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An enhanced variance estimator with power transformation

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ABSTRACT

In this study, we propose an enhanced variance estimator with power transformation to estimate the population variance of the study variable. The properties (Bias and Mean Square Error) of the proposed estimator were derived up to the first order of approximation using the Taylor series technique. The conditions for the proposed estimator to be better than existing related estimators are established. The empirical study was conducted using two natural populations and the result revealed that the proposed estimator is more efficient.

Keywords: Population variance, Sampling, Auxiliary variable, Mean Square Error, Bias

1. INTRODUCTION

The ratio method of estimation for the finite population was developed by (Isaki, 1983) in the presence of population variance of the auxiliary variable. This estimation procedure is used when there is a positive correlation between the study and the auxiliary variable. Auxiliary information is very useful for the enhancement of an estimator and that is why authors used it often during the development of estimators, in this case (Audu et al., 2016; Singh et al., 2007; Singh et al., 2011; Ahmed et al., 2003; Gupta and Shabbir, 2008; Singh and Solanki, 2013; Solanki and Singh, 2013; Adejumobi et al., 2022; Upadhyaya and Singh, 1999; Yadav and Kadilar, 2013; Kadilar and Cingi, 2006; Subramani and Kumarapandiyan, 2012; Adejumobi and Yunusa, 2022).

Consider $\varphi=(1,2,\ldots,N)$ as a population of size N and X, Y be auxiliary and study variable having values $(X_i,Y_i)\in R>0$ on the i[th] unit of $\varphi_i\,(1<i\leq N)$, we assume a positive correlation $\rho>0$ between the auxiliary and study variables. Let Considering be the finite population variance of X and Y respectively s_x^2 and s_y^2 be the sample variance based on the random sample of size n chosen without replacement.

Let us define the following relation for error terms as

$$e_0=\frac{s_y^2-S_y^2}{S_y^2},\quad e_1=\frac{s_x^2-S_x^2}{S_x^2}\ \text{such that}$$

$$s_y^2=S_y^2(1+e_0),\quad s_x^2=S_x^2(1+e_1)$$

$$E(e_0)=E(e_1)=0,\quad E(e_0^2)=\gamma(\lambda_{40}-1),\quad E(e_1^2)=\gamma(\lambda_{04}-1),$$

$$E(e_0e_1)=\gamma(\lambda_{22}-1),\quad \gamma=\frac{1}{n}$$

In this study, we proposed an enhanced variance estimator with power transformation for the estimation of population variance of the study variable.

Estimators in literature

The usual unbiased variance estimator is given by:

$$t_0=s_y^2=(n-1)^{-1}\sum_{i=1}^{n}(y_i-\bar{y})^2 \tag{1}$$

The variance is given as:

$$\operatorname{var}(t_0)=\gamma S_y^4(\lambda_{40}-1) \tag{2}$$

Isaki, (1983) proposed the ratio estimator for population variance as

$$t_1=s_y^2\left(\frac{S_x^2}{s_x^2}\right) \tag{3}$$

The mean square error is given by

$$MSE(t_1)=\gamma S_y^4\left[(\lambda_{40}-1)+(\lambda_{04}-1)-2(\lambda_{22}-1)\right] \tag{4}$$

Singh et al., (2009) proposed a ratio-type exponential estimator for the estimation of finite population variance as

$$MSE(t_2)=\gamma S_y^4\left[(\lambda_{40}-1)+\frac{(\lambda_{04}-1)}{4}-(\lambda_{22}-1)\right] \tag{5}$$

2. MATERIALS AND METHODS

Proposed estimator

Having studied the estimators in literature and following estimation strategy of (Yunusa et al., 2021) population mean estimator, we proposed an enhanced variance estimator in the form

$$t_{AM}=2^{-1}s_y^2\left[\left(\frac{s_x^2}{S_x^2}\right)^{\alpha}+\exp\left(\frac{S_x^2-s_x^2}{S_x^2+s_x^2}\right)\right] \tag{6}$$

where, α is the constant to be determined.

Express t_{AM} in terms of error $e_i^{'s}\ (i=0,1)$ we have

$$t_{AM}=2^{-1}S_y^2(1+e_0)\left[\left(\frac{S_x^2(1+e_1)}{S_x^2}\right)^{\alpha}+\exp\left(\frac{S_x^2-S_x^2(1+e_1)}{S_x^2+S_x^2(1+e_1)}\right)\right] \tag{7}$$

Simplify (7) we have

$$t_{AM}=2^{-1}S_y^2(1+e_0)\left[(1+e_1)^{\alpha}+1-\frac{e_1}{2}+\frac{3e_1^2}{8}\right] \tag{8}$$

$$t_{AM}=\frac{S_y^2(1+e_0)}{2}\left[1+\alpha e_1+\frac{\alpha(\alpha-1)e_1^2}{2}+1-\frac{e_1}{2}+\frac{3e_1^2}{8}\right] \tag{9}$$

$$t_{AM}=S_y^2(1+e_0)\left[1+\left(\alpha-\frac{1}{2}\right)e_1+\left(\frac{\alpha^2-\alpha}{2}+\frac{3}{8}\right)e_1^2\right] \tag{10}$$

Subtract S_y^2 from both sides and simplify to first order of approximation, we have

$$t_{AM}-S_y^2=S_y^2\left[e_0+\left(\alpha-\frac{1}{2}\right)e_1+\left(\alpha-\frac{1}{2}\right)e_0e_1+\left(\frac{\alpha^2-\alpha}{2}+\frac{3}{8}\right)e_1^2\right] \tag{11}$$

$$Bias\left(t_{AM}\right)=S_y^2\gamma\left[\left(\alpha-\frac{1}{2}\right)\left(\lambda_{22}-1\right)+\left(\frac{\alpha^2-\alpha}{2}+\frac{3}{8}\right)\left(\lambda_{04}-1\right)\right] \tag{12}$$

Squaring and taking expectation of equation (11), we obtain the mean square error up to first order of approximation as:

$$MSE\left(t_{AM}\right)=\gamma S_y^4\left[\left(\lambda_{40}-1\right)+\left(\alpha-\frac{1}{2}\right)^2\left(\lambda_{04}-1\right)+2\left(\alpha-\frac{1}{2}\right)\left(\lambda_{22}-1\right)\right] \tag{13}$$

Differentiating (13) with respect to α, equate to zero and solve for α, we obtain

$$\alpha=\frac{1}{2}-\frac{\left(\lambda_{22}-1\right)}{\left(\lambda_{04}-1\right)} \tag{14}$$

Substituting (14) into (13) to obtain the minimum mean square error (MSE) of t_{AM} is

$$MSE\left(t_{AM}\right)_{\min}=\gamma S_y^4\left[\left(\lambda_{40}-1\right)-\frac{\left(\lambda_{22}-1\right)^2}{\left(\lambda_{04}-1\right)}\right] \tag{15}$$

Efficiency Comparison

In this section, the estimator t_{AM} is more efficient than t_0, t_1 and t_2, if the following conditions are satisfied

$MSE\left(t_{AM}\right)_{\min}<\operatorname{var}\left(t_0\right)$ if

$$\frac{\left(\lambda_{22}-1\right)^2}{\left(\lambda_{04}-1\right)}>0 \tag{16}$$

$MSE\left(t_{AM}\right)_{\min}<MSE\left(t_1\right)$ if

$$\left(\lambda_{04}-1\right)>\left(\lambda_{22}-1\right)\left(2-\frac{\left(\lambda_{22}-1\right)}{\left(\lambda_{04}-1\right)}\right) \tag{17}$$

$MSE\left(t_{AM}\right)_{\min}<MSE\left(t_2\right)$ if

$$\left(\lambda_{04}-1\right)>\left(\lambda_{22}-1\right)\left(1-\frac{\left(\lambda_{22}-1\right)}{\left(\lambda_{04}-1\right)}\right) \tag{18}$$

3. EMPIRICAL STUDY

In this section, in order to elucidate the performance of the proposed estimator efficiency over t_0, t_1 and t_2, we use two natural populations.

Table 2 shows the mean square errors (MSEs) and percentage relative efficiencies (PREs) of the proposed estimator and existing related estimators and. It can be observed that the proposed estimator has the minimum mean square error (min MSE) and higher percentage relative efficiency. This reveals that it is more efficient than its counterparts.

Table 1 Dataset used for the empirical study

Parameters	Population 1	Population 2
N	34	69
n	15	40
$\bar{X}$	208.88	4591.07
$\bar{Y}$	199.44	4514.89
C_x	0.72	1.38
C_y	0.75	1.35
λ_{40}	3.6161	7.66
λ_{04}	2.8266	9.84
λ_{22}	3.0133	8.19
ρ_{yx}	0.98	0.96

Table 2 Mean Square Error of the proposed and existing estimators

Estimators	Population 1		Population 2	
	MSE	PRE	MSE	PRE
t_0	873086634.77	100.00	2.29794E+14	100.00
t_1	13886748.57	628.72	3.8644E+13	594.64
t_2	19758683.55	441.87	2.4363E+13	396.43
t_3^d	13249882.6	658.94	2.80177E+13	820.17

4. CONCLUSION

In this study, we proposed an enhanced variance estimator and derived its properties (Bias and MSE) as well as efficiency conditions of t_{AM} being efficient than t_0, t_1 and t_2. The empirical study revealed that t_{AM} is more efficient with the evident of minimum MSE and higher PRE. Hence, the proposed estimator t_{AM} is recommended in practical use.

Informed consent
Not applicable.

Ethical approval
Not applicable.

Conflicts of interests
The authors declare that there are no conflicts of interests.

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Data and materials availability
All data associated with this study are present in the paper.

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