sg-R_0 iff $sgcl\{x\} \subseteq G$ whenever $x \in G \in SGO(X)$.

(ii) weakly sg-R_0 iff $\cap sgcl\{x\} = \phi$.

(iii) sg-R_1 iff for $x,y \in X$ such that $sgcl\{x\} \neq sgcl\{y\}$ $\exists$ disjoint U; $V \in SGO(X)$ such that $sgcl\{x\} \subseteq U$ and $sgcl\{y\} \subseteq V$.

Example 5.1: Let X = {a, b, c, d} and

(i) $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}$ then X is sgR_0 and weakly sgR_0.

(ii) $\tau = \{\phi, \{a, b\}, \{a, b, c\}, X\}$ then X is sgR_1.

Remark 5.1:

(i) r-$R_i \Rightarrow R_i \Rightarrow$ g $R_i \Rightarrow$ sgR_i, i = 0, 1.

(ii) Every weakly-R_0 space is weakly sg R_0.

5.6. Lemma 5.1

Every sgR_0 space is weakly sgR_0.

Converse of the above Theorem is not true in general by the following Examples.

Example 5.2:

(i) Let X = {a, b, c, d} and $\tau = \{\phi, \{a\}, \{a, b\}, \{c, d\}, \{a, c, d\}, X\}$, then X is weakly sg$R_0$ but not sgR_0.

(ii) Let X = {a, b, c} and $\tau = \{\phi, \{a, b\}, X\}$, then X is sg$R_0$ and R_0.

5.7. Theorem 5.1

Every sg-regular space X is sg_2 and sg-R_0.
Proof: Let X be sg-regular and let $x \neq y \in X$. By Lemma 4.1, {x} is either sg-open or sg-closed. If {x} is sg-open, {x} is sg-open and hence sg-clopen. Thus {x} and X - {x} are separating sg-open sets. Similar argument, for {x} is sg-closed gives {x} and X - {x} are separating sg-closed sets. Thus X is sg_2 and sg-R_0.

5.8. Theorem 5.2

X is sg-R_0 iff given $x \neq y \in X$; $sgcl\{x\} \neq sgcl\{y\}$.
Proof: Let X be sg-R_0 and let let $x, \neq y \in X$. Suppose U is a sg-open set containing x but not y, then $y \in sgcl\{y\} \subset X\text{-}U$ and so $x \notin sgcl\{y\}$. Hence $sgcl\{x\} \neq sgcl\{y\}$.
Conversely, let $x, \neq y \in X$ such that $sgcl\{x\} \neq sgcl\{y\} \Rightarrow sgcl\{x\} \subset X\text{-}sgcl\{y\} = U$(say) a sg-open set in X. This is true for every sgcl{x}. Thus $\cap\, sgcl\{x\} \subseteq U$ where $x \in sgcl\{x\} \subseteq U \in SGO(X)$, which in turn implies $\cap\, sgcl\{x\} \subseteq U$ where $x \in U \in SGO(X)$. Hence X is sgR_0.

5.9. Theorem 5.3

X is weakly sgR_0 iff $Ker_{\{sg\}}\{x\} \neq X$ for any $x \in X$.
Proof: Let $x_0 \in X$ such that $ker_{\{sg\}}\{x_0\} = X$. This means that x_0 is not contained in any proper sg-open subset of X. Thus $x_0 \in sgcl\{x\}$ of every singleton set. Hence $x_0 \in \cap sgcl\{x\}$, a contradiction.
Conversely assume $Ker_{\{sg\}}\{x\} \neq X$ for any $x \in X$. If there is an $x_0 \in X$ such that $x_0 \in \cap\{sgcl\{x\}\}$, then every sg-open set containing x_0 must contain every point of X. Therefore, the unique sg-open set containing x_0 is X. Hence $Ker_{\{sg\}}\{x_0\} = X$, which is a contradiction. Thus X is weakly sg-R_0.

5.10. Theorem 5.4

The following statements are equivalent:
(i) X is sg-R_0 space.
(ii) For each $x \in X$, $sgcl\{x\} \subset Ker_{\{sg\}}\{x\}$
(iii)For any sg-closed set F and a point $x \notin F$, $\exists\, U \in SGO(X)$ such that $x \notin U$ and $F \subset U$.
(iv) Each sg-closed set F can be expressed as $F = \cap\{G$: G is sg-open and $F \subset G\}$.
(v) Each sg-open set G can be expressed as $G = \cup\{A$: A is sg-closed and $A \subset G\}$.
(vi) For each sg-closed set F, $x \notin F$ implies $sg\text{-}cl\{x\} \cap F = \phi$.
Proof: (i) ⇒ (ii) For any $x \in X$, we have $Ker_{\{sg\}}\{x\} = \cap\{U: U \in SGO(X)$ and $x \in U\}$. Since X is sg-R_0, each sg-open set containing x contains sgcl{x}. Hence $sgcl\{x\} \subset Ker_{\{sg\}}\{x\}$.
(ii) ⇒ (iii) Let $x \notin F \in SGC(X)$. Then for any $y \in F$; $sgcl\{y\} \subset F$ and so $x \notin sgcl\{y\} \Rightarrow y \notin sgcl\{x\}$ that is $\exists\, U_y \in SGO(X)$ such that $y \in U_y$ and $x \notin U_y\ \forall\, y \in F$. Let $U = \cup\{U_y: U_y \in SGO(X, y)$ and $x \notin U_y\}$. Then $U \in SGO(X)$ such that $x \notin U$ and $F \subset U$.
(iii) ⇒ (iv) Let $F \in SGO(X)$ and $N = \cap\{G$: G is sg-open and $F \subset G\}$. Then $F \subset N \to (1)$.
Let $x \notin F$, then by (iii) $\exists\, G \in SGO(X)$ such that $x \notin G$ and $F \subset G$.
Hence $x \notin N$ which implies $x \in N \Rightarrow x \in F$. Hence $N \subset F \to (2)$.
Therefore from (1) and (2), each sg-closed set $F = \cap\{G$: G is sg-open and $F \subset G\}$
(iv) ⇒ (v) obvious.
(v) ⇒ (vi) Let $x \notin F \in SGC(X)$. Then $X\text{-}F = G \in SGO(X, x)$. Then by (v), $G = \cup\{A$: A is sg-closed and $A \subset G\}$, and so $\exists\, M \in SGC(X)$ such that $x \in M \subset G$; and hence $sgcl\{x\} \subset G$ which implies $sgcl\{x\} \cap F = \phi$.
(vi) ⇒ (i) Let $x \in G \in SGO(X)$. Then $x \notin (X\text{-}G)$, which is a sg-closed set. Therefore by (vi) $sgcl\{x\} \cap (X\text{-}G) = \phi$, which implies that $sgcl\{x\} \subseteq G$. Thus X is sgR_0 space.

5.11. Theorem 5.5

Let f: $X \to Y$ be a sg-closed one-one function. If X is weakly sg-R_0, then so is Y.

5.12. Theorem 5.6

If X is weakly sg-R_0, then for every space Y, $X \times Y$ is weakly sg-R_0.
Proof: $\cap sgcl\{(x,y)\} \subseteq \cap\{sgcl\{x\} \times sgcl\{y\}\} = \cap[sgcl\{x\}] \times [sgcl\{y\}] \subseteq \phi \times Y = \phi$. Hence $X \times Y$ is sgR_0.

5.13. Corollary 5.1

(i) If X and Y are weakly sgR_0, then $X \times Y$ is weakly sgR_0.
(ii) If X and Y are sgR_0, then $X \times Y$ is weakly sgR_0.
(iii) If X is sgR_0 and Y are weakly R_0, then $X \times Y$ is weakly sgR_0.

5.14. Theorem 5.7

X is sgR_0 iff for any $x, y \in X$, $sgcl\{x\} \neq sgcl\{y\} \Rightarrow sgcl\{x\} \cap sgcl\{y\} = \phi$.
Proof: Let X is sgR_0 and $x, y \in X$ such that $sgcl\{x\} \neq sgcl\{y\}$ .Then $\exists\, z \in sgcl\{x\}$ such that $z \notin sgcl\{y\}$ (or $z \in sgcl\{y\}$) such that $z \notin sgcl\{x\}$. There exists $V \in SGO(X)$ such that $y \notin V$ and $z \in V$; hence $x \in V$. Therefore, $x \notin sgcl\{y\}$. Thus $x \in [sgcl\{y\}]^c \in SGO(X)$, which implies $sgcl\{x\} \subset [sgcl\{y\}]^c$ and $sgcl\{x\} \cap sgcl\{y\} = \phi$. The proof for otherwise is similar.
Sufficiency: Let $x \in V \in SGO(X)$. Let $y \notin V$, i.e., $y \in V^c$. Then $x \neq y$ and $x \notin sgcl\{y\}$. Hence $sgcl\{x\} \neq sgcl\{y\}$. But $sgcl\{x\} \cap sgcl\{y\} = \phi$. Hence $y \notin sgcl\{x\}$. Therefore $sgcl\{x\} \subset V$.

5.15. Theorem 5.8

X is sgR_0 iff for any $x, y \in X$, $Ker_{\{sg\}}\{x\} \neq Ker_{\{sg\}}\{y\} \Rightarrow Ker_{\{sg\}}\{x\} \cap Ker_{\{sg\}}\{y\} = \phi$.
Proof: If X is sgR_0, by Lemma 5.3 for any $x, y \in X$ if $Ker_{\{sg\}}\{x\} \neq Ker_{\{sg\}}\{y\}$ then $sgcl\{x\} \neq sgcl\{y\}$. Assume that $z \in Ker_{\{sg\}}\{x\} \cap Ker_{\{sg\}}\{y\}$. By $z \in Ker_{\{sg\}}\{x\}$ and Lemma 5.2, it follows that $x \in sgcl\{z\}$. Since $x \in sgcl\{z\}$, $sgcl\{x\} = sgcl\{z\}$. Similarly, we have $sgcl\{y\} = sgcl\{z\} = sgcl\{x\}$. This is a contradiction. Therefore, we have $Ker_{\{sg\}}\{x\} \cap Ker_{\{sg\}}\{y\} = \phi$.
Conversely, let $x, y \in X$, s.t. $sgcl\{x\} \neq sgcl\{y\}$, then by Lemma 5.3, $Ker_{\{sg\}}\{x\} \neq Ker_{\{sg\}}\{y\}$. By hypothesis $Ker_{\{sg\}}\{x\} \cap Ker_{\{sg\}}\{y\} = \phi$ which implies $sgcl\{x\} \cap sgcl\{y\} = \phi$. But $z \in sgcl\{x\}$ implies that $x \in Ker_{\{sg\}}\{z\}$ and hence $Ker_{\{sg\}}\{x\} \cap Ker_{\{sg\}}\{z\} \neq \phi$ Therefore by Theorem 5.7 X is a sgR_0 space.

5.16. Theorem 5.9

The following properties are equivalent:
(i) X is a sg-R_0 space.
(ii) For any $A \neq \phi$ and $G \in SGO(X)$ such that $A \cap G \neq \phi\ \exists\, F \in SGC(X)$such that $A \cap F \neq \phi$ and $F \subset G$.

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Proof: (i) $\Rightarrow$ (ii): Let $A \neq \phi$ and $G \in SGO(X)$ such that $A \cap G \neq \phi$. There exists $x \in A \cap G$. Since $x \in G \in SGO(X)$, sgcl$\{x\} \subset G$. Set F = sgcl$\{x\}$, then $F \in SGC(X)$, $F \subset G$ and $A \cap F \neq \phi$
(ii) $\Rightarrow$ (i): Let $G \in$ SGO(X) and $x \in$ G. By (2), sgcl$\{x\} \subset$ G. Hence X is sg-R_0.

5.17. Theorem 5.10

The following properties are equivalent:
(i) X is a sg-R_0 space;
(ii) $x \in$ sgcl$\{y\}$ iff $y \in$ sgcl$\{x\}$, for any points x and y in X.
Proof: (i) $\Rightarrow$ (ii): Assume X is sgR_0. Let $x \in$ sgcl$\{y\}$ and D be any sg-open set such that $y \in D$. Now by hypothesis, $x \in D$. Therefore, every sg-open set which contain y contains x. Hence $y \in$ sgcl$\{x\}$.
(ii) $\Rightarrow$ (i): Let U be a sg-open set and $x \in U$. If $y \notin$ U, then $x \notin$ sgcl$\{y\}$ and hence $y \notin$ sgcl$\{x\}$. This implies that sgcl$\{x\} \subset U$. Hence X is sgR_0.

5.18. Theorem 5.11

The following properties are equivalent:
(i) X is a sgR_0 space;
(ii) If F is sg-closed, then F = $Ker_{\{sg\}}(F)$;
(iii) If F is sg-closed and $x \in$ F, then $Ker_{\{sg\}}\{x\} \subseteq F$;
(iv) If $x \in$ X, then $Ker_{\{sg\}}\{x\} \subset$ sgcl$\{x\}$.
Proof: (i) $\Rightarrow$ (ii): Let $x \notin F \in SGC(X) \Rightarrow (X-F) \in SGO(X, x)$. For X is sg$R_0$, sgcl$(\{x\}) \subset (X-F)$. Thus sgcl$(\{x\}) \cap F = \phi$ and $x \notin Ker_{\{sg\}}(F)$. Hence $Ker_{\{sg\}}(F) = F$.
(ii) $\Rightarrow$ (iii): $A \subset B \Rightarrow Ker_{\{sg\}}(A) \subset Ker_{\{sg\}}(B)$. Therefore, by (2) $Ker_{\{sg\}}\{x\} \subset Ker_{\{sg\}}(F) = F$.
(iii) $\Rightarrow$ (iv): Since $x \in$ sgcl$\{x\}$ and sgcl$\{x\}$ is sg-closed, by (3) $Ker_{\{sg\}}\{x\} \subset$ sgcl$\{x\}$.
(iv) $\Rightarrow$ (i): Let $x \in$ sgcl$\{y\}$. Then by Lemma 5.2 $y \in Ker_{\{sg\}}\{x\}$. Since $x \in$ sgcl$\{x\}$ and sgcl$\{x\}$ is sg-closed, by (iv) we obtain $y \in Ker_{\{sg\}}\{x\} \subseteq$ sgcl$\{x\}$. Therefore $x \in$ sgcl$\{y\}$ implies $y \in$ sgcl$\{x\}$. The converse is obvious and X is sgR_0.

5.19. Corollary 5.2

The following properties are equivalent:
(i) X is sgR_0. (ii) sgcl$\{x\}$ = $Ker_{\{sg\}}\{x\} \forall x \in X$.
Proof: Straight forward from Theorems 5.4 and 5.11.

Recall that a filterbase F is called sg-convergent to a point x in X, if for any sg-open set U of X containing x, there exists $B \in$ F such that $B \subset$ U.

5.20. Lemma 5.4

Let x and y be any two points in X such that every net in X sg-converging to y sg-converges to x. Then $x \in$ sgcl$\{y\}$.
Proof: Suppose that x_n = y for each $n \in N$. Then $\{x_n\}_{n \in N}$ is a net in sgcl$\{(\{y\})\}$. Since $\{x_n\}_{n \in N}$ sg-converges to y, then $\{x_n\}_{n \in N}$ sg-converges to x and this implies that $x \in$ sgcl$\{y\}$.

5.21. Theorem 5.12

The following statements are equivalent:
(i) X is a sgR_0 space;
(ii) If x, $y \in X$, then $y \in$ sgcl$\{x\}$ iff every net in X sg-converging to y sg-converges to x.
Proof: (i) $\Rightarrow$ (ii): Let x, $y \in X$ such that $y \in$ sgcl$\{x\}$. Suppose that $\{x_\alpha\}_{\alpha \in I}$ is a net in X such that $\{x_\alpha\}_{\alpha \in I}$ sg-converges to y. Since $y \in$ sgcl$\{x\}$, by Thm. 5.7 we have sgcl$\{x\}$ = sgcl$\{y\}$. Therefore $x \in$ sgcl$\{y\}$. This means that $\{x_\alpha\}_{\alpha \in I}$ sg-converges to x.
Conversely, let x, $y \in$ X such that every net in X sg-converging to y sg-converges to x. Then $x \in$ sg-cl$\{y\}$[by 5.4]. By Thm. 5.7, we have sgcl$\{x\}$ = sgcl$\{y\}$. Therefore $y \in$ sgcl$\{x\}$.
(ii) $\Rightarrow$ (i): Let x, $y \in X$ such that sgcl$\{x\} \cap$ sgcl$\{y\} \neq \phi$. Let $z \in$ sgcl$\{x\} \cap$ sgcl$\{y\}$. So $\exists$ a net $\{x_\alpha\}_{\alpha \in I}$ in sgcl$\{x\}$ such that $\{x_\alpha\}_{\alpha \in I}$ sg-converges to z. Since $z \in$ sgcl$\{y\}$, then $\{x_\alpha\}_{\alpha \in I}$ sg-converges to y. It follows that $y \in$ sgcl$\{x\}$. Similarly we obtain $x \in$ sgcl$\{y\}$. Thus sgcl$\{x\}$ = sgcl$\{y\}$. Hence X is sgR_0.

5.22. Theorem 5.13

(i) Every subspace of sgR_1 space is again sgR_1.
(ii) Product of any two sgR_1 spaces is again sgR_1.
(iii) X is sgR_1 iff given $x \neq y \in$ X, sgcl$\{x\} \neq$ sgcl$\{y\}$.
(iv) Every sg$_2$ space is sgR_1.
(v) If X is sg-R_1, then X is sg-R_0.
(vi)X is sg-R_1 iff for x, $y \in$ X, $Ker_{\{sg\}}\{x\} \neq Ker_{\{sg\}}\{y\}$, $\exists$ disjoint U; $V \in SGO(X)$ such that sgcl$\{x\} \subset U$ and sgcl$\{y\} \subset V$.

The converse of (iv) is not true. However, we have the following result.

5.23. Theorem 5.14

Every sg$_1$ and sgR_1 space is sg$_2$.
Proof: Let $x \neq y \in X$. Since X is sg$_1$; $\{x\}$ and $\{y\}$ are sg-closed sets such that sgcl$\{x\} \neq$ sgcl$\{y\}$. Since X is sgR_1, there exists disjoint sg-open sets U and V such that $x \in U$; $y \in V$. Hence X is sg$_2$.

5.24. Corollary 5.3

X is sg$_2$ iff it is sgR_1 and sg$_1$.

Theorem 5.15: The following are equivalent
(i) X is sg-R_1. (ii) $\cap$sgcl$\{x\}$ = $\{x\}$. (iii)For any $x \in X$, $\cap$sg[nbds$\{x\}$] = $\{x\}$.
Proof: (i) $\Rightarrow$ (ii) Let $y \neq x \in X$ such that $y \in$ sgcl$\{x\}$. Since X is sgR_1, $\exists$ $U \in SGO(X)$ such that $y \in U$, $x \notin U$ or $x \in U$, $y \notin U$. In either case $y \notin$ sgcl$\{x\}$. Hence $\cap$sgcl$\{x\}$ = $\{x\}$.
(ii) $\Rightarrow$ (iii) If $y \neq x \in X$, then $x \notin \cap$sgcl$\{y\}$, so there is a sg-open set containing x but not y. Therefore $y \notin \cap$sg[nbds$\{x\}$]. Hence $\cap$sg[nbds$\{x\}$] = $\{x\}$.
(iii) $\Rightarrow$ (i) Let $x \neq y \in X$. by hypothesis, $y \notin \cap$sg[nbds$\{x\}$] and $x \notin \cap$sg[nbds$\{y\}$], which implies sgcl$\{x\} \neq$ sgcl$\{y\}$. Hence X is sg-R_1.

5.25. Theorem 5.16

The following are equivalent:

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(i) X is sg-R_1.
(ii) For each pair x, y$\in$X with sgcl{x} $\neq$ sgcl{y}, $\exists$ a sg-open, sg-closed set V s.t. x$\in$V and y$\notin$V, and
(iii)For each x $\neq$ y$\in$X with sgcl{x} $\neq$ sgcl{y}, $\exists$ a sg-continuous function *f:* X$\to$[0, 1] s.t. *f*(x) = 0 and *f*(y) = 1.
Proof: (i) $\Rightarrow$ (ii) Let x, y$\in$X with sgcl{x} $\neq$ sgcl{y}, $\exists$ disjoint U; W$\in$SGO(X) such that sgcl{x}$\subset$U and sgcl{y}$\subset$W and V = sgcl(U) is sg-open and sg-closed such that x$\in$V and y$\notin$V.
(ii) $\Rightarrow$ (iii) Let x, y$\in$ X with sgcl{x} $\neq$ sgcl{y}, and let V be sg-open and sg-closed such that x$\in$V and y$\notin$V. Then *f:* X$\to$ [0, 1] defined by *f*(z) = 0 if z$\in$V and *f*(z) = 1 if z$\notin$V satisfied the desired properties.
(iii) $\Rightarrow$ (i) Let x, y$\in$ X such that sgcl{x} $\neq$ sgcl{y}, let *f:* X$\to$ [0, 1] such that *f* is sg-continuous, *f*(x) = 0 and *f*(y) = 1. Then U = f^{-1}([0, 1/2)) and V = f^{-1}((1/2, 1]) are disjoint sg-open and sg-closed sets in X, such that sgcl{x}$\subset$U and sgcl{y}$\subset$V.

6. SG-C_i AND SG-D_i SPACES, i = 0,1,2:

6.1. Definition 6.1

X is said to be a
(i) sg-C_0 space if for each pair of distinct points x, y of X there exists a sg-open set G whose closure contains either of the point x or y.
(ii) sg-C_1 space if for each pair of distinct points x, y of X there exists a sg-open set G whose closure containing x but not y and a sg-open set H whose closure containing y but not x.
(iii)sg-C_2 space if for each pair of distinct points x, y of X there exists disjoint sg-open sets G and H such that G containing x but not y and H containing y but not x.
Note: sg-C_2 $\Rightarrow$ sg-C_1 $\Rightarrow$ sg-C_0. Converse need not be true in general as shown by.
Example 6.1: Let X = {a, b, c, d} and
(i) τ = {ϕ, {a}, {b}, {a, b}, X} then X is sg-C_i, i = 1, 2.
(ii) τ = {ϕ, {a}, {a, b}, {c, d}, {a, c, d}, X} then X is not sg-C_i, i = 0, 1, 2.

6.2. Theorem 6.1

(i) Every subspace of sg-C_i space is sg-C_i.
(ii) Every sg_i spaces is sg-C_i.
(iii)Product of sg-C_i spaces are sg-C_i.

6.3. Theorem 6.2

Let (X, τ) be any sg-C_i space and A $\neq$ ϕ $\subseteq$ X then A is sg-C_i iff (A, $\tau_{/A}$) is sg-C_i.

6.4. Theorem 6.3

(i) If X is sg-C_1 then each singleton set is sg-closed.
(ii)In an sg-C_1 space disjoint points of X has disjoint sg- closures.

6.5. Definition 6.2

A$\subset$ X is called a sg-Difference(Shortly sgD-set) set if there are two U, V$\in$ SGO(X) such that U $\neq$ X and A = U-V.

Clearly every sg-open set U different from X is a sgD-set if A = U and V = ϕ.

6.6. Definition 6.3

X is said to be a
(i) sg-D_0 if for any pair of distinct points x and y of X there exist a sgD-set in X containing x but not y or a sgD set in X containing y but not x.
(ii) sg-D_1 if for any pair of distinct points x and y in X there exist a sgD-set of X containing x but not y and a sgD-set in X containing y but not x.
(iii)sg-D_2 if for any pair of distinct points x and y of X there exists disjoint sgD-sets G and H in X containing x and y respectively.
Example 6.2: Let X = {a, b, c} and τ = {ϕ, {b}, {a, c}, X} then X is sgD_i, i = 0, 1, 2.
Remark 6.2: (i) If X is rT_i, then it is sg_i, i = 0, 1, 2 and converse is false.
(ii) If X is sg_i, then it is $sg_{\{i-1\}}$, i = 1, 2.
(iii) If X is sg_i, then it is sg-D_i , i = 0, 1, 2.
(iv) If X is sg-D_i, then it is sg-$D_{\{i-1\}}$, i = 1, 2.

6.7. Theorem 6.4

The following statements are true:
(i) X is sg-D_0 iff it is sg_0.
(ii) X is sg-D_1 iff it is sg-D_2.

Corollary 6.1: If X is sg-D_1, then it is sg_0.
Proof: Remark 6.1(iv) and Theorem 6.2(i)

6.8. Definition 6.4

A point x$\in$X which has X as the unique sg-neighborhood is called sg.c.c point.

6.9. Theorem 6.5

For an sg_0 space X the following are equivalent:
(i) X is sg-D_1; (ii) X has no sg.c.c point.
Proof: (i) $\Rightarrow$ (ii) Since X is sg-D_1, then each point x of X is contained in a sgD-set O = U - V and thus in U. By Definition U $\neq$ X. This implies that x is not a sg.c.c point.
(ii) $\Rightarrow$ (i) If X is sg_0, for each x $\neq$ y$\in$ X, at least one of them, x (say) has a sg-nbd U containing x but not y. Thus U $\neq$ X is a sgD-set. If X has no sg.c.c point, y is not a sg.c.c point, so there exists a sg-nbd V of y such that V $\neq$ X. Thus y$\in$(V-U) but not x and V-U is a sgD-set. Thus X is sg-D_1.

6.10. Corollary 6.2

A sg_0 space X is sg-D_1 iff there is a unique sg.c.c point in X.
Proof: Only uniqueness is sufficient to prove. If x_0 and y_0 are two sg.c.c points in X then since X is sg_0, at least one of x_0 and y_0 say x_0, has a sg-neighborhood U such that $x_0\in$ U and $y_0\notin$ U, hence U $\neq$ X, x_0 is not a sg.c.c point, a contradiction.

6.11. Definition 6.5

X is sg-symmetric if for x and y in X, $x \in sgcl\{y\}$ implies $y \in sgcl\{x\}$.

6.12. Theorem 6.6

X is sg-symmetric iff {x} is sgg-closed for each $x\in X$.

Proof: Assume that $x\in sgcl\{y\}$ but $y\notin sgcl\{x\}$. This means that $[sgcl\{x\}]^c$ contains y. This implies that $sgcl\{y\}\subset [sgcl\{x\}]^c$. Now $[sgcl\{x\}]^c$ contains x which is a contradiction.

Conversely, If $\{x\}\subset E\in SGO(X)$ but $sgcl\{x\}\not\subset E$, then sgcl{x} and E^c are not disjoint. Let y belongs to their intersection. Now we have $x\in sgcl\{y\}\subset E^c$ and $x\notin E$. But this is a contradiction.

6.13. Corollary 6.3

If X is a sg_1, then it is sg-symmetric.

Proof: Follows from theorem 2.2(ii), Remark 6.2 and theorem 6.6.

6.14. Corollary 6.4

The following are equivalent:

(i) X is sg-symmetric and sg_0

(ii) X is sg_1.

Proof: By Corollary 6.3 and Remark 6.1 it suffices to prove only (i) $\Rightarrow$ (ii). Let $x \neq y$ and by sg_0, we may assume that $x\in G_1\subset\{y\}^c$ for some $G_1\in SGO(X)$.Then $x\notin sgcl\{y\}$ and hence $y\notin sgcl\{x\}$. There exists a $G_2\in SGO(X)$ such that $y\in G_2 \subset\{x\}^c$ and X is a sg_1 space.

6.15. Theorem 6.7

For a sg-symmetric space X the following are equivalent:

(i) X is sg_0; (ii) X is sg-D_1; (iii) X is sg_1.

Proof: (i) $\Rightarrow$ (iii) Corollary 6.4 and **(iii) $\Rightarrow$ (ii) $\Rightarrow$ (i)** Remark 6.1.

6.16. Theorem 6.8

(i) If f:X$\rightarrow$ Y is a sg-irresolute surjective function and $E\in SGD(Y)$, $f^{-1}(E)\in SGD(X)$.

(ii) If Y is sg-D_1 and f: X $\rightarrow$ Y is sg-irresolute and bijective, then X is sg-D_1.

6.18. Theorem 6.9

X is sg-D_1 iff for each pair of distinct points x , y in X there exist a sg-irresolute surjective function f: X$\rightarrow$ Y, where Y is a sg-D_1 space such that f(x) and f(y) are distinct.

Proof: Necessity. For every $x \neq y\in X$, it suffices to take the identity function on X.

Sufficiency. Let $x \neq y\in X$. By hypothesis, $\exists$ a sg-irresolute, surjective function f: X onto a sg-D_1 space Y s.t. $f(x) \neq f(y)$. Therefore, $\exists$ disjoint G_x; $G_y\in SGD(Y)$ s.t. $f(x)\in G_x$ and $f(y)\in G_y$. Then by Theorem 6.8(i), $f^{-1}(G_x)$ and $f^{-1}(G_y)\in SGD(X)$ containing x and y respectively. Therefore X is sg-D_1 space.

6.19. Corollary 6.5

Let $\{X_\alpha/\alpha\in I\}$ be any family of topological spaces. If X_α is sg-D_1 for each $\alpha\in I$, so is ΠX_α. **Proof:** Let $(x_\alpha)\neq(y_\alpha)\in\Pi X_\alpha$. Then there exists an index $\beta\in I$ s.t. $x_\beta \neq y_\beta$. The natural projection $P_\beta:\Pi X_\alpha\rightarrow X_\beta$ is almost continuous and almost open and $P_\beta((x_\alpha)) = P_\beta((y_\alpha))$. Since X_β is sg-D_1, ΠX_α is sg-D_1.

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