



# Real physical meaning of the spin of electron

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## ABSTRACT

Further study of quantum mechanics led us to some new comprehensions. The spin of electron is no longer spin of electron in quantum mechanics, but the effect of relativity. The Bohr orbital angular momentum is corresponded to what was to be said the spin of electron. Its direction in an orbit decides the signs of wave function  $\psi$ , i.e.  $\psi+$ , or  $\psi-$ . The  $d_{z^2}$  orbital is deduced to be the only magnetic one in all types of orbits.

**Keywords:** Electron spin, Wave function, Magnetism, Polarization.

## 1. INTRODUCTION

It is well known that, quantum mechanics is a successful theory for describing the state of electron. Its applications were obtained satisfied results (Ni, 1979). In despite of that, there were still a few flaws in comprehensions. The main problems are the physical meanings of the spin of electron and the signs of wave function  $\psi$ . The answers are studied in this paper.

## 2. HISTORY REVIEW

In non-relativistic situations, for a free electron, the energy ( $E$ ) and the momentum ( $p$ ) has the relations as (Ni, 2003),

$$E = \frac{1}{2m} \mathbf{p}^2 \dots\dots\dots(1)$$

To be undergone a functor treatment, substituting  $E$  with  $i\hbar \frac{\partial}{\partial t} \psi(x, y, z, t)$ , ( $\equiv \hat{H} \Psi(x, y, z, t)$ ), and  $p$  with

$-i\hbar \nabla \psi(x, y, z, t)$ , ( $\equiv \hat{\mathbf{p}} \Psi(x, y, z, t)$ ), the famous Schrödinger equation was obtained as,

$$i\hbar \frac{\partial}{\partial t} \psi(x, y, z, t) = -\frac{\hbar^2}{2m} \nabla^2 \psi(x, y, z, t) \dots\dots\dots (2)$$

Considering the relativistic effect for a free electron, the energy ( $E$ ) and the momentum ( $p$ ) has following relation as,

$$E^2 = c^2 p^2 + m^2 c^4 \dots\dots\dots(3)$$

In order to avoid the embarrassed double differential situations (which Klein-Gordon did) which caused by  $E^2$ , giant Dirac rewrote equation (3) as follow gleamy styles,

$$E = \sqrt{c^2 \mathbf{p}^2 + m^2 c^4}$$

$$E\psi = \hat{H}_D \psi$$

$$\hat{H}_D = c\boldsymbol{\alpha} \cdot \hat{\mathbf{p}} + \beta mc^2 \dots\dots\dots (4)$$

In order to satisfy equation (4), the coefficients  $\boldsymbol{\alpha}$  and  $\beta$  may not be ordinary constants; they must be the matrix functors. Where,

$$\boldsymbol{\alpha}_i = \begin{bmatrix} 0 & \sigma_i \\ \sigma_i & 0 \end{bmatrix}, \beta = \begin{bmatrix} I & 0 \\ 0 & -I \end{bmatrix} \dots\dots\dots (5)$$

In non-relativistic situations, the angular momentum,  $\mathbf{L}$ , has relation with  $\mathbf{p}$  and radius  $\mathbf{r}$  as,

$$\mathbf{L} = \mathbf{r} \times \mathbf{p}, \text{ then, } \hat{\mathbf{L}} = \mathbf{r} \times \hat{\mathbf{p}}$$

In non-relativistic situations, functors  $\hat{H}$  and  $\hat{\mathbf{L}}$  are commutative, i.e.  $[\hat{\mathbf{L}}, \hat{H}] = 0$ . However, in relativistic situations,  $\hat{H}_D$  and  $\hat{\mathbf{L}}$  are not commutative. i.e.,

$$[\hat{\mathbf{L}}_i, \hat{H}_D] = i\hbar c (\boldsymbol{\alpha} \times \hat{\mathbf{p}})_i \neq 0$$

Fortunately, Dirac discovered that,

$$[\frac{1}{2} \hbar \boldsymbol{\Sigma}_i, \hat{H}_D] = -i\hbar c (\boldsymbol{\alpha} \times \hat{\mathbf{p}})_i \dots\dots\dots (6)$$

$$\text{where, } \boldsymbol{\Sigma}_i = \begin{bmatrix} \sigma_i & 0 \\ 0 & \sigma_i \end{bmatrix}, \sigma_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \sigma_y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}, \sigma_z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, i = x, y, z$$

So, Dirac observed sensitively that, the relativistic  $\hat{H}_D$  ought be commutative with the angular momentum in relativistic situations,

expressed by  $\hat{\mathbf{J}}$ . Evidently, it has the form as,

$$\hat{\mathbf{J}} = \hat{\mathbf{L}} + \frac{1}{2} \hbar \boldsymbol{\Sigma} = \hat{\mathbf{L}} + \hat{\mathbf{S}}, \text{ (where, } \boldsymbol{\Sigma} = \begin{bmatrix} \boldsymbol{\sigma} & 0 \\ 0 & \boldsymbol{\sigma} \end{bmatrix}) \dots\dots (7)$$

On account of,  $\hat{J}_i = \hat{L}_i + \hat{S}_i$ ,  $[\hat{J}_i, \hat{H}_D] = 0$ ,  $\hat{J}_i^2 = i\hbar \hat{J}_i$ ,  $\hat{J}^2 = i\hbar \hat{J}$ , Thus,  
 $[\hat{J}, \hat{H}_D] = \frac{1}{i\hbar} [\hat{J}^2, \hat{H}_D] = \frac{1}{i\hbar} [\sum \hat{J}_i^2, \hat{H}_D] = \frac{1}{i\hbar} \sum [\hat{J}_i^2, \hat{H}_D] = \frac{1}{i\hbar} \sum i\hbar [\hat{J}_i, \hat{H}_D] = 0$

Therefore, it had the result and solutions as,

$$\hat{S} = \frac{1}{2} \hbar \Sigma$$

$$\hat{S}_i^2 \Psi(x, y, z, t) = S_i^2 \Psi(x, y, z, t), S_i^2 = \frac{1}{4} \hbar^2, (i = x, y, z)$$

$$S_x = S_y = S_z = \pm \frac{1}{2} \hbar, \mathbf{S} = \frac{\sqrt{3}}{2} \hbar$$

### 3. DISCUSSION

#### 3.1. Question the Spin of Electron

Equation (7) is the famous Dirac relation. It is only an expression by mathematics. Then, a question is that how can we comprehend it. Although  $\hat{S}$  was a functor of rotation, however, to be a functor of rotation do not means the spin of electron. In fact that, saying it to be a spin of electron is only a guess. Because that, after the guess, it had to be appended another embarrassed saying that, the spin of electron has no actually physical meaning. Because it would led to a fantastic result that, the speed at edge would exceed the speed of light for a spin electron as if the electron radius was considered as about  $2.15 \times 10^{-15}$  m. Or, the fantastic result is that, the electron radius would be inconceivably large (calculated to be  $2.90 \times 10^{-11}$  m according to equation (17)) and near approximately the radius of an hydrogen atom ( $5.29 \times 10^{-11}$  m).

Here, we proposed a reasonable saying as follows.

Functor  $\hat{J}$  is thought to be the effect of relativity of  $\hat{L}$ .  $\frac{1}{2} \hbar \Sigma$  or  $\hat{S}$  is the correction term of relativistic effect (Xu, 2000). The eigenvalue of  $\hat{J}$  is  $J$ . It can be expressed as,

$$J = L + S$$

$J$ ,  $L$ , and  $S$  are vectors.  $J$  is a constant; it holds its value in every direction in global space.

$S$  can be seen as a variable in an instant; it holds its absolute value on 8 directions in space. I.e.,

$$S_i = \begin{pmatrix} x \\ y \\ z \end{pmatrix}_i = \begin{pmatrix} \frac{\hbar}{2} \\ \frac{\hbar}{2} \\ \frac{\hbar}{2} \end{pmatrix}_1 ; \begin{pmatrix} \frac{\hbar}{2} \\ \frac{\hbar}{2} \\ -\frac{\hbar}{2} \end{pmatrix}_2 ; \begin{pmatrix} \frac{\hbar}{2} \\ \frac{\hbar}{2} \\ \frac{\hbar}{2} \end{pmatrix}_3 ; \begin{pmatrix} \frac{\hbar}{2} \\ \frac{\hbar}{2} \\ -\frac{\hbar}{2} \end{pmatrix}_4 ; \begin{pmatrix} -\frac{\hbar}{2} \\ \frac{\hbar}{2} \\ \frac{\hbar}{2} \end{pmatrix}_5 ; \begin{pmatrix} -\frac{\hbar}{2} \\ \frac{\hbar}{2} \\ -\frac{\hbar}{2} \end{pmatrix}_6 ; \begin{pmatrix} -\frac{\hbar}{2} \\ \frac{\hbar}{2} \\ \frac{\hbar}{2} \end{pmatrix}_7 ; \begin{pmatrix} -\frac{\hbar}{2} \\ \frac{\hbar}{2} \\ -\frac{\hbar}{2} \end{pmatrix}_8 \dots \dots \dots (8)$$

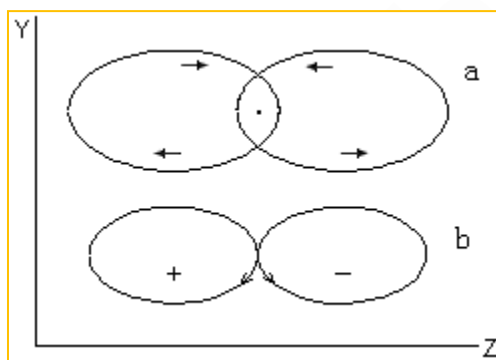
In terms of quantum mechanics, it ought to say it has the same opportunities on above 8 directions.

#### 3.2. Bohr Orbital Angular Momentum, $L_B$

It is well known that, the electron has the wave-particle duality. Owing to the splendid success of quantum mechanics, we nearly forget the particle character of electron in the field of quantum mechanics. Now, it's time to recall all of it. It must be confirmed that, for electron the wave function  $\psi$  was just the description in wave. Or, it ought to say that it was a description statistically, and not the real action. That's why we can't explain the dispersion force between noble gases in terms of wave function except the instantaneous dipoles in terms of particle character. Another good example is that, though the diffraction experiments of electron and light were completely explained in terms of pure wave theory, we still dare not say that the electron or light is only a wave.

Perhaps we must imitate what N. Bohr was done in his atomic model. Where, Bohr introduced quantum conditions of  $n\hbar$  to his classical particle description of electron. Bohr's atomic model also explained the spectrum of hydrogen atom successfully. If the relativistic effect was considered, the results would be as precise as quantum mechanics. So, it is imprudent to deny the rationalities of Bohr's model as it was usually treated by public. Well then, following Bohr's footprints, we add the conception of particle characters in wave function  $\psi$ .

Electron is an elementary particle. It goes around atomic nuclear in an atom or molecule, and has its running orbit (Bohr orbit). Because of wave character, the running orbit of electron is not exactly as it was to be in classical physics, but a changeable. The running orbit would change by itself wave character and environmental disturbance. We know that, as a circular running electron for a single run, it must have its characters of rotary angular momentum. We expressed it as  $L_B$ . The orbital angular momentum in quantum mechanics is expressed as  $L_{QM}$ ; and the statistical orbital angular momentum (in a period of processing time  $\Delta t$ ) is expressed as  $L$  here. They have following relations,



**Figure 1** The sketch map of an electron running and exchanging in two Bohr's orbits

$$L_B = n\hbar, (n = 1, 2, 3, \dots, \infty) \dots\dots\dots(9)$$

$$L_{QM} = \pm\sqrt{l(l+1)}\hbar \dots\dots\dots (10)$$

$$L = \int_0^{\Delta t} dL_B \dots\dots\dots (11)$$

Considering the relativistic effect, equation (11) would be expressed as  $J = \int_0^{\Delta t} dL_B'$ .

It is clear that, the  $L$  (or  $J$ ) is the statistical result of many single  $L_B$  (or  $L_B'$ ). Every  $L_B$  (or  $L_B'$ ) may have their different directions. It must be emphasized that  $L_{QM}$  has two opposite values. The two opposite values have same opportunities that existed, and correspond to  $\psi_+$  and  $\psi_-$  respectively. In equation (11), if the instantaneous processing time  $\Delta t$  is small enough, the value of  $L$  would be near with  $L_B$ ; if the instantaneous  $\Delta t$  is large enough, the value of  $L$  would be near with  $L_{QM}$ . The value of  $L$  is between  $L_B$  and  $L_{QM}$  depending upon the processing time  $\Delta t$ .  $L_B$ , the Bohr orbital angular momentum for a single run, is just the physical quantity that was to be said as the spin of electron. For s orbital, because its  $\psi$  is symmetric as a sphere, vectors of  $L_B$  are counteracted. So, it gave a result of  $L = 0$ . For p, d, and f orbital, the vectors of  $L_B$  can't be counteracted in a whole, they gave the statistical results. Its maximum value is  $L_{QM}$ . The shape of Bohr orbit is ought to be a round or ellipse. Though the orbit is a plane curve, because of the wave character and environmental disturbance, the orbit would become a three-dimensional figure. The Bohr orbit may has certain relations with orbit  $\psi$  in quantum mechanics. However, it must be mentioned that, it can't be misunderstood that taking the orbit  $\psi$  as a simply statistical result of Bohr orbit.

### 3.3. The Physical Meaning of $\pm$ signs in $\psi$

The circular running electron has its Bohr orbital magnetic moment correspondingly. It can be expressed as,

$$\mathbf{M}_B = -(e/2m_e)\mathbf{L}_B = n\mu_B,$$

Where,  $\mu_B$  is the Bohr's magneton, its absolute value is  $9.274 \times 10^{-24}$  J/K,  $n$  is the main quantum number.

An orbital magnetic moment ( $\mathbf{M}_B$ ) has a proper value. It can attract another orbital magnetic moment (which has the opposite direction) to be near with and running in the same orbit by overcoming the repulsion of electron charge. That's just the reasons of Pauli's rule.  $\psi_+$ , and  $\psi_-$  were appeared in the calculation of the solid part of  $\psi$ . It means they had opposite direction of  $\mathbf{L}_B$  in different regions. The positive value and negative value are correspondingly. We can't say that a positive value has any superiorities than the negative one. For two s orbits of electron, they may have the same positive value of  $\psi$  in their individual system.

If they were introduced in one system, their  $\mathbf{L}_B$  would have different directions. If their  $\mathbf{L}_B$  had same direction, we could describe them as positive  $\psi_+$ . If their  $\mathbf{L}_B$  had opposite direction, we'd like to describe one of them as positive  $\psi_+$ , whereas the other must be  $\psi_-$ . If the two s orbitals interacted with each other, it would appear two conditions:

$$\psi_1 = \psi_+ + \psi_-; \psi_2 = \psi_+ - \psi_- \dots\dots\dots (12)$$

Where,  $\psi_1$  and  $\psi_2$  were corresponded to bonding orbital and antibonding orbital respectively in quantum chemistry.

It needs to mentioned that, there are no minus sign appeared in equation (12). That's a reasonable thing and was just we anticipated. Because there was no minus operation existence for two to be closer  $\psi$ . For p orbits of electron, it's  $\psi$  has the shape like "8". It may be treated as an electron that running in an orbit of ellipse. Let the focus of ellipse to be at the original point of a polar coordinates. When the electron running near the nuclear (original point), it is easy to change into its opposite orbit (showed in Figure 1a) because of its wave characters. As the inertia action, the electron got an opposite direction of  $\mathbf{L}_B$  naturally. One part has the positive value, the other negative. Due to the wave characters, the electron could act with its fields of tracks. In other words, the two parts in figure 1a could interact with each other. The action can be expressed in mathematics. The function of the orbits (ellipse) of two parts in figure 1a can be expressed in polar coordinates as,

$$r_1 = b^2/a(1 - \epsilon \cos\theta)^{-1} \dots\dots\dots (13)$$

$$r_2 = -b^2/a(1 - \epsilon \cos(\pi - \theta))^{-1} \dots\dots\dots (14)$$

Where,  $a$  is the radius of semi-major axis of ellipse;  $b$  the semi-minor axis;  $\epsilon$  the eccentricity of ellipse.

Combing (13) and (14), we have,

$$r = r_1 + r_2 = b^2/a(1 - \epsilon \cos\theta)^{-1} - b^2/a(1 - \epsilon \cos(\pi - \theta))^{-1} \dots\dots\dots (15)$$

The graphics of equation (15) was expressed in Figure 1b. It is just the picture like  $p_z$  orbital (solid part) which we are familiar in quantum chemistry. At the center (the tangent original point), the electron has equal opportunities to go left or right. It corresponds to  $\psi_+$  or  $\psi_-$ .

### 3.4. The Magnetism of Orbitals (s, p, d, f)

For a running electron in an atom, the properties of electron were showed us as a statistical result of Bohr's orbit. The magnetic moment ( $\mathbf{M}$ ) of an electron in an orbit depends on the statistical orbital angular momentum ( $\mathbf{L}$ ). They have the relation as,

$$\mathbf{M} = -(e/(2m_e))\mathbf{L} \dots\dots\dots (16)$$

If the processing time,  $\Delta t$ , is large enough, it would has  $\mathbf{L} = \mathbf{L}_{QM}$ . If the processing time,  $\Delta t$ , is small enough, the value of  $\mathbf{L}$  would be near with  $\mathbf{L}_B$ , the  $\mathbf{M}$  with  $\mathbf{M}_B$  even for s orbital, i.e., every orbit including s would has their magnetism in such case.

For s orbit of electron,  $l = 0$ ,  $\mathbf{M} = 0$ . So, s orbital has no magnetism. That's just the properties we see for s-block and ds-block elements. In addition, it doesn't contradict the prediction that CuO and CuS molecules would have their magnetism.

For p orbits of electron,  $l = 1$ ,  $\mathbf{M}_+ = 2^{1/2}eh/(4\pi m_e)$ ,  $\mathbf{M}_- = -2^{1/2}eh/(4\pi m_e)$ . However,  $\mathbf{M}_+$  and  $\mathbf{M}_-$  can't be counteracted to zero exactly, because  $\mathbf{M}_+$  and  $\mathbf{M}_-$  acted in different regions of space from the point of view of an atom self. But in a whole, to the surrounding space outside the atom, it seems like that two semi-electrons running in one orbit. The magnetism was counteracted thoroughly. So, p orbital has nearly no magnetism. That's just we were familiar with the elements in p-block.

For d orbits of electron,  $l = 2$ ,  $\mathbf{M}_+ = 6^{1/2}eh/(4\pi m_e)$ ,  $\mathbf{M}_- = -6^{1/2}eh/(4\pi m_e)$ . Similarly with above p orbital, the magnetism of most d orbital ( $d_{xy}$ ,  $d_{xz}$ ,  $d_{yz}$ ,  $d_{x^2-y^2}$ ) was counteracted near zero except the  $d_{z^2}$  orbital.

Because for  $d_{z^2}$  orbital, the  $\psi_+$  and  $\psi_-$  had different distributions in space; they had quite different shape and size and had no plane of inverse (in sign) symmetry in them. So, after partial counteracted, the  $d_{z^2}$  orbital would have some residual magnetism in all. As we known that, all the metals of element in d-block are paramagnetic.

For f orbits of electron,  $l = 3$ ,  $\mathbf{M}_+ = 12^{1/2}eh/(4\pi m_e)$ ,  $\mathbf{M}_- = -12^{1/2}eh/(4\pi m_e)$ . The situations are as the same as p orbital. If the f orbital of f-block elements were alike with hydrogen atom, the  $\psi_{\pm}$  in all the seven f orbital is inverse (in sign) symmetric with respect to reflection. The magnetism in all the f orbital were counteracted near zero with no exceptions even for  $f_{z(5z^2-3r^2)}$  orbital. Actually, the metals of element in f-block are also paramagnetic; that's may because of the easy transitions of electron into the extremely adjacent 5d or 6d empty orbits at normal temperature.

Those experiments (the deflexion of Ag atom in magnetic field—the experiment by Stern–Gerlach in 1922; and the similar experiment of H atom—by Phipps–Taylor in 1927) which were considered to be the evidence of electron spin were just confirmed the existence of the instant magnetism of electron in an atom. The experiments are also in agreement with our illative result as in a case for  $\Delta t$  to be small enough. Apparently, if we still held the old conception of electron spin, the problem of magnetism would not be explained as well as above statements.

### 3.5. The Real Spin of Electron

As a particle, electron could have a degree of freedom of its spin (the real spin). If the spin has probably the same energy ( $E$ ) with kinetic energy, the maximum angular momentum ( $L_s$ ) of spin could be estimated as,

$$E = 2^{-1}(2mr^2/5)(L_s m^{-1}r^{-2})^2 = 13.6Z^2/n^2$$

$$L_s = \pm (5mE)^{1/2}r \dots\dots\dots (17)$$

$$\begin{aligned} L_s &= \pm (5 \times 9.1095 \times 10^{-31} \times 2.18 \times 10^{-18} Z^2/n^2)^{1/2}r \\ &= \pm 3.15 \times 10^{-24}rZ/n = \pm 3.15 \times 10^{-24} \times 2.15 \times 10^{-15}Z/n \\ &= \pm 6.77 \times 10^{-39}Z/n \text{ J} \cdot \text{s} \end{aligned}$$

$$L_s = \pm 6.42 \times 10^{-5}L_B, \text{ (as } Z = n = 1)$$

Where,  $m$ , and  $r$  are the rest mass and the radius of electron (Mei & Mei 2013),  $Z$  and  $n$  are the charge of nuclear and the main quantum number respectively.  $2.18 \times 10^{-18} \text{ J} = 13.6 \text{ eV}$ .

It showed that, the real spin angular momentum of electron is far more less than the orbital one.

## 4. TAGS

It is a complex and difficult thing to deduce the shape of other d or f orbit of wave function  $\psi$  from particle characters or Bohr orbit. It is as well as an unnecessary thing. We just want to insist on the idea that the particle characters of electron must be added into modern quantum mechanics. It would play a proper role and produce necessary results. Especially the correct understanding of the spin of electron would help us studying the manipulation or polarization of the spin of electron in materials.

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