DISCOVERY

Modified novel family of log exponential estimators utilizing auxiliary attributes

Mojeed Abiodun Yunusa[1], Awwal Adejumobi[2*], Ahmed Audu[1]

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Author Affiliation:
[1]Department of Statistics, Usmanu Danfodiyo University, Sokoto, Nigeria
[2]Department of Mathematics, Kebbi State University of Science and Technology, Aliero, Nigeria

[*]**Corresponding author**
Department of Mathematics, Kebbi State University of Science and Technology, Aliero
Nigeria
Email: awwaladejumobi@gmail.com

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ABSTRACT

In this paper, we implied a modified novel family of log-exponential ratio estimators for the estimation of the population mean of the variable of interest in the presence of auxiliary attribute. We acquire the biases and mean square errors (MSEs) of the suggested estimators as well as the efficiency conditions for which the proposed estimators are more efficient theoretically. Empirical study was conducted using two datasets and the results revealed that the proposed estimators are more efficient.

Keywords: Sampling, Coefficient of Variation, Bias, Mean Square Error, Auxiliary Attribute

1. INTRODUCTION

One of the main aims in a sample survey is the estimation of parameters of the population like population mean, variance, standard deviation, coefficient of variation, etc. These parameters are estimated with the use of a statistic or estimator. One of some is the sample mean estimator which is an unbiased estimator which is an unbiased estimator of population mean. This estimator demands no use of auxiliary information. Information can either be qualitative or quantitative when the auxiliary information is in the form of an attribute, that is if the auxiliary information is not available in the form of quantitative. Then the estimator uses information on the attribute for the purpose of precision. The information in the form of an attribute can be such as the height of a person as a function of sex, that is, male or female. The efficiency of a dog is a function of a particular breed of that dog. Many authors have developed estimators when the information is the form of quantitative like Perri, (2007), Singh and Kumar, (2011), Lu, (2013), Audu and Adewara, (2017) and Yunusa et al., (2021). Also, authors have developed estimators in the presence of auxiliary attributes and they include Naik and Gupta, (1996), Singh et al., (2007), Singh, (2008), Singh and Solanki, (2012), Sharma et al., (2013), Sharma and Singh, (2015), Zaman and Kadilar, (2019), Zaman, (2020), Audu et al., (2021) and Adejumobi et al., (2022).

In this paper, some modified novel family of log-exponential estimators are proposed to estimate the population mean for the interest using information on auxiliary attribute.

Let y_i be i^{th} population characteristic ϕ_i and φ_i is the case of possessing certain attribute. If i^{th} unit has the desired characteristic, it assumes 1 and 0 otherwise, that is;

$$\varphi_i = \begin{cases} 1 & \text{if } i^{th} \text{ unit of the population assumes the attribute} \\ 0, & \text{elsewhere} \end{cases}$$

Let $G = \sum_{i=1}^{N} \varphi_i$ and $g = \sum_{i=1}^{n} \phi_i$ be the total count of the units that assume certain attributes in the population and the sample, respectively. And $P = \frac{G}{N}$ and $p = \frac{g}{n}$ are the ratio of these units respectively;

$C_y = \frac{S_y}{\bar{Y}}$, $C_\phi = \frac{S_\phi}{P}$, $\rho_{y\phi} = \frac{S_{y\phi}}{S_y S_\phi}$, $\beta_{2(\phi)} = \frac{\mu_4}{\delta^4}$ are the population coefficient of variation of study variable and auxiliary attribute, bi-serial correlation between study variable, attribute and kurtosis. The variance of the sample mean $\bar{y}$ is;

$$\operatorname{var}(\bar{y}) = \gamma C_y^2 \tag{1}$$

Existing Estimators

Naik and Gupta, (1996) presented ratio and product estimators of the population mean of the study variable Y in the presence of auxiliary attribute as:

$$t_{NG1} = \bar{y}\frac{P}{p} \tag{2}$$

$$t_{NG2} = \bar{y}\frac{p}{P} \tag{3}$$

The Biases and Mean Square Error of t_{NG1} and t_{NG2} are given by:

$$Bias(t_{NG1}) = \bar{Y}\gamma\left(C_\phi^2 - \rho_{y\phi}C_y C_\phi\right) \tag{4}$$

$$Bias(t_{NG2}) = \bar{Y}\gamma\rho_{y\phi}C_y C_\phi \tag{5}$$

$$MSE(t_{NG1}) = \gamma\bar{Y}^2\left(C_y^2 - 2\rho_{y\phi}C_y C_\phi + C_\phi^2\right) \tag{6}$$

$$MSE(t_{NG2}) = \gamma\bar{Y}^2\left(C_y^2 + 2\rho_{y\phi}C_y C_\phi + C_\phi^2\right) \tag{7}$$

Singh et al., (2007b) pursuing Bahl and Tuteja, (1991), propose exponential ratio and product type estimators in the presence of auxiliary attributes as:

$$t_{S1} = \bar{y}\exp\left(\frac{P-p}{P+p}\right) \tag{8}$$

$$t_{S2} = \bar{y}\exp\left(\frac{p-P}{p+P}\right) \tag{9}$$

The biases and MSEs of these estimators are respectively given by:

$$Bias\left(t_{S1}\right) = \gamma\bar{Y}\left(\frac{3}{8}C_\phi^2 - \frac{1}{2}\rho_{y\phi}C_y C_\phi\right) \tag{10}$$

$$Bias\left(t_{S2}\right) = \gamma\bar{Y}\left(\frac{1}{2}\rho_{y\phi}C_y C_\phi - \frac{1}{8}C_\phi^2\right) \tag{11}$$

$$MSE(t_{S1}) = \gamma\bar{Y}^2\left(C_y^2 + \frac{C_\phi^2}{4} - \rho_{y\phi}C_y C_\phi\right) \tag{12}$$

$$MSE(t_{S2}) = \gamma\bar{Y}^2\left(C_y^2 + \frac{C_\phi^2}{4} + \rho_{y\phi}C_yC_\phi\right) \tag{13}$$

where $\bar{Y} = \frac{1}{N}\sum_{i=1}^{N} Y_i$ and $\gamma = \left(\frac{1}{n} - \frac{1}{N}\right)$

Zaman and Kadilar, (2019a) proposed the family of exponential ratio type estimators in the presence of auxiliary attribute as;

$$t_i = \bar{y}\exp\left(\frac{(kP+l)-(kp+l)}{(kP+l)+(kp+l)}\right), \qquad i = 1,2,\ldots,7 \tag{14}$$

where $k \neq 0$ and l are either real number or function of the unknown parameters of the attribute C_ϕ, $\beta_{2(\phi)}$, $\rho_{y\phi}$

The Bias and MSE of the estimator are given as follows:

$$Bias(t_i) = \gamma\bar{Y}\left(\theta_i^2C_\phi^2 - \theta_i\rho_{y\phi}C_yC_\phi\right) \quad i = 1,2,\ldots.,7 \tag{15}$$

$$MSE(t_i) = \gamma\bar{Y}^2\left(C_y^2 + \theta_1^2C_\phi^2 - 2\theta_i\rho_{y\phi}C_yC_\phi\right) \quad i = 1,2\ldots.,7 \tag{16}$$

where, $\theta_1 = \frac{P}{2(P+\beta_{2(\phi)})}$, $\theta_2 = \frac{P}{2(P+C_\phi)}$, $\theta_3 = \frac{P}{2(P+\rho_{\phi y})}$; $\theta_4 = \frac{\beta_{2(\phi)}P}{2(\beta_{2(\phi)}P+C_\phi)}$; $\theta_5 = \frac{C_\phi P}{2(C_\phi P+\beta_{2(\phi)})}$;

$\theta_6 = \frac{\rho_{y\phi}P}{2(\rho_{y\phi}P+C_\phi)}$; $\theta_7 = \frac{\rho_{y\phi}P}{2(\rho_{y\phi}P+\beta_{2(\phi)})}$

2. METHODOLOGY

Proposed Estimators

Having studied the work of Zaman and Kadilar, (2019), we proposed the following modified novel family of log-exponential type estimators for $\bar{Y}$ as:

$$T_{m1} = \bar{y}\left[\left(\frac{P}{p-P}\right)\log\left(\frac{p}{P}\right)\exp\left(\frac{P-p}{P+p+2\beta_{2(\phi)}}\right)\right]^{1/4} \tag{17}$$

$$T_{m2} = \bar{y}\left[\left(\frac{P}{p-P}\right)\log\left(\frac{p}{P}\right)\exp\left(\frac{P-p}{P+p+2C_\phi}\right)\right]^{1/4} \tag{18}$$

$$T_{m3} = \bar{y}\left[\left(\frac{P}{p-P}\right)\log\left(\frac{p}{P}\right)\exp\left(\frac{P-p}{P+p+2\rho_{y\phi}}\right)\right]^{1/4} \tag{19}$$

$$T_{m4} = \bar{y}\left[\left(\frac{P}{p-P}\right)\log\left(\frac{p}{P}\right)\exp\left(\frac{\beta_{2(\phi)}(P-p)}{\beta_{2(\phi)}(P+p)+2C_\phi}\right)\right]^{1/4} \tag{20}$$

$$T_{m5} = \bar{y}\left[\left(\frac{P}{p-P}\right)\log\left(\frac{p}{P}\right)\exp\left(\frac{C_\phi(P-p)}{C_\phi(P+p)+2\beta_{2(\phi)}}\right)\right]^{1/4} \tag{21}$$

$$T_{m6} = \bar{y}\left[\left(\frac{P}{p-P}\right)\log\left(\frac{p}{P}\right)\exp\left(\frac{\rho_{y\phi}(P-p)}{\rho_{y\phi}(P+p)+2C_\phi}\right)\right]^{1/4} \tag{22}$$

$$T_{m7}=\bar{y}\left[\left(\frac{P}{p-P}\right)\log\left(\frac{p}{P}\right)\exp\left(\frac{\rho_{y\phi}(P-p)}{\rho_{y\phi}(P+p)+2\beta_{2(\phi)}}\right)\right]^{1/4} \tag{23}$$

The above estimators can be written in general form as:

$$T_{mi}=\bar{y}\left[\left(\frac{P}{p-P}\right)\log\left(\frac{p}{P}\right)\exp\left(\frac{(kP+l)-(kp+l)}{(kP+l)+(kp+l)}\right)\right]^{1/4}, \quad i=1,2,\ldots,7 \tag{24}$$

To obtain the bias and mean square error expressions of these estimators, we define the following relations:

$$e_0=\frac{\bar{y}}{\bar{Y}}-1,\ \ e_1=\frac{p}{P}-1 \text{ such that } \bar{y}=\bar{Y}(1+e_0),\quad p=P(1+e_1)$$

$$E(e_0)=E(e_1)=0,\ E(e_0^2)=\gamma C_y^2,\ E(e_1^2)=\gamma C_\phi^2 \text{ and } E(e_0e_1)=\gamma\rho_{y\phi}C_yC_\phi$$

Expressing equation (24) in terms of $e_k,(k=0,1)$ and reduce terms up to the first order of approximation.

$$T_{mi}=\bar{Y}(1+e_0)\left[\left(\frac{P}{P(1+e_1)-P}\right)\log\left(\frac{P(1+e_1)}{P}\right)\exp\left(-\theta_ie_1(1+\theta_ie_1)^{-1}\right)\right]^{1/4} \tag{25}$$

where, $\theta_i=\dfrac{kP}{2(kP+l)},\quad i=1,2,\ldots,7$

$$T_{mi}=\bar{Y}(1+e_0)\left[\frac{1}{e_1}\left(e_1-\frac{e_1^2}{2}\right)\exp\left(-\theta_ie_1+\theta_i^2e_1^2\right)\right]^{1/4} \tag{26}$$

$$T_{mi}=\bar{Y}(1+e_0)\left[\left(1-\frac{e_1}{2}\right)\left(1-\theta_ie_1+\theta_i^2e_1^2+\frac{\theta_i^2e_1^2}{2}\right)\right]^{1/4} \tag{27}$$

$$T_{mi}=\bar{Y}(1+e_0)\left[\left(1-\frac{e_1}{2}\right)\left(1-\theta_ie_1+\frac{3\theta_i^2e_1^2}{2}\right)\right]^{1/4} \tag{28}$$

$$T_{mi}=\bar{Y}(1+e_0)\left[1-\left(\theta_i+\frac{1}{2}\right)e_1+\frac{(3\theta_i^2+\theta_i)}{2}e_1^2\right]^{1/4} \tag{29}$$

$$T_{mi}=\bar{Y}(1+e_0)\left[1-\frac{1}{4}\left(\theta_i+\frac{1}{2}\right)e_1+\frac{1}{32}\left(45\theta_i^2+13\theta_i-\frac{3}{4}\right)e_1^2\right] \tag{30}$$

$$T_{mi}=\bar{Y}\left[1+e_0-\frac{1}{4}\left(\theta_i+\frac{1}{2}\right)e_1+\frac{1}{32}\left(45\theta_i^2+13\theta_i-\frac{3}{4}\right)e_1^2-\frac{1}{4}\left(\theta_i+\frac{1}{2}\right)e_0e_1\right] \tag{31}$$

By subtracting $\bar{Y}$ from both sides, we obtain

$$T_{mi}-\bar{Y}=\bar{Y}\left[e_0-\frac{1}{4}\left(\theta_i+\frac{1}{2}\right)e_1+\frac{1}{32}\left(45\theta_i^2+13\theta_i-\frac{3}{4}\right)e_1^2-\frac{1}{4}\left(\theta_i+\frac{1}{2}\right)e_0e_1\right] \tag{32}$$

Taking the expectation of equation (32), we obtain the bias of the estimator as:

$$Bias(T_{mi})=\gamma\bar{Y}\left[\frac{1}{32}\left(45\theta_i^2+13\theta_i-\frac{3}{4}\right)C_\phi^2-\frac{1}{4}\left(\theta_i+\frac{1}{2}\right)\rho_{y\phi}C_yC_\phi\right] \tag{33}$$

Squaring and taking the expectation of equation (32), we obtain the mean square error of the estimator as:

$$MSE(T_{mi})=\gamma\bar{Y}\left[C_y^2+\frac{1}{16}\left(\theta_i+\frac{1}{2}\right)^2C_\phi^2-\frac{1}{2}\left(\theta_i+\frac{1}{2}\right)\rho_{y\phi}C_yC_\phi\right] \tag{34}$$

Efficiency Comparison

In this section, the mean square error of the existing estimators is compared with reducing $T_i^{'s}$ mean square error for efficiency.

i. $T_i^{'s}$ is more efficient than $\bar{y}$ if

$$MSE\left(T_i\right)-\operatorname{var}\left(\bar{y}\right)<0$$

$$\theta_i<8\rho_{y\phi}C_y-\frac{1}{2} \tag{35}$$

ii. $T_i^{'s}$ is more efficient than t_{NG1} if

$$MSE\left(T_i\right)-MSE\left(t_{NG1}\right)<0$$

$$\left(4\theta_i^2+4\theta_i-15\right)C_\phi<16\left(2\theta_i-3\right)\rho_{y\phi}C_y \tag{36}$$

iii. $T_i^{'s}$ is more efficient than t_{NG2} if

$$MSE\left(T_i\right)-MSE\left(t_{NG2}\right)<0$$

$$\left(4\theta_i^2+4\theta_i-15\right)C_\phi<16\left(2\theta_i+5\right)\rho_{y\phi}C_y \tag{37}$$

iv. $T_i^{'s}$ is more efficient than t_i if

$$MSE\left(T_i\right)-MSE\left(t_i\right)<0$$

$$\left(4\theta_i^2+4\theta_i-15\right)C_\phi<16\left(2\theta_i-7\right)\rho_{y\phi}C_y \tag{38}$$

Empirical Study

In this section, an empirical study will be conducted to enlighten the enactment of the proposed estimators over the existing ones.

Population 1

$$N=89,\ n=20,\ \bar{Y}=3.3596,\ P=0.1236,\ \beta_{2(\phi)}=3.492$$

$$C_y=0.6008,\ \rho_{y\phi}=0.766,\ C_\phi=2.6779$$

Population 2

$$N=111,\ n=30,\ \bar{Y}=29.279,\ P=0.117,\ \beta_{2(\phi)}=3.898$$

$$C_y=0.872,\ \rho_{y\phi}=0.797,\ C_\phi=2.758$$

Table 1 MSEs and PREs of the proposed and existing estimators

Estimators	Population 1		Population 2	
	MSE	PRE	MSE	PRE
$\bar{y}$	0.1579	100.00	15.8557	100.00
t_{NG1}	2.2168	7.1226	94.5320	16.7728
t_{NG2}	4.3742	3.6098	254.4077	6.2324
t_{S1}	0.4030	38.1811	15.5403	102.0300
t_{S2}	1.4817	10.6567	95.4782	16.6066
t_1	0.1404	112.4644	14.7247	107.6810
t_2	0.1357	116.3596	14.2948	110.9194

t_3	0.0981	160.9582	11.3891	139.2182
t_4	0.0982	160.7943	10.9827	144.3700
t_5	0.1171	134.8420	13.0318	121.6693
t_6	0.1404	112.4644	14.5909	108.6684
t_7	0.1442	109.5007	14.9436	106.1036
T_{m1}	0.070953	222.5417	8.197212	193.4280
T_{m2}	0.070626	223.5721	8.141027	194.7629
T_{m3}	0.067992	232.2332	7.737865	204.9105
T_{m4}	0.067995	232.2230	7.677221	206.5292
T_{m5}	0.069357	227.6627	7.971349	198.9086
T_{m6}	0.070954	222.5385	8.179812	193.8394
T_{m7}	0.071216	221.7198	8.225563	192.7620

From table 1, it can be observed that the proposed estimator $T_{mi},\ i=1,2,...,7$ outmatch the existing estimators considered in the study with the evidence of minimum mean square error (MSE) and higher percentage relative efficiency (PRE).

3. CONCLUSION

In this study, we propose some modified novel family of exponential estimators in the presence of an auxiliary variable. The properties (biases and mean square error) of the estimator were derived. The empirical study revealed that the proposed estimators are more efficient. With this conclusion, we recommend the use of the estimator in practical situation. In forthcoming studies, we hope to extend the proposed estimator attribute presented in this paper to stratified and cluster sampling.

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Informed consent

Not applicable.

Ethical approval

Not applicable.

Conflicts of interests

The authors declare that there are no conflicts of interests.

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Data and materials availability

All data associated with this study are present in the paper.

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