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DISCOVERY

Quantifying the impact of the co-existence steady-state solution on the type of stability using a numerical simulation approach

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ABSTRACT

The interaction between species has been a long standing scientific research both for mathematical ecologists and environmental statisticians, biologists or scientists to mention a few. In the present study activity we have indentified the full potential of using a numerical simulation which is computationally efficient to differentiate types of stability due to a variation of a co- existence steady state solution. The novel results which we have achieved on the implementation of this method have not been seen elsewhere; they are presented and discussed in this paper.

Keywords: species, interaction, environmental, numerical simulation, steady state

1. INTRODUCTION

The interaction between two populations dates back to the earlier formulation of Lotka-Volterra system of continuous non- linear first order ordinary differential equations. In the theory of competing species, the co-existence steady state solution plays a significant role in the survival of two competing species (Burden and Faires, 2001). Since such a steady state solution is a point in the phase plane, varying one co-ordinate and fixing the other co-ordinate can have an impact on the type of stability for the two competing species. This proposed idea cannot be successfully tackled using an analytical method. Therefore, for the purpose of this study, we have utilized the method of a numerical simulation to quantify the impact of one co-ordinate of co-existence steady state solution on the type of stability.

2. MATHEMATICAL FORMULATION

Following (Murray, 2001, Yan and Ekaka-a, 2001, Ford *et al.*, 2010, Kot, 2001, Beeby, 1993). We have considered the following system of continuous non-linear first order differential equation;

$$\frac{dx-1(t)}{dt} = x-1(t)(4-0.0003x-1(t)-0.0004x-2(t) \quad (1)$$

$$\frac{dx-2(t)}{dt} = x-2(t)(2-0.0002x-1(t)-0.0001x-2(t) \quad (2)$$

Subject to the ordinary initial conditions $x_1(0) = x_{10} > 0$ and $x_2(0) = x_{20} > 0$

Method of Analysis

We have obtained a co-existence steady state solution (8000, 4000) by using the standard analytical method of Crammer's Rule to solve the above equations (1) and (2). This steady state solution indicates that for the two competing species to survive together, x_1 species will have a population size of 8000 whereas the x_2 species will have a population size of 4000. For the purpose of this study, we first fixed x_2 population size and vary the x_1 population size and using this variations to study the stability of the expected co-existence steady state solution. Secondly, we fixed x_1 population size and vary x_2 population size and similarly study the stability of the co-existence steady state solution. The results that we have obtained on the application of this method are presented and discussed below.

3. RESULTS AND DISCUSSIONS

The results that we have obtained upon the implementation of the above method of analysis are fully presented and discussed in this section.

Table1 Evaluating the impact of x_1 between 800 and 4400 on the type of stability

Example	x_1	x_2	Eigenvalue λ_1	Eigenvalue λ_2	Type of stability
1	800	4000	2.1505	0.8095	Unstable
2	1200	4000	2.0367	0.8095	Unstable
3	1600	4000	1.9284	0.3916	Unstable
4	2000	4000	0.1754	1.8246	Unstable
5	2400	4000	1.7245	-0.0445	Unstable
6	2800	4000	1.6274	-0.2674	Unstable
7	3200	4000	-0.4927	1.5327	Unstable
8	3600	4000	-0.7200	1.400	Unstable
9	4000	4000	-0.9489	1.3489	Unstable
10	4400	4000	-1.1792	1.2592	Unstable

What can we learn from Table 1? From Table 1, the ten empirical examples of the two steady state solutions (x_1, x_2) are unanimously unstable having two positive eigen values and eigen values of opposite signs. By the mathematical concept of a quantitative behavior of solution trajectories, two positive eigen values contribute to the unbound growth of the solution trajectories (hence the steady state solution is said to be unstable). In this context, two eigen values of opposite signs signify that the positive eigen value is growing unbound and faster than the negative eigen value that is contributing to the decaying behavior of the solution trajectories. A similar observation has been made for Table 2.

Table 2 Evaluating the impact of x_1 between 4800 and 8400 on the type of stability

Example	X_1	x_2	Eigenvalue λ_1	Eigenvalue λ_2	Type of stability
11	4800	4000	-1.4106	1.1706	Unstable
12	5200	4000	-1.6429	1.0829	Unstable
13	5600	4000	-1.8761	0.9961	Unstable
14	6000	4000	-2.1100	0.9100	Unstable
15	6400	4000	-2.3444	0.8244	Unstable
16	6800	4000	-2.5794	0.7394	Unstable
17	7200	4000	-2.8148	0.6548	Unstable
18	7600	4000	-3.0506	0.5706	Unstable
19	8000	4000	-3.2868	0.4868	Unstable
20	8400	4000	-3.5233	0.4033	Unstable

However, the co-existence steady state solutions continue to be unstable up to the steady state solution (10000, 4000), after which the instability is lost (Table 3 & 4). From our analysis, we have observed that the instability is lost between the steady state solutions (10000, 4000) and (10200, 4000). After this bifurcation phase, each steady state solution is consistently stable having two negative eigen values that contribute to the decaying behavior of the solution trajectories of $x_1(t)$ and $x_2(t)$.

Table 3 Evaluating the impact of x_1 between 8800 and 11800 on the type of stability

Example	x_1	x_2	Eigenvalue λ_1	Eigenvalue λ_2	Type of stability
21	8800	4000	-3.7600	0.3200	Unstable
22	9200	4000	-3.9978	0.2370	Unstable
23	9600	4000	-4.2342	0.1542	Unstable
24	10000	4000	-4.4716	0.0716	Unstable
25	10200	4000	-4.7091	-0.0109	Stable
26	10400	4000	-4.9468	-0.0932	Stable
27	10800	4000	-5.1847	-0.1753	Stable
28	11200	4000	-5.4227	-0.2573	Stable
29	11400	4000	-5.6608	-0.3392	Stable
30	11800	4000	-5.8991	-0.4209	Stable

Table 4 Evaluating the impact of x_1 between 12800 and 16400 on the type of stability

Example	x_1	x_2	Eigenvalue λ_1	Eigenvalue λ_2	Type of stability
31	12800	4000	-6.1374	-0.5026	Stable
32	13200	4000	-6.3758	-0.5842	Stable
33	13600	4000	-6.6143	-0.6657	Stable
34	14000	4000	-6.8529	-0.7471	Stable
35	14400	4000	-7.0915	-0.8285	Stable
36	14800	4000	-7.3302	-0.9098	Stable
37	15200	4000	-7.5690	-0.9910	Stable
38	15600	4000	-7.8078	-1.0722	Stable
39	16000	4000	-8.0467	-1.1533	Stable
40	16400	4000	-8.2857	-1.2343	Stable

What is our next task? Having varied x_1 and fixed x_2 to construct several steady state solutions that were unstable (dominant) and stable in the minor, we would like to explore further on the variation of x_2 while x_1 is fixed. The results of this new idea are shown in Tables 5-8.

Table 5 Evaluating the impact of x_2 between 400 and 2200 on the type of stability

Example	x_1	x_2	Eigenvalue λ_1	Eigenvalue λ_2	Type of stability
1	8000	400	-1.1358	0.4958	Unstable
2	8000	600	-1.2853	0.5253	Unstable
3	8000	800	-1.4271	0.5471	Unstable
4	8000	1000	-1.5630	0.5630	Unstable
5	8000	1200	-1.6942	0.5742	Unstable
6	8000	1400	-1.8215	0.5815	Unstable
7	8000	1600	-1.9455	0.5855	Unstable
8	8000	1800	-2.0668	0.5856	Unstable
9	8000	2000	-2.1856	0.5856	Unstable
10	8000	2200	-2.3024	0.5824	Unstable

Table 6 Evaluating impact of x_2 between 2400 and 4200 0n the type of stability

Example	x_1	x_2	Eigenvalue λ_1	Eigenvalue λ_2	Type of stability
11	8000	2400	-2.4172	0.5772	Unstable
12	8000	2600	-2.5304	0.5704	Unstable
13	8000	2800	-2.6420	0.5620	Unstable
14	8000	3000	-2.7523	0.5523	Unstable
15	8000	3200	-2.8613	0.5413	Unstable
16	8000	3400	-2.9692	0.5292	Unstable
17	8000	3600	-3.0760	0.5760	Unstable
18	8000	3800	-3.1818	0.5018	Unstable
19	8000	4000	-3.2868	0.4868	Unstable
20	8000	4200	-3.3909	0.4709	Unstable

Table 7 Evaluating the impact of x_2 between 4400 and 6200 on the type of stability

Example	x_1	x_2	Eigenvalue λ_1	Eigenvalue λ_2	Type of stability
21	8000	4400	-3.4942	0.4542	Unstable
22	8000	4600	-3.5968	0.4368	Unstable
23	8000	4800	-3.6987	0.4187	Unstable
24	8000	5000	-3.8000	0.4000	Unstable
25	8000	5200	-3.9007	0.3807	Unstable
26	8000	5400	-4.0007	0.3607	Unstable
27	8000	5600	-4.1003	0.3403	Unstable
28	8000	5800	-4.1993	0.3193	Unstable
29	8000	6000	-4.2978	0.2978	Unstable
30	8000	6200	-4.3959	0.2759	Unstable

Table 8 Evaluating the impact of x_2 between 6400 and 8200 on the type of stability

Example	x_1	x_2	Eigenvalue λ_1	Eigenvalue λ_2	Type of stability
31	8000	6400	-4.4935	0.2535	Unstable
32	8000	6600	-4.5907	0.2307	Unstable
33	8000	6800	-4.6875	0.2075	Unstable
34	8000	7000	-4.7839	0.1839	Unstable

35	8000	7200	-4.8800	0.1600	Unstable
36	8000	7400	-4.9757	0.1357	Unstable
37	8000	7600	-5.0711	0.1111	Unstable
38	8000	7800	-5.1661	0.0086	Unstable
39	8000	8000	-5.2608	0.0608	Unstable
40	8000	8200	-5.3553	0.0353	Unstable

What contribution has the present analysis made over the earlier formulation? While the original formulation simply described the deterministic interaction between two competing species, the in-depth study of the impact of the co-existence steady state solution on the type of stability was not considered. In order to extend this idea, we have utilized a sound mathematical reasoning to investigate the effect of the co-existence steady state solution on the type of stability. On the basis of this present analysis, we can clearly mention that the present numerical analysis is a cutting edge contribution over the previous mathematically track-table and ecological track-table system of competing species.

4. CONCLUSION AND FURTHER RESEARCH

The key achievement of this important study depends on its significance contribution: we have utilized the method of numerical simulation to clearly study the effect of varying the co-existence of the steady state solution on the type of stability. In particular when the x_1 population size is varied for a fixed x_2 population size, we have found several significant dominant unstable steady state solutions that consequently change to a few instances of stable co-existence steady state solutions. This aspect of bifurcation analysis has the potential to influence ecosystem functioning, planning and sustainable development. The full details of other parameter variations in the context of pure competition and other types of interactions such as mutualism, commensalism and predation will be the subjects of our further research.

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