Electro conductivity of generalized MHD Burgers’ fluid flow due to a sinusoidal accelerating plate

Ngiangia AT and Nwabuzor PO

Department of Physics, University of Port Harcourt, P M B 5323 Choba, Port Harcourt, Nigeria

Corresponding author
Department of Physics, University of Port Harcourt, P M B 5323 Choba, Port Harcourt, Nigeria
Email: peter_nwabuzor@yahoo.com

Article History
Received: 16 July 2019
Accepted: 04 September 2019
Published: September 2019

Citation
Ngiangia AT and Nwabuzor PO. Electro conductivity of generalized MHD Burgers’ fluid flow due to a sinusoidal accelerating plate. Science & Technology, 2019, 5, 195-204

Publication License
This work is licensed under a Creative Commons Attribution 4.0 International License.

General Note
Article is recommended to print as color digital version in recycled paper.

ABSTRACT
In this paper, we present the solution to the MHD Burgers’ fluid due to a sinusoidal accelerating plate using the method of fractional derivative combined with numerical method for the hydrodynamics equations and Laplace transform for the energy equation. The result revealed that the fractional parameters $\alpha$ improved the fluid flow but the fractional parameter $\beta$ impeded on the flow of the fluid. This clearly indicates that they both had opposite effect on the flow of the fluid. However, the relaxation time and the retardation time both increased the shear stress profile of the fluid.

1. INTRODUCTION
Generalized Burgers fluid falls into the category of Non-Newtonian fluids, referred to as viscoelastic fluids. They flow like fluids and deform like solids [1] or they exhibit both elastic and viscous properties. Examples are gels and pastes. The viscosity of these classes...
of fluids decreases with stress. They are used in damping noise, absorbing shock and as exotic lubricants just to mention few. The flow behavior index of Newtonian fluids is unity while that of non-Newtonian fluids is not. Therefore for effective and reliable theoretical study or description of viscoelastic fluids flow and heat transfer, the use of fractional derivative to model the governing equations is of necessity. Advances of fractional differential equations are being given attention due to its application to nonlinear complex systems arising in life sciences, fluid mechanics electrochemistry and physics [2-8]. The rheological characteristics of viscoelastic fluid using the application of fractional differential equations were reinforced [4-5]. The exponential stretching sheet of viscoelastic fluid flow in two dimensions was tackled by Khan [9] and Sahoo and Poncet [10].

Liu et al. [11] tackled MHD flow and the transfer of heat of a generalized Burgers’ fluid due to an exponential accelerating plate with radiation effect. They found out that the non-Newtonian effects are stronger at larger values of time and the greater the value of the time the higher the temperature. Sultan et al. [12] worked on flow of generalized Burger fluid between parallel walls induced by rectified sine pulse stress. They observed that as the bottom plate is set into oscillation with respect to y-coordinate, amplitude of oscillations decreases from the plate. The velocity increases from zero to a maximum in the middle of the channel.

Khan et al. [13] studied the stokes’ second problem of magneto hydrodynamics flow in a burgers’ fluid using the Laplace transform technique. The expressions of velocity field and tangential stress are developed when the relaxation time satisfies a certain condition. The obtained closed form solutions are presented in the form of simple or multiple integrals in terms of Bessel functions. They found out that the velocity decreases whereas the shear stress increases when the Hartman number is increased. Salah et al. [14] worked on steady state solution for magneto hydrodynamics rotating flow of generalized burgers’ fluid in a porous medium by means of Fourier transform. They observed that the steady-state solution corresponds to the motion of generalized burgers’ fluid due to the constant acceleration of an infinite flat plate.

Aziz et al. [15] tackled MHD Steady state solution for rotating flow of burgers’ fluid. They obtained a result showing the effects of material parameters on MHD generalized Burgers’ fluid on the real point of velocity profile. Furthermore, Ibraheem and Abdulhadi [16] by using discrete Laplace transform of the sequential fractional derivative studied the pressure gradient influence on MHD flow for generalized Burgers’ fluid with slip condition. Subsequently, Zhang et al. [17], investigated heat and mass transfer of MHD flow of Burgers fluid with effects of the second order slip and their results shows that the influences of the fractional parameters α and β on the flow are to each other opposite. Another interesting discovery they made is that the impact trends of the relaxation time λ1 showed that the profile of the velocity increases but the retardation time λ2 decreased the velocity profile of the fluid. An interesting work of Zheng et al. [18-20] obtained the fractional energy equation of a generalized Maxwell fluid. Khan et al. [21-22] treated MHD flows of a generalized Oldroyd-B fluid and a generalized Burgers’ fluid. Furthermore, the related velocity distributions were calculated using Fourier transform together with Laplace transform techniques for the fractional calculus. The above study indicates that the magnetic field provides a force, which resists the flow since it is applied in the transverse direction.

Khan et al. [23-25], examined the use of integral transform in analysis rotating flow, accelerating flow and oscillating flow of generalized Burgers fluid flow. The influence of differential pressure on generalized burgers fluid flow was also tackled by Ghada et al. [26]. Some scholars in the past have worked on MHD flows of viscoelastic fluid. They include but not limited to Khan et al. [27-28], Ezzat [29] and Zhang et al. [17]. Ngiangia and Orukari [30] examine rudimentary characteristics of viscoelastic fluids and deduced that increase in relaxation constant and friction/viscosity, reduces the distance of the fluid flow while increase in the rational frequency increase the distance of the fluid flow.

All the literatures cited in this work, none has incorporated the influence of electro conductivity, which is an impending factor in fluid flow phenomena and enhances the effect of applied magnetic field if present, is yet to be tackled. The aim in this work is to examine the effect or otherwise of electro conductivity and radiation on generalized Burgers’ fluid flow due to sinusoidal accelerating plate. The solution of the fractional model is obtained by assuming that the Grünwald–Letnikov fractional derivative is equivalent to the Riemann–Liouville fractional derivative and Laplace transform used to determine the solution of the energy equation.

2. MATHEMATICAL FORMULATION OF THE PROBLEM

The flow of an incompressible generalized Burgers’ fluid due to a sinusoidal accelerating plate is considered. It is assumed that the flow occupies the space \( y > 0 \). The velocity and shear stress field are thus:

\[ U = u(y,t), \quad \tau = \tau(y,t) \]
Where $\mathbf{U}$ is the fluid velocity and $\mathbf{i}$ is the unit vector in the $x$ - direction. The initial condition $s(y,0) = 0$. Also $\tau_{xx} = \tau_{yy}$ and 

\[ \tau_{xy} = \tau_{yx} \]

Following [31], the fractional constitutive model for generalized Burgers’ fluid is given as 

\[
( 1 + \lambda_1^\alpha D_t^\alpha + \lambda_2^\alpha D_t^{2\alpha} ) \mathbf{\tau} = \mu ( 1 + \lambda_1^\beta D_t^\beta ) \frac{\partial u}{\partial y} \tag{2}
\]

where $\mu, \lambda_1, \lambda_2$ are respectively the dynamic viscosity, relaxation time and retardation time.

The fractional derivative of order $a > 0$ is defined as 

\[
D^\alpha f(x) = \frac{1}{\Gamma(m-a)} \int_0^x \left[ \frac{d^m}{d(x-t)^m} f(t) \right] dt, \quad m-1 < a < m
\]

\[
\frac{d^m}{dx^m} f(t), \quad a = m
\]

Where $\Gamma$ is gamma function.

Since the flow is not influenced by the pressure gradient, the governing equations of the problem reduced to 

\[
\frac{\partial u}{\partial y} = 0
\]

\[
\rho \frac{\partial u}{\partial t} = \mu \frac{\partial^2 u}{\partial y^2} - (\sigma_0 + \beta_0^2)
\]

\[
\rho C_p \frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial y^2} - \frac{\partial q}{\partial y}
\]

Where $\rho, C_p, \mu$ are the density, specific heat capacity and viscosity respectively.

Substituting eq (2) into eq (5) the hydrodynamical equations, takes the form 

\[
( 1 + \lambda_1^\alpha D_t^\alpha + \lambda_2^\alpha D_t^{2\alpha} ) \frac{\partial u'}{\partial t'} = \nu ( 1 + \lambda_2^\beta D_t^\beta ) \frac{\partial^2 u'}{\partial y^2} - \sigma_0\left[ 1 - \frac{\lambda_1^\alpha D_t^\alpha + \lambda_2^\alpha D_t^{2\alpha}}{\mu} u' \right] - \frac{\sigma \beta^2}{\rho} \left[ 1 - \frac{\lambda_1^\alpha D_t^\alpha + \lambda_2^\alpha D_t^{2\alpha}}{\mu} \right] u'
\]

\[
( 1 + \lambda_1^\alpha D_t^\alpha + \lambda_2^\alpha D_t^{2\alpha} ) \mathbf{\tau}' = \mu ( 1 + \lambda_2^\beta D_t^\beta ) \frac{\partial u'}{\partial y'}
\]

\[
\rho C_p \frac{\partial T}{\partial t'} = k \frac{\partial^2 T}{\partial y^2} - \frac{\partial q}{\partial y'}
\]

According to [32] for optically thin medium with relatively low density, 

the radiative heat flux is given by 

\[
\frac{\partial q}{\partial y'} = 4 \delta^2 (T - T_0)
\]

where $\delta$ is the radiation absorption coefficient.

With the initial and boundary conditions given as 

\[
u = \frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} = 0, \quad t = 0
\]

\[
\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} = 0, \quad \tau = \frac{\partial \tau}{\partial t} = 0
\]

\[
\left. u(0,t) = u_0 \sin \alpha y \quad t > 0 \right)
\]

\[
T(y,0) = T_{\infty} \quad y > 0
\]
\[ T(0, t) = T_\infty \text{Sinh} y \quad t > 0 \]
\[ T(y, t) \to T_\infty \quad y \to \infty \]

Applying the following dimensionless parameters

\[
\begin{align*}
\theta &= \frac{T}{T_\infty}, u = \frac{u'}{u_0}, y = \frac{y'}{\nu}, t = \frac{t'}{\nu}, u' = \frac{u'}{\mu}, \lambda_1 &= \frac{\lambda'_1 u}{\nu}, \lambda_2 &= \frac{\lambda'_2 (u')^2}{\nu}, \\
\lambda_3 &= \lambda'_3 \frac{u'}{\nu}, \mu = \frac{\mu}{\nu}, M = \frac{\sigma B_0^2}{\rho u_0^2}, N = \frac{\nu \tau'}{\mu u}, N = \frac{4 \delta^2 y^5 c v}{\rho C_p k}, \\
\sigma_0 &= \frac{\sigma_0 a v' t'}{u}
\end{align*}
\]

Equation (7), (8) and (9) can be rewritten as

\[
(1 + \lambda_1^a D_t^a + \lambda_2^a D_t^2a) \frac{\partial u}{\partial t} = \nu (1 + \lambda_3^\beta D_t^\beta) \frac{\partial^2 u}{\partial y^2} - \sigma_0 (1 - (1 + \lambda_1 D_t^a + \lambda_2 D_t^{2a}) u - M (1 + \lambda_1 D_t^a + \lambda_2 D_t^{2a}) u \tag{12}
\]

\[
(1 + \lambda_1^a D_t^a + \lambda_2^a D_t^{2a}) \tau = \mu (1 + \lambda_3^\beta D_t^\beta) \frac{\partial u}{\partial y} \tag{13}
\]

\[
\frac{\partial \theta}{\partial t} = \rho r^{-1} \frac{\partial^2 \theta}{\partial y^2} - N \theta \tag{14}
\]

If the fractional derivative model is used to replace the time derivative term, equation (14) takes the form

\[
\frac{\partial^a \theta}{\partial t^a} = \rho r^{-1} \frac{\partial^2 \theta}{\partial y^2} - N \theta \tag{15}
\]

Applying the Laplace transform on equation (15), we get

\[
L \frac{\partial^a \theta}{\partial t^a} = \rho r^{-1} L \frac{\partial^2 \theta}{\partial y^2} - NL \theta \tag{16}
\]

\[
L \theta(y, t) = \rho r^{-1} (S^2 \bar{f}(s) - S - 1) - NS \bar{f}(s)
\]

Simplification and obtaining the inverse Laplace transform of equation (16), gives the solution

\[
\theta(y, t) = \rho r^{-1} (Cosh\sqrt{N}y - \frac{1}{N}Sinh\sqrt{N}y) \tag{17}
\]

To solve equations (12) and (23) following Zhan et al [17] the assumption that, under the given initial conditions, the Grunwald-Letnikov fractional derivative is equivalent to the Riemann-Liouville fractional derivative. Based on the G1 algorithm, we define the following

\[
t_n = n \delta(n = 0, 1, 2, \ldots Q) \]
\[
y_i = i h(i = 0, 1, 2, \ldots W) \]
where $\delta = \frac{t}{Q}$ is the time step if we take $u_i$ to be the numerical approximation, to $u_i(t_n)$ and also discretize the fractional derivative of order $P$ as

$$D_i^p f(t_n) \approx \delta^{-p} \sum_{k=0}^{n} W_k^{(p)} f(t_n - k\delta)$$

(20)

where $W_k^{(p)} (k = 0, 1, 2, ..., n)$ are the Grunwald-Letnikov Coefficients and defined by

$$W_i^p = 1, W_k = (1 - \frac{s + 1}{k}) W_{k-1}^{(p)}, k = 1, 2, 3,...$$

(21)

$$\frac{\partial u}{\partial t} = \frac{u_i - u_{i-1}}{\delta} + 0(\delta)$$

(22)

Equations (20) and (22) is substituted into equations (12) and (13), we get

$$\frac{u_i^n - u_{i-1}^{n-1}}{\delta} + \lambda_1 \delta^{-1} \sum_{k=0}^{n} W_k^{(a+1)} u_{i-n+k} + \lambda_2 \delta^{-2a} \sum_{k=0}^{n} W_k^{(2a+1)} u_{i-n+k} = \mu \left[ u_{i+1}^{n} - 2u_i^n + u_{i-1}^n \right] / h^2$$

$$+ \lambda_3 \delta^{-b} \sum_{k=0}^{n} W_k^{(b+1)} u_{i-n+k-2} - \sigma_0 \left[ 1 - (u_i^n + \lambda_1 \delta^{-a} \sum_{k=0}^{n} W_k^{(a)} u_{i-n+k}) + \lambda_2 \delta^{-2a} \sum_{k=0}^{n} W_k^{(2a)} u_{i-n+k} \right] / u - m$$

(23)

$$u_i^n + \lambda_1 \delta^{-a} \sum_{k=0}^{n} W_k^{(a)} u_{i-n+k} + \lambda_2 \delta^{-2a} \sum_{k=0}^{n} W_k^{(2a)} u_{i-n+k}$$

$$- \lambda_3 \delta^{-b} \sum_{k=0}^{n} W_k^{(b)} u_{i-n+k} - \sigma_0 \left[ 1 - (u_i^n + \lambda_1 \delta^{-a} \sum_{k=0}^{n} W_k^{(a)} u_{i-n+k}) + \lambda_2 \delta^{-2a} \sum_{k=0}^{n} W_k^{(2a)} u_{i-n+k} \right] / u - m$$

(24)

Figure 1 Velocity distribution for different $\alpha$. $\alpha = 0.2, 0.4, 0.5, 0.6, 0.8, 1.0.$
3. RESULTS AND DISCUSSION

The velocity, shear stress and temperature solutions where obtained by numerically solving equations (17), (23) - (24). To study the various effects of the materials parameters on the distribution of velocity, temperature, space step and shear stress fields, the numerical results are plotted in Figure 1-10.

Figure 1-4 clearly shows the effect of the fractional parameter (α and β) together with the relaxation and the retardation time (λ₁ and λ₃). Figure 1 shows that when the fractional parameter α is increased there is a corresponding increase in the velocity distribution of the fluid. This indicates that the fractional parameter α improves on the flow of the fluid. But we noticed a reverse behavior for β on the velocity profile as shown in Figure 2. These results are in strong agreement with the findings of Zhang et al. [17]. The retardation time λ₃ and the relaxation time λ₁ effect on the velocity profile are shown in Figure 3 – Figure 4.

![Figure 2](image2.png)

**Figure 2** Velocity distribution for different β. β = 0.3, 0.5, 0.7, 0.9, 1.2.

From Figure 3 we noticed that the velocity profile increases with an increase in the relaxation time λ₁ which best describes the stress relaxation characteristics of viscoelastic fluid. It is observed from Figure 4 that the Retardation time λ₃ enhances the flow. It is seen from figure 4 that an increase in the retardation time increases the velocity distribution of the flow.

![Figure 3](image3.png)

**Figure 3** Velocity distribution for different λ₁. λ₁ = 1.0, 5.0, 10.0, 15.0, 20.0.
Figure 4 Velocity distribution for different $\lambda_3$. $\lambda_3 = 2.0, 7.0, 12.0, 17.0, 23.0$.

The illustration of the influence of the Prandtl number and the Radiation term on the temperature profile of the fluid are shown in Figure 5 and figure 6. Figure 5 illustrates that lowering the temperature distribution is as a result of larger values of the Pr parameter. A different reaction was observed in Figure 6 were the radiation term N when increased, brought about an increase in the temperature distribution.

Figure 5 Temperature distributions for different Prandtl (Pr). $Pr = 0.41, 0.51, 0.61, 0.71, 0.81$.

Figure 6 Temperature distributions for different Radiation term (N). $N = 0.97, 1.97, 2.97, 3.97, 4.97$. 
From Figure 7, the relaxation time $\lambda_1$ when increased lead to an increase in the shear stress of the fluid. A similar reaction occurred in Figure 8 when the retardation time $\lambda_3$ was improved, the shear stress profile also increased.

**Figure 7** shear stress distributions for varying Retardation time parameter $\lambda_3$.

$\lambda_3 = 2.0, 7.0, 12.0, 17.0, 23.0$.

**Figure 8** shear stress distributions for varying Relaxation time parameter $\lambda_1$.

$\lambda_1 = 1.0, 5.0, 10.0, 15.0, 20.0$.

Figure 9-10 showed the space step profile for both of the fractional parameters. Figure 9 showed that when the fractional parameter $\beta$ is increased the space step profile decreases. However, the behavior of the fractional parameter $\alpha$ was different in the space step. When the fractional parameter $\alpha$ increases the space step increases accordingly.

**Figure 9** Space step distribution for varying fractional parameter $\beta$. $\beta = 0.3, 0.5, 0.7, 0.9, 1.2$. 
4. CONCLUSION

This paper numerically provides an analysis for an incompressible generalized Burgers’ fluid due to a sinusoidal accelerated plate. The solutions were obtained by using fractional derivative combined with numerical methods and the Laplace transform. The various effect of the fractional parameter, relaxation time, retardation time, Prandtl number and the radiation number on the temperature profile, velocity profile the space step and the shear stress profile are analyzed. The results gotten shows that the fractional parameters on the velocity profile are opposite. While, the relaxation and the retardation time have the same effect on the velocity profile. The Prandtl number had a different effect on the on temperature distribution with the radiation number. The relaxation and the retardation time had the same effect on the shear stress of the fluid. However, we noticed the fractional parameter had different reaction on the space step profile.

REFERENCE