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Advection and its applications: Trajectories over Busia County in Kenya

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ABSTRACT
Advection is defined as a conservative transport of a substance by bulk motion. The substances include pollutants, enthalpy or any material that contains thermal energy. This paper introduces the concept of advection as applied in air pollution modelling of possible pollutants using Hybrid Single Particle Lagrangian Integrated Trajectory Model (HYSPLIT). A three dimensional (3-D) advection equation is specified and a graphical output of a forward air trajectory given over Busia County. The study reveals transboundary flow of air pollutants to the Eastern parts of Uganda and across counties in Western, Central, Rift valley and Eastern Kenya respectively. An intercounty environmental monitoring policy framework is recommended in this study due to the transcounty nature of air pollution issues.

Keywords: Kenya, Busia County, Advection, HYSPLIT

1. INTRODUCTION
Study Back ground
Trajectories can be used to investigate atmospheric flow patterns over an area of study [1]. Research by [2] used forward trajectories to locate the flow of dust from the source lake to other areas several kilometers away. In another study by [3], identification of dust

transport pathways from Lake Eyre, Australia was carried out using a dispersion model that was also used to investigate the flow of North American dust from the Bodélé depression to neighboring regions [4]. In the study, it was revealed that dust affected kilometers of regions form the source location. Air carrying dust particles can be simulated using this approach as investigated by [5] where dust from southern Kalahari Desert was found to be moving into the nile along a forward trajectory. Another investigation was conducted by [6] to ascertain the transport and dispersion of total suspended particles over Nairobi city revealing that the pollutant was dispersed more than 100 km from the city and reduced concentrations of the same in the city. The transport of pollutants can take place to several kilometers away from a coal minimg plant along trajectories [7]. Air pollution is a growing concern in third world cities and municipalities due to poor carrying capacity, inefficient waste disposal systems among others [8]. Busia County is one of the counties in kenya of transboundary air pollution concern based on studies by [1] where air trajectories indicate flow patterns from Bungoma to Uganda across Busia County and dispersion to other regions in Kenya. Busia town is vulnerable to air pollution, just like any municipality as described by [8].

2. METHODOLOGY

Area of study

Busia County is one of the 47 kenyan Counties neighboring Bungoma, Kakamega and Siaya Counties respectively in western Kenya. The County is characterized by a tropical climate with average temperatures of 22 degrees centigrade and average rainfall of 1691 mm per annum. The coordinates of the county are 0.4347° N and 34.2422° E respectively. The map of study area is shown in figure 1 below. This study makes attempts to examine the concept of advection and uses a HYSPLIT model to predict the flow of pollutants in Busia County, Kenya.

Figure 1 Map of the study area (Source; [9]).

Three Dimensional Advection Equations

Advection is defined as a conservative transport of a substance by bulk motion. The substances include pollutants, enthalpy or any material that contains thermal energy. The fluid is considered the vector field and the material transported is the scalar field distributed over a space. To make a mathematical model of physical phenomena, it is essential to understand the behavior of equation. The advection equation in 3-D is given by the following equation:

$c_t + uc_x + vc_y + wc_z = \eta_x c_{xx} + \eta_y c_{yy} + \eta_z c_{zz}$ ....(1)

where $(x,y,z,t)\epsilon\Omega\times(0,T]$ with initial conditions $c(x,y,z,0)=c_0(x,y,z),(x,y,z)\epsilon\Omega$. The Dirichlet's boundary conditions are therefore smooth functions $c(x,y,z,t)$ that represent heat, diffusion, whereas u,v,w are constants that denotes the convective velocities and η_x,η_y,η_z denotes diffusion coefficients in the direction of (x,y,z) directions. Therefore equation 1 above has both advection and diffusion term. The equation above is probably one of the simplest non-linear PDE for which it is possible to obtain an exact solution. Its analytical solution takes the form

$c(x,y,z,t)=e^{t+x+z}$ where we take $c_x=2, c_y=1, c_z=1$ and $\eta_x=\eta_y=\eta_z=1$ ....(2)

But then, the best solution to the equation 1 is the numerical solution approach with numerical approximations. We assume that wind and tensor components moves horizontally such that equation 1 takes the general form;

$c_t+uc_x+vc_y=\eta_x c_{xx}+\eta_y c_{yy}+\eta_z c_{zz}$..(3)

The time derivative finite difference is found by using a finite difference method in which the approximation is obtained using a Taylor series. The solution domain over a time interval [0,T] is given by a mesh of grid lines that are uniformly spaced that are parallel to the space and time coordinate. We make approximations at the points of intersections using the Forward in Time, Centre in Space with first order errors as follows:

$$c_t=\frac{c^{m+1}{}_{i,j,k}}{\Delta t}, c_x=\frac{c^m{}_{i,j+1,k}-c^m{}_{i,j-1,k}}{2\Delta x},\ c_y=\frac{c^m{}_{i+1,j,k}-c^m{}_{i-1,j,k}}{2\Delta x}, c_{xx}=\frac{c^m{}_{i,j+1,k}-2c^m{}_{i,j,k}+c^m{}_{i,j-1,k}}{(\Delta x)^2}, c_{yy}=\frac{c^m{}_{i+1,j,k}-2c^m{}_{i,j,k}+c^m{}_{i-1,j,k}}{(\Delta y)^2} \text{ and } c_{yy}=$$

$$\frac{c^m{}_{i,j,k}+-2c^m{}_{i,j,k}+c^m{}_{i,j,k-1}}{(\Delta z)^2}\text{.................................}(4)$$

Substituting these finite differences in equation we have;

$$\frac{c^{m+1}{}_{i,j,k}}{\Delta t}+u\frac{c^m{}_{i,j+1,k}-c^m{}_{i,j-1,k}}{2\Delta x}+v\frac{c^m{}_{i+1,j,k}-c^m{}_{i-1,j,k}}{2\Delta x}=\eta_x\frac{c^m{}_{i,j+1,k}-2c^m{}_{i,j,k}+c^m{}_{i,j-1,k}}{(\Delta x)^2}+\eta_y\frac{c^m{}_{i+1,j,k}-2c^m{}_{i,j,k}+c^m{}_{i-1,j,k}}{(\Delta y)^2}+$$

$$\eta_z\frac{c^m{}_{i,j,k}+-2c^m{}_{i,j,k}+c^m{}_{i,j,k-1}}{(\Delta z)^2}\text{.............}(5)$$

$$c^{m+1}{}_{i,j,k}=\left(\frac{\eta_x\Delta t}{(\Delta x)^2}+\frac{u\Delta t}{2\Delta x}\right)c^m{}_{i,j-1,k}+\left(\frac{\eta_y\Delta t}{(\Delta y)^2}+\frac{v\Delta t}{2\Delta y}\right)c^m{}_{i-1,j,k}+2\frac{\eta_z\Delta t}{(\Delta z)^2}c^m{}_{i,j,k-1}+\left(\frac{\eta_x\Delta t}{(\Delta x)^2}-\frac{u\Delta t}{2\Delta x}\right)c^m{}_{i,j-1,k}+\left(\frac{\eta_y\Delta t}{(\Delta y)^2}-\frac{v\Delta t}{2\Delta y}\right)c^m{}_{i-1,j,k}+(1-$$

$$2\frac{\eta_z\Delta t}{(\Delta z)^2}-2\frac{\eta_y\Delta t}{(\Delta y)^2}-2\frac{\eta_x\Delta t}{(\Delta x)^2})c^m{}_{i,j,k}\ \text{.............}(6)$$

This simplifies to:

$$c^{m+1}{}_{i,j,k}=2\frac{\eta_x\Delta t}{(\Delta x)^2}c^m{}_{i,j-1,k}+2\frac{\eta_y\Delta t}{(\Delta y)^2}c^m{}_{i-1,j,k}+2\frac{\eta_z\Delta t}{(\Delta z)^2}c^m{}_{i,j,k-1}+c^m{}_{i,j,k}-2\frac{\eta_z\Delta t}{(\Delta z)^2}c^m{}_{i,j,k}-2\frac{\eta_y\Delta t}{(\Delta y)^2}c^m{}_{i,j,k}-2\frac{\eta_x\Delta t}{(\Delta x)^2}c^m{}_{i,j,k}$$

....(7)

The difference scheme is stable if

$$c^m{}_{i,j,k}-2\frac{\eta_z\Delta t}{(\Delta z)^2}c^m{}_{i,j,k}-2\frac{\eta_y\Delta t}{(\Delta y)^2}c^m{}_{i,j,k}-2\frac{\eta_x\Delta t}{(\Delta x)^2}c^m{}_{i,j,k}\le 0\text{..}(8)$$

The solution obtained here is in a box form.

According to [1], the advection of a particle or puff is computed from the average of the three-dimensional velocity vectors at the initial-position P(t) and the first-guess position P'(t+Δt). The velocity vectors are linearly interpolated in both space and time. The first guess position is

$$P'(t+\Delta t) = P(t) + V(P,t)\,\Delta t \quad (9)$$

and the final position is $P(t+\Delta t) = P(t) + 0.5\,[\,V(P,t) + V(P',t+\Delta t)\,]\,\Delta t$ ....(10)

HYSPLIT (Hybrid Single Particle Lagrangian Integrated Trajectory Model) uses an intergrated (Hybrid of) lagrangian and eularian approaches as outlined in [1]. The eulerian langrangian approaches are defined in [10].

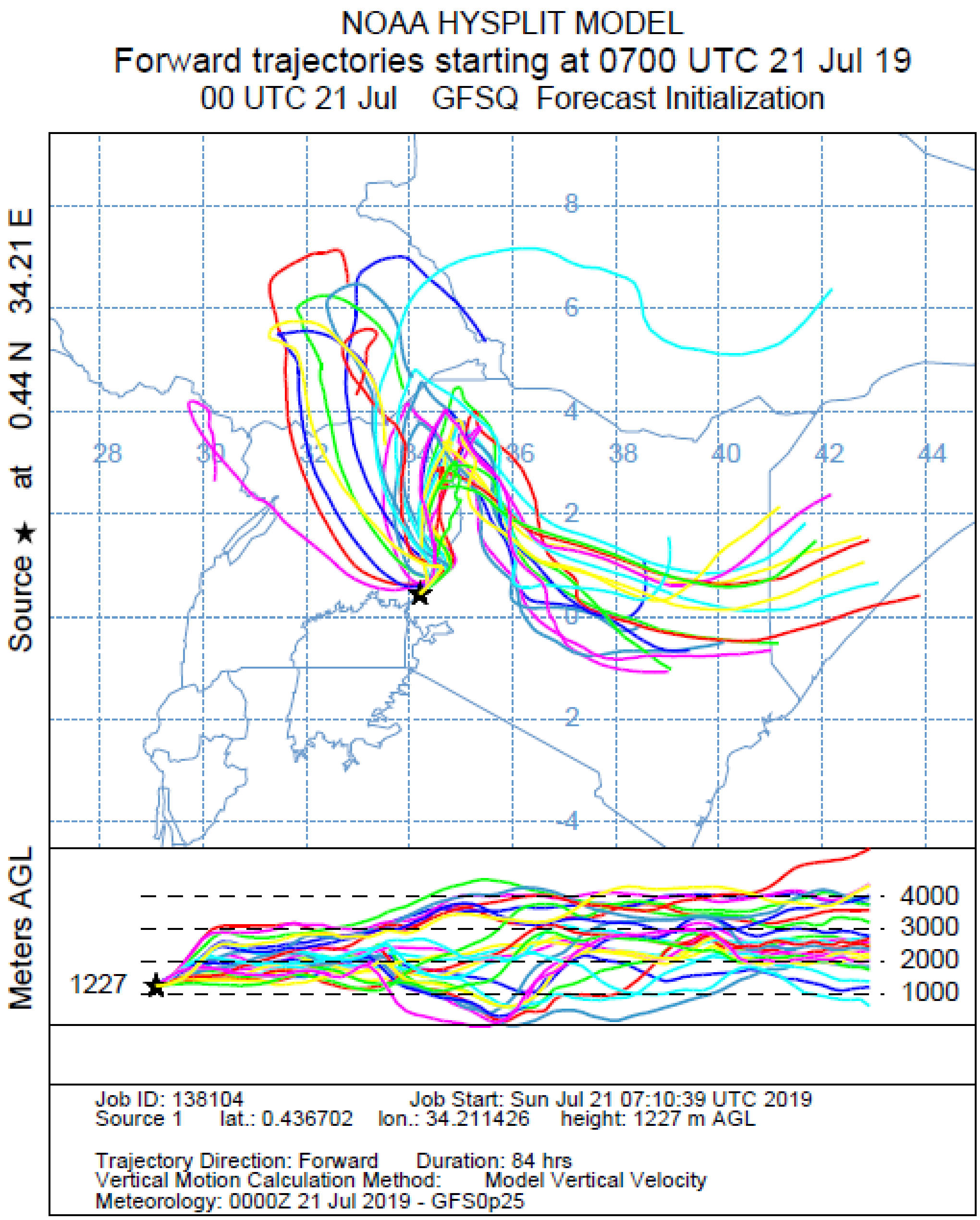

Figure 2 Forward trajectory of Busia County in Kenya

Simulation of atmospheric pollutants

HYSPLIT was used according to the criteria outlined by [13]. Normal trajectory was computed from the Global Data Assimilation System (GDAS) archive at a resolution of single degree spanning 2006 to present at source location 0°437 N; 34°2143 E corresponding to the area of study using a 7-day archived meteorological file at 1227 meters above sea level. A forward trajectory was generated. Model Vertical velocity approach was employed in the study at 1227 meters above ground level using the Global Forecast Data (GFD).

3. RESULTS AND DISCUSSION

Forward trajectory over the area of study

Figure (2) shows a forward trajectory over Busia County in Kenya. It can be observed that "percels" of air that possibly contains pollutants are dispersed up to 4000 meters above ground level to Eastern Uganda and 16 counties namely Bungoma, Kakamega, Vihiga, Kisumu, Uasin Gishu, Nandi, Baringo, Nyeri, Murang'a, Embu, Muranga, Kirinyaga, Tharaka, Garissa, Isiolo and Wajir respectively.

4. CONCLUSION

The study affirms the transboundary nature of air pollution issues and therefore not characteristic of a single county. The study recommends a concerted air pollution control policy action by all counties and National Government to reduce air pollution.

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