DISCOVERY

57(303), March, 2021

Modeling blood flow with unilateral stenosis based on interpenetration model

Umurdin Dalabaev

To Cite:
Umurdin Dalabaev. Modeling blood flow with unilateral stenosis based on interpenetration model. *Discovery*, 2021, 57(303), 225-229

Author Affiliation:
University of World Economy and Diplomacy, Uzbekistan
E-mail: udalabaev@mail.ru

Peer-Review History
Received: 19 December 2020
Reviewed & Revised: 20/December/2020 to 30/January/2021
Accepted: 31 January 2021
Published: March 2021

Peer-review
External peer-review was done through double-blind method.

ABSTRACT

The paper deals with the numerical simulation of the flow of Newtonian fluid by the control volume method, which simulates the flow of blood in a blood vessel with stenosis. An interpenetrating model of two-phase media is used to describe the process. Based on this model, under appropriate simplifying assumptions, we obtain the Navier-Stokes equations in the free (outside the stenosis region) zone, and in the stenotic region, we obtain a system of equations generalizing the filtration equation. Stenosis is viewed as a porous medium. The influence of the Reynolds number and the degree of stenosis on the nature of the flow on the behavior of blood is investigated.

Keywords: Blood flow; stenosis; Navier-Stokes equation

INTRODUCTION

Arterial stenosis occurs due to the deposition of cholesterol on its walls. Stenoses restrict blood flow. This change in blood flow caused by stenosis can lead to the development of arterial disease and arterial deformity.

With stenosis of the artery, the nature of the blood flow in the lower part of the artery changes significantly. These changes are highly dependent on the degree of stenosis. Due to the presence of moderate or severe stenosis in the lower course, the course is severely disrupted and, therefore, the course becomes unevenly complex.

Many studies have been investigated to understand the nature of blood flow disturbances in stenosis. Both experimental and numerical methods have been used by various researchers to determine the development of stenosis and its magnitude [1-4].

It is assumed here that the movement in the vessels is laminar. To simulate the flow of blood in vessels with stenosis, we use an interpenetrating model. We consider stenosis as the second phase in an interpenetrating model for describing heterogeneous media with a fixed zero velocity. The area free from stenosis, the blood flow is described by the Navier-Stokes equations. We represent stenosis as a porous medium.

Flow description model

Consider an interpenetrating model describing the flows of two-phase media in a Cartesian coordinate system (two-dimensional problem) [6-7]:

$$\rho_i \frac{\partial u_i^j}{\partial t} + \sum_{k=1}^{2} \rho_i u_i^k \frac{\partial u_i^j}{\partial x_k} = -f_i \frac{\partial p}{\partial x_j} + \mu_i \sum_{k=1}^{2} \left(\frac{1}{3}\delta_j^k + 1 \right) \frac{\partial}{\partial x_k} \left(f \frac{\partial u_i^j}{\partial x_k} \right) + \\ + \mu_i \sum_{k=1}^{2} \left(1 - \frac{5}{3}\delta_j^k \right) \frac{\partial}{\partial x_k} \left(f_i \frac{\partial u_i^{3-j}}{\partial x_{3-k}} \right) + K\left(u_{3-i}^j - u_i^j \right) + \rho_i g_i^j, \tag{1}$$

$$\frac{\partial \rho_i}{\partial t} + \sum_{k=1}^{2} \frac{\partial \left(\rho_i u_i^k \right)}{\partial x_k} = 0, \tag{2}$$

$$f_1 + f_2 = 1. \tag{3}$$

Here, u_i^j – the j-th velocity component, p – pressure, f_i – volumetric concentration of the i – phase ($f_i = \rho_i / \rho_{0i}$), δ_j^k – is the Kronecker symbol, μ_i – the viscosity of the i – phase, K – the phase interaction coefficient, ρ_i, ρ_{0i} – are the reduced and true density of the i – phase (ρ_{0i} – const), g_i^j – the j-th component of the i – phase mass force.

If we represent in the equations that the second phase is stationary, we obtain a system of equations describing the flow of a liquid through a stationary porous layer:

$$\rho \frac{\partial u^j}{\partial t} + \sum_{k=1}^{2} \rho u^k \frac{\partial u^j}{\partial x_k} = -f \frac{\partial p}{\partial x_j} + \mu \sum_{k=1}^{2} \left(\frac{1}{3}\delta_j^k + 1 \right) \frac{\partial}{\partial x_k} \left(f \frac{\partial u^j}{\partial x_k} \right) + \\ + \mu \sum_{k=1}^{2} \left(1 - \frac{5}{3}\delta_j^k \right) \frac{\partial}{\partial x_k} \left(f \frac{\partial u^{3-j}}{\partial x_{3-k}} \right) - Ku_j + \rho g^j, \tag{4}$$

$$\sum_{k=1}^{2} \frac{\partial \left(\rho u^k \right)}{\partial x_k} = 0. \tag{5}$$

As accepted in porous media, we take the interaction coefficient in the form:

$$K = \frac{150(1-f)^2}{d^2 f^2}.$$

Here d – is the characteristic size of the porous layer.

It is easy to see that at $f \equiv 1$ equations (4), (5) transforms into the Navier-Stokes equations. When $1 \neq f < 1$ we get a system of filtration equations. Let us write equations (4) - (5) in dimensionless form (stationary case):

$$fu\frac{\partial u}{\partial x}+fv\frac{\partial u}{\partial x}=-\frac{f}{\mathrm{Re}}\frac{\partial p}{\partial x}+\frac{4}{3\mathrm{Re}}\frac{\partial}{\partial x}\left(f\frac{\partial u}{\partial x}\right)+$$
$$+\frac{1}{\mathrm{Re}}\frac{\partial}{\partial y}\left(f\frac{\partial u}{\partial y}\right)-\frac{2}{3\mathrm{Re}}\frac{\partial}{\partial x}\left(f\frac{\partial v}{\partial y}\right)+\frac{1}{\mathrm{Re}}\frac{\partial}{\partial y}\left(f\frac{\partial v}{\partial x}\right)-\frac{D^2}{\mathrm{Re}}\frac{(1-f)^2}{f^3}u, \quad (6)$$

$$fu\frac{\partial v}{\partial x}+fv\frac{\partial v}{\partial x}=-\frac{f}{\mathrm{Re}}\frac{\partial p}{\partial y}+\frac{1}{\mathrm{Re}}\frac{\partial}{\partial x}\left(f\frac{\partial v}{\partial x}\right)+$$
$$+\frac{4}{3\mathrm{Re}}\frac{\partial}{\partial y}\left(f\frac{\partial u}{\partial y}\right)+\frac{1}{\mathrm{Re}}\frac{\partial}{\partial x}\left(f\frac{\partial v}{\partial y}\right)-\frac{2}{3\mathrm{Re}}\frac{\partial}{\partial y}\left(f\frac{\partial v}{\partial x}\right)-\frac{D^2}{\mathrm{Re}}\frac{(1-f)^2}{f^3}v, \quad (7)$$

$$\frac{\partial(f\,u)}{\partial x}+\frac{\partial(f\,v)}{\partial x}=0. \quad (8)$$

where u, v— are the longitudinal and transverse components of the velocity, x, y — are the Cartesian coordinates, Re = $hU\rho/\mu$— is the Reynolds number, U— is the characteristic velocity, h — is the characteristic size, $D=\sqrt{\alpha}h/d$ is the dimensionless number, d — is the characteristic size of the porous medium , α — coefficient of proportionality.

For the numerical solution of the system of equations (6) - (8), the control volume method [8] with appropriate generalizations was applied. An uneven coordinated mesh is built by thickening near the stenosis.

BOUNDARY CONDITIONS

At the entrance to the vessel, a parabolic law of the velocity distribution is given, at outputs a soft condition, on the walls of the vessel, the no-slip condition:

$$x=0,\quad u=6y(1-y),\quad v=0,\quad p=p^0.$$

With sufficient distance, we set the soft condition

$$x=L,\qquad \frac{\partial u}{\partial x}=\frac{\partial v}{\partial x}=0.$$

STENOSIS MODELING

Consider to determine the contribution of stenosis in blood vessels based on the model (6-8). As a form of stenosis, let us take the form given in [9]. We believe that the stenosis is asymmetrical (Fig. 1). The form of stenosis is determined by the equation

$$y(x)=\frac{h_0}{2}\left[1+\cos\frac{2\pi}{L_0}\left(x-d-\frac{L_0}{2}\right)\right].$$

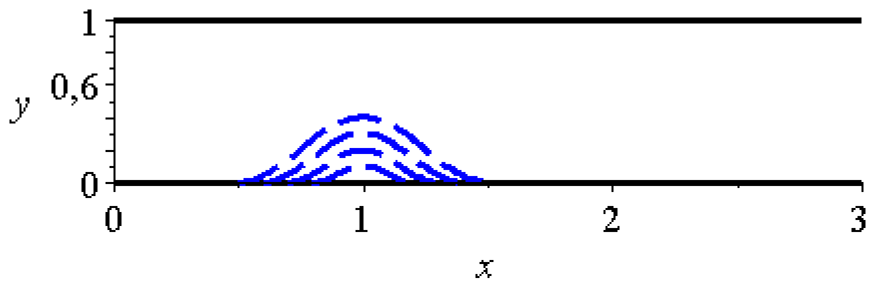

Figure: 1. Area of flow with stenosis

Here *d* is the distance from the entrance to the vessel, h_0 is the fullness of the vessel, L_0 is the width of the stenosis

NUMERICAL EXPERIMENTS

In the calculations, the parameters of stenosis were taken: $d = 0,5;$ $L_0 = 1.$ The parameter h_0 varied. The calculation was carried out in the area $[0,1]\mathrm{x}[0,L]$ $(L = 4).$ Number of nodes 50x30.

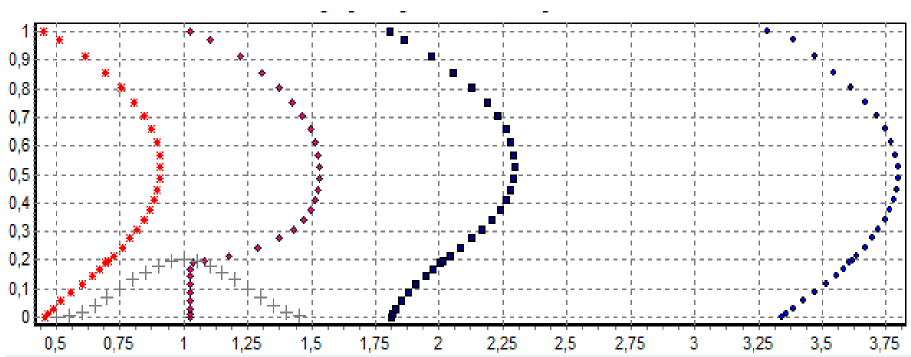

Figure: 2 Velocity distribution over the sections of the vessel, the fullness of the vessel is 20%

In fig. 2 shows the distribution of the velocity in different sections: the asterisks represent the distribution of the velocity in the section: diamond - in cross section $x = 0,45$; rectangles - $x = 1,025$ and circles - $x = 3,337$. The crosses represent the border of the stenosis.

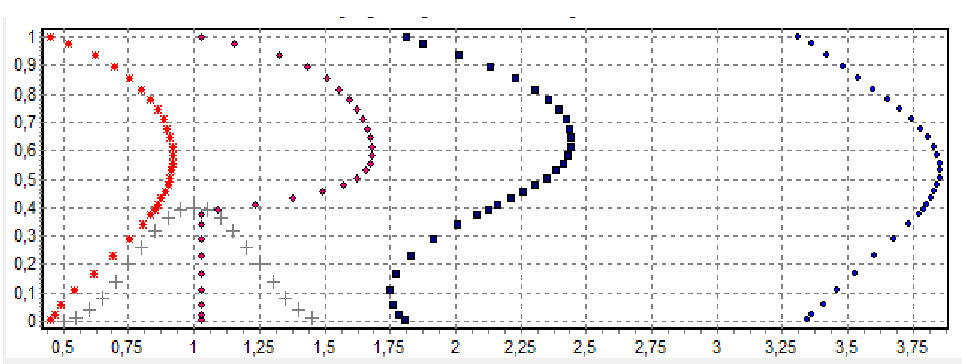

Figure: 3 Velocity distribution over the sections of the vessel, the fullness of the vessel is 40%

Figure 3 shows the calculation results for the same parameter values. The difference lies in the fullness of the vessel. In this case, there is a circulating flow behind the stenosis in the area $x \in [1,3; 2,56]$. For example, in Fig. 3 the graph of the velocity distribution corresponding to the cross section $x = 1,809$ in the lower part is observed reverse flow. However, with the same parameters, but with porosity $f = 0,6$, there is no circulation flow.

With an increase in the filling of the vessel (Fig. 4), an increase in the zone of reverse flows is observed. The secondary flow covers the zone $x \in [1,25;3,8]$.

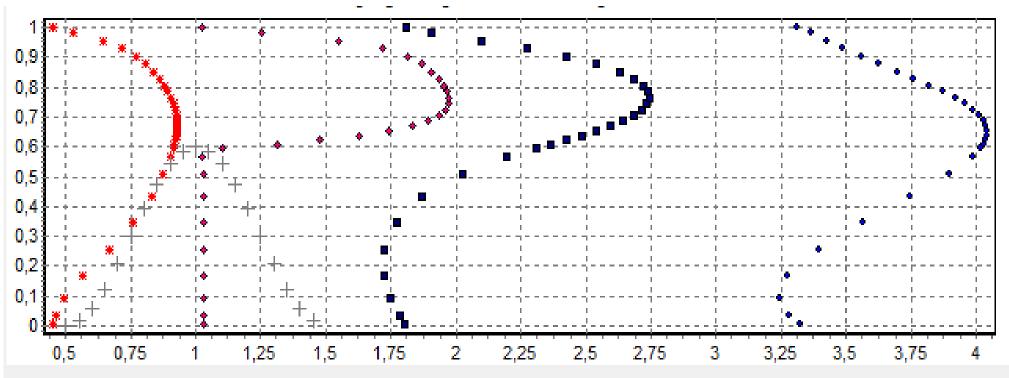

Figure: 4 Velocity distribution over the cross-sections of the vessel, the fullness of the vessel is 60%

CONCLUSION

On the basis of an interpenetrating model of the flow of heterogeneous media, a mathematical description of the flow of blood with porous stenosis is proposed. For two-dimensional problems, a discrete model of the equation is constructed using the control volume method. The results of calculations and the influence of grid parameters, reflecting the legitimacy of the application of the model and the proposed numerical approach, are presented.

Funding

This study has not received any external funding.

Conflict of Interest

The authors declare no conflicts of interests any matter related to this paper.

Data and materials availability:

All data associated with this study are present in the paper.

REFERENCES AND NOTES

1. Deplano, E., Siouffi, M., 1999. Experimental and numerical study of pulsatile flows through stenosis: wall shear stress analysis. Journal of Biomechanics, 32, 1081–1090.
2. Tu, C., Deville, M., 1996. Pulsatile flow of non-Newtonian fluids through arterial stenosis. Journal of Biomechanics 29 (7), 899–908.
3. F.Kh. Tazyukov, Jafar M. Hassan, Kh.A. Khalaf, B.A. Snigerev, Safaa H. Abdul Rahman, Blood flow in a symmetrical blood artery with stenosis. Russian Journal of Biomechanics. 2012. T. 16, № 1 (55): 46–56
4. Fernando A. Morales,, Ralph E.Showalterb, A Darcy–Brinkman model of fractures in porous media, Journal of Mathematical Analysis and Applications, J. Math. Anal. Appl. 452 (2017) 1332–135800
5. Dalabaev U. On an approach for viscous flow modeling about porous media, Thermophysics and Aeromechanics, v.5, No 1, Novosibirsk, 1998, p.45
6. Fayzullaev D.F. Laminar motion of multiphase media in pipelines. Tashkent: FAN,, 1966
7. R.I. Nigmatulin. Dynamics of Multiphase Media, Volume 1 1st Edition, Moscow: Nauka, 1978. 336 p
8. Suhas V. Patankar. Numerical Heat Transfer and Fluid Flow, Hemisphere Publishing Corporation, 1980
9. Young D.F. Fluid mechanics of arterial stenosis. J. Biomech. Eng. 1979, 101, 157–173.