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Distribution of number and density of electrons in quadrupole Penning trap

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ABSTRACT

In ion traps a single charged particle to few millions are trapped and studied to characterize them. The number distribution of the trapped electrons with trap voltage is measured. The data acquired is fitted to a polynomial function and reveals the distribution of number of the trapped electrons from 50,000 to 3,00,000 for variation of trap voltage from 2 V to 9 V at filament current of 1 A. The total number of particles per unit volume of a trap is the number density of plasma. There are no direct methods to measure it exactly, but can be estimated by analysis of shape of detection signal and space charge shifts. The peak of signal amplitude or the product of the peak of signal amplitude and the full width at half maximum of signal are directly proportional to the number of trapped electrons. The analysis of resonance absorption signals of electrons yield the number density of trapped electrons. The number density of trapped electrons estimated from both space charge shifts and analysis of signal shape of resonance absorption methods are comparable in million ranges. In this work we present results on distribution of number of the trapped electrons with

trap voltage. The number densities estimated from analysis of shape of detection signal and space charge shifts are comparable in the range$\sim 10 \times 10^{6} mm^{-3} - \sim 20 \times 10^{6} mm^{-3}$.

Key words: Number density, Resonance absorption signal, Space charge shifts, non-neutral plasma of electrons, Quadrupole Penning trap

PACS: 52.25.b: Plasma properties, 52.27.Jt: Non neutral plasmas, 25.70.Ef : Resonances

1. INTRODUCTION

In ion traps from a single charged particle to few million particles are trapped and studied to characterize various parameters. A single particle was trapped in many experiments and its fundamental properties were studied. We have measured the number distribution of the trapped electrons with trap voltage (Durgesh Datar, et al., 2016). The data is fitted to a polynomial function and reveals the distribution of number of the trapped electrons from 50,000 to 3,00,000 for variation of trap voltage from 2 V to 9 V at filament current of 1 A. It is not always possible to trap a single particle due to limitations of designs of traps, surface imperfections and other anharmonicities, however a very large number of particles are trapped. The electrons whose energies lower than the depth of the trap get confined in the trap. The total area under resonance absorption signals measure the total number of electrons being trapped (Durgesh Datar, et al., 2016; K T Satyajit, 2010). The total number of particles per unit volume of a trap is called the number density. It is necessary to measure the number density to characterize the plasma of trapped electrons. There are no direct methods to measure the number density of trapped electrons exactly, but it can be estimated by space charge shifts and analysis of shape of detection signal (K T Satyajith, et al., 2009; Soumen Bhattacharya, et al., 2006). The electric potential of space charge alters the harmonic potential inside the trap and also shifts the trapping potential and frequencies (Jeffries J, et al. 1983).

We develop a model in which the axial oscillations of electrons are considered like damped harmonic oscillators (Gaboriaud, et al., 1981). The trapped electrons are probed by an external RF field coupled to detection circuit. We can see that the trapped electron system is electrically equivalent to that of series LCR circuit (X Peng, et al., 1995). The data was acquired, the shape of detection signal was analyzed and peak signal amplitude versus time was plotted (K T Satyajith, et al., 2009; Soumen Bhattacharya, et al., 2006). The peak of signal amplitude or the product of the peak of signal amplitude and the full width at half maximum of signals are directly proportional to the number of trapped electrons (K T Satyajith, et al., 2009; Soumen Bhattacharya, et al., 2006). The total number of trapped electrons was measured by fitting the RF absorption signal to the relative electron response signal.

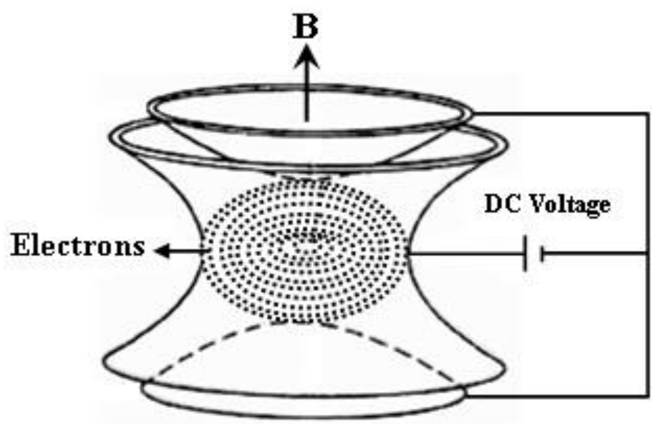

Figure 1 Quadrupole Penning trap with cloud of electrons

2. THEORY

The quadrupole Penning trap is designed with three-electrode infinite hyperboloid revolution of structure, which consists of two end-cap electrodes and a ring electrode. The equation of hyperbola of ring electrode is $\frac{r^2}{r_0^2} - \frac{z^2}{z_0^2} = +1$ and that of two similar end-cap electrodes are given by $\frac{r^2}{r_0^2} - \frac{z^2}{z_0^2} = -1$(F G Major, et al., 2002; P K Ghosh, 1995; R D Knight, 1983). Where $r_0 = \sqrt{2}\, z_0 = 7mm$, r_0 is the inner radius of the ring electrode in the radial plane and $z_0 = 5mm$ is half of the vertical distance between the two end-cap electrodes as shown in figure 1. The Penning trap uses a time-independent, spatially homogeneous uniform magnetic field along positive z-axis of the trap. The end-cap electrodes are biased with a positive dc voltage for positive ion confinement or negative dc voltage for negative ion or electron confinement with respect to the ring electrode (F G Major, et al., 2002; P K Ghosh, 1995). Therefore there is purely quadrupolar potential in quadrupole Penning trap (R D Knight, 1983).

Number distribution of trapped electrons:

The number distribution of the trapped electrons varies with the trap voltage (Durgesh Datar, et al., 2016; K T Satyajit, 2010). The distribution function in terms of variation of electric current with voltage is given by (Durgesh Datar, et al., 2016; K T Satyajit, 2010).

$$f(E) = -\frac{1}{e}\frac{dI}{dV}$$

The electrons whose energies lower than the depth of the trap get confined in the trap. The area under resonance signal measures the total number of electrons being trapped. The data of resonance signal versus voltage is obtained and fitted to a polynomial function and the number distribution of the trapped electrons with trap voltage is measured.

Estimation of the number density of trapped electrons:

There are two methods to estimate the number density of electrons trapped in Penning trap viz. (i) Measurement of space charge shifts and (ii) Analysis of the shapes of resonance absorption signals.

Space charge and image charge shifts:

The ions trapped in Penning trap interact through Coulomb force and modify the motion, this is known as space charge effect. The space charge effects cause small shift in Eigen frequencies of electrons viz. axial, pure cyclotron, modified cyclotron and magnetron frequencies in the trap. The image charges are induced across trap electrodes and anharmonicty in the trap also cause small shift in Eigen frequencies. A very large number ($\sim 10^6$) of charged particles are trapped in Penning trap, the additional space charge potential of a trapped ion cloud shifts the Eigen frequencies ($f_z, f_c, f_c^{'}$ & f_m) of individual electrons are called the space charge shifts (Beck D et al., 2001). The opposite charges induced inside the trap on the surfaces of trap electrodes by oscillating trapped electrons inside the trap are called image charges. The image charges create an electric field that interacts with the stored electrons and shifts its motional frequencies ($f_z, f_c, f_c^{'}$ & f_m) is called the image charge shift. Space charge shifts the Eigen frequencies of the single particle motion in the trap (Wineland D J and Dehmelt H G, 1975). Image charges create an electric field and also shift the axial frequency of the trapped electrons. Frequency shifts also caused by anharmonicity in the trap. The space charge shifts and asymmetry of trap cause additional loss, and limit the trapping of maximum ion density in the central region of the trap (Yu I, et al., 2006). The electric potential of space charge alters the harmonic potential inside the trap and also shifts the trapping potential and frequencies (Jeffries J et al., 1983). When a cloud of electrons is trapped, then a single electron inside the cloud experiences an additional potential due to other electrons called the space charge potential ($U^{'}$). It can be derived similar to the quadrupolar potential of the trap. The trapped ion cloud assumes almost a spherical shape with constant, homogeneous charge density in the radial direction (D F A Winters, et al., 2006). The number of electrons trapped is given by

$$N = \frac{12\epsilon_0 V}{qd^2}\Delta U \qquad (1)$$

The number density of electrons trapped is given by

$$n = \frac{N}{V} = \frac{12\epsilon_0}{qd^2}\Delta U \qquad (2)$$

The number density from analysis of the shape of resonance signal:

We develop a model in which the axial oscillations of trapped charged particles are considered like damped harmonic oscillators (Gaboriaud, et al., 1981). The trapped electrons are probed by an external RF field. The electron-electron interactions, collisions between electrons and back ground gas particles and imperfections in the trap are included in the damping term. The centre of mass of cloud of trapped electrons exhibit forced damped harmonic oscillations along the symmetry axis of the trap and the equation of oscillatory motion of trapped electrons is given by (X Peng, et al., 1995)

$$\ddot{z} + \gamma\dot{z} + \omega_z^2 z = \gamma^{'}\left(\frac{eV_0}{2mz_0}\right) \leftrightarrow \frac{\ddot{q}}{L_i} + \left(\frac{R_i}{L_i}\right)\dot{q} + \frac{q}{L_iC_i} = \frac{V_0}{L_i}$$

Where, γ is the damping constant, $\gamma^{'}$=0.86 is a correction factor for non-parallel electrodes, ω_z is radial axial angular frequency, e is charge of electron and m is mass of electron, V_0 is the probing voltage applied across the end-cap electrodes and z_0 is the position co-ordinate of centre of mass of electron cloud (Gaboriaud, et al.,1981). The external RF excitation frequency (f_0) matches with the axial oscillation frequency (f_z) of electron, then resonance occurs and the electrons offer minimum impedance. The relative electron response signal is given by (X Peng, et al., 1995)

$$Y = \left[1 - \frac{V_0}{V_{00}}\right]$$

Where $V_{00} = \omega_0 RCV_i$ is the value of V_0 at resonance frequency of detection tank circuit in the absence of the electron cloud in the trap. The peak of signal amplitude (PSA) or the product of the peak of signal amplitude and the full width at half maximum (PSA×FWHM) of detection of signal electrons are directly proportional to the number of trapped electrons (K T Satyajith, et al., 2009; Soumen Bhattacharya, et al., 2006).

3. EXPERIMENTAL PROCEDURES

Number distribution of trapped electrons:

The number of electrons at the centre of the trap is measured using nondestructive technique at different trapping potentials. The axial oscillatory motion of the electrons is monitored through the Lab VIEW program. The number of electrons at the centre of the trap is a function of the storage potential which results a graph of the storage potential versus area under detection signal of electrons. The strength of resonance absorption signal of electrons is directly proportional to the number of electrons trapped in ion trap (K T Satyajith, et al., 2009; Soumen Bhattacharya, et al., 2006). Initially the storage potential is held at 10 V and then reduced to a potential V, which causes electrons with energy higher than V leave the trap. After a certain time t, the detection is carried out and the signal strength is recorded. Subsequently measurements are made by reducing the potential V in steps of 0.5 V. The signal strength is measured and the area under the signal is found at different voltages. The strength of resonance absorption signal of electrons is directly proportional to the number of electrons trapped in ion trap are recorded.

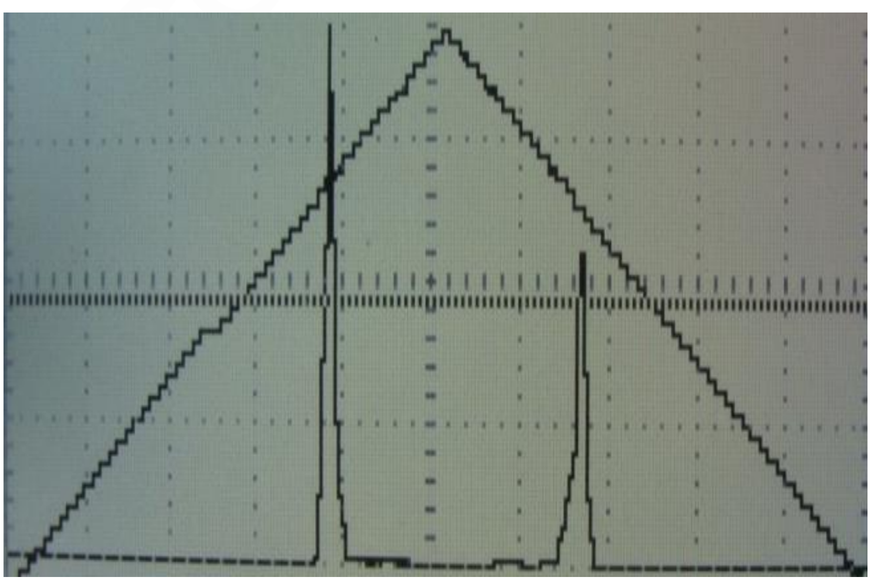

Figure 2 A pair of clear and sharp signals of electrons on CRO at trapping voltages

The number density from space charge shifts:

The space charge shifts modify the axial frequency of trapped electrons, as it depends on trapping voltage. As external tank circuit coupled to the motion of the centre of mass of an electron cloud is weakly excited by external RF field, the RF source is tuned and axial angular frequency of the trapped electrons is brought into resonance with the frequency of the detection circuit (K T Satyajith, et al., 2009; Soumen Bhattacharya, et al., 2006). When we ramp the potential of the trap at a constant resonant frequency of the detection circuits, then the electrons absorb energy from the detection tank circuit and the circuit undergoes damping. The electron absorption signal appears at different voltage values of the ramp. The trapping voltage varies depending on the effect of space charges, which in turn depends on total number of electrons being trapped. The detection circuit is modulated and demodulated consecutively; then the output of the detection circuit is a resonance absorption signal. The amplitude of the resonance absorption signal is directly proportional to the number of trapped electrons (K T Satyajith, et al., 2009; Soumen Bhattacharya, et al., 2006). We measure the shift in the trap potential ($\sim 1V$) at the axial frequency of about 10 MHz of trapped electrons, which is same as that of the resonant frequency of the tank circuit. The effect of space charges is measured from the amount of shift in the trap potential as seen in figure 2 (Beck D et al., 2001). The stored electron cloud can be considered as uniform spherical cloud of plasma with constant density in the radial plane. The potential of space charge alters the harmonic potential in the trap, which shifts the trap potential. The number density of electrons trapped is given by (K T Satyajith, et al., 2009; Soumen Bhattacharya, et al., 2006).

$$n = \frac{12\epsilon_0}{ed^2}\Delta U \qquad \left\{\because \Delta U = \frac{ned^2}{12\epsilon_0}, U = 0.667V\right\} \qquad (3)$$

Where, $n = \frac{N}{V}$ is the number density and e is the charge of electrons. The number of electrons trapped is given by

$$N = \frac{12\epsilon_0 V}{ed^2}\Delta U = \frac{12 \times 8.854 \times 10^{-12} V}{1.6 \times 10^{-19} \times 99 \times 10^{-6}}\Delta U \qquad (4)$$

Where V is the volume of the spherical cloud of trapped electrons and it is difficult to measure exactly. The bias voltage and filament current are varied in suitable steps and the corresponding shift in the trap voltage is noted. The energy of the trapped electrons which are oscillating along axial direction i.e. symmetry axis of the trap at resonance is of the order of an eV and is given by $E_k = \frac{1}{2}m\omega_z^2 R^2$(K T Satyajith, et al., 2009; Soumen Bhattacharya, et al., 2006). Where E_k is the kinetic energy, ω_z is the axial angular frequency of oscillations, $R_{max} = 5mm$ is the amplitude of the axial angular oscillations (K T Satyajith, et al., 2009). We have considered $R_{max} = 5mm$ to be approximately the radius of almost spherical electron cloud with the identical resonant frequency (K T Satyajith, et al., 2009). If the radius of spherical cloud of electrons, R=5 mm, $V = 523.33\ mm^3 = 523.33 \times 10^{-9} m^3$.

$$N = 3.5103 \times 10^6 (\Delta U) \qquad (5)$$

$$n = \frac{N}{V} = \frac{3.5103 \times 10^6}{523.33 \times 10^{-9}}(\Delta U) = 6.7076 \times 10^{15}(\Delta U)m^{-3} = 6.7076 \times 10^6 (\Delta U)mm^{-3} \qquad (6)$$

The number density from analysis of the shape of resonance signal:

A passage of small dc voltage of 6V is passed through the thoriated- tungsten filament continuously and electrons are generated by thermionic emission. A varying triangular dc voltage (-10V to +10V) called ramp is applied to the ring electrode. The external tank circuit coupled to the motion of the centre of mass of an electron cloud is weakly excited by external RF field. The RF source is tuned and axial frequency f_z of the trapped electrons is brought into resonance with the frequency f_0 of the tank circuit ($f_z = f_0$) (K T Satyajith, et al., 2009; Soumen Bhattacharya, et al., 2006). At resonance, the trapped electrons absorb energy from RF source; that causes voltage drop across detection tank circuit. The voltage is amplified, demodulated by a tuned amplifier and followed by an amplitude demodulator. A clear and sharp resonance absorption signal of electrons appears on CRO. A Lab VIEW program is run continuously for desired period of time to acquire the data of signal of electrons. In each detection cycle 10 signals are scanned and acquired at the rate of 10,000 samples per second. The data was acquired, the shape of detection signal was analyzed and peak signal amplitude versus scanning time was plotted (K T Satyajith, et al., 2009; Soumen Bhattacharya, et al., 2006). The scanning time axis was transformed to the corresponding frequency axis using the relation between axial frequency and trap voltage $\left(f_z = \frac{1}{2\pi}\sqrt{\frac{4qU}{md^2}}\right)$ and peak signal amplitude axis was transformed to relative electron response signal height (K T Satyajith, et al., 2009;

Soumen Bhattacharya, et al., 2006). The number density of trapped electrons was estimated from integrating the relative electron response signal being fitted to least square fit (K T Satyajith, et al., 2009; Soumen Bhattacharya, et al., 2006). The axial angular frequency of an electron is directly proportional to the square root of trap potential of an electron at resonance i.e. $f_z \propto \sqrt{U_0}$.

$$U_0 = \frac{md^2\omega_z^2}{4e} = \frac{9.1 \times 10^{-31} \times 99 \times 10^{-6} \times 4\pi^2 \times (10.957 \times 10^6)^2}{4 \times 1.6 \times 10^{-19}} = 0.667V \qquad (7)$$

4. RESULTS

The distribution of number of the trapped electrons varies from 50,000 to 3,00,000 for variation of trap voltage from 2 V to 9 V at filament current of 1 A as shown in figure 3. The number density of trapped electrons estimated from both space charge shifts and analysis of shape of resonance absorption methods are entered in table 1 and table 2, it can be observed that both are comparable in the range from $\sim 10 \times 10^6 mm^{-3}$ to $\sim 20 \times 10^6 mm^{-3}$.

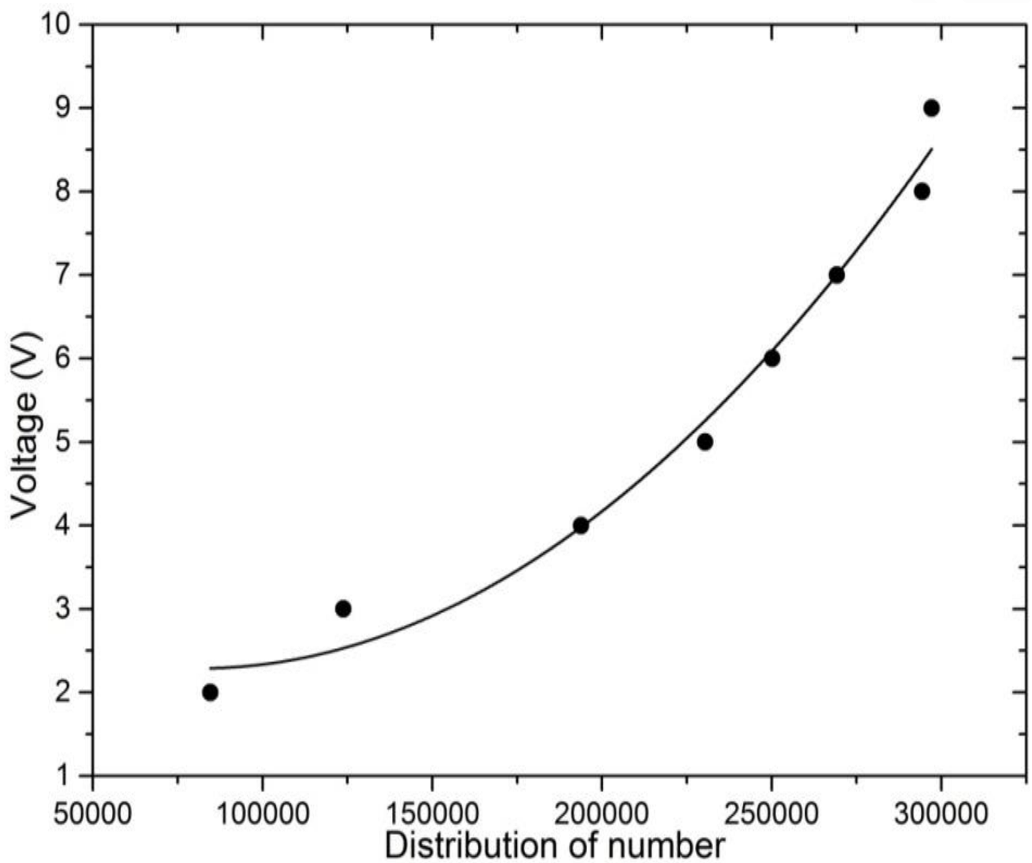

Figure 3 Number distribution of electrons versus trap voltage at magnetic field of 500G

Table 1 Number density of electrons estimated from space charge shifts

Filament current (A)	Bias Voltage (V)	Trap Voltage $U^{'}(\mathrm{V})$	Shift in trap voltage, $\Delta U(\mathrm{V}) = U^{'} - U$	Number (*N*) (Space charge shift) $\times 10^6$ (mm^{-3})	**Number Density(*n*) (Space charge shift) $\times 10^6$ (mm^{-3})**
1.30	0.1	2.21	1.54	5.39	**10.33**
1.30	0.5	2.43	1.76	6.17	**11.79**
1.35	0.1	2.60	1.93	6.78	**12.96**

1.35	0.5	2.74	2.07	7.27	**13.89**
1.40	0.1	2.92	2.25	7.88	**15.06**
1.40	0.5	3.12	2.45	8.61	**16.45**
1.45	0.1	3.36	2.69	9.44	**18.04**
1.45	0.5	3.47	2.80	9.83	**18.78**
1.50	0.1	3.62	2.98	10.46	**19.99**
1.50	0.5	3.72	3.05	10.68	**20.43**

Table 2 Comparison of number density of electrons from space charge shift (column 4) and resonance absorption signals (column 6)

Filament current (A)	Bias Voltage (V)	Number (*N*) (Resonance Absorption)×10^6	**Number Density (*n*) (ResonanceAbsorption) ×10^6 (mm^{-3})**	Number (*N*) (Space charge shift) ×10^6 (mm^{-3})	**Number Density(*n*) (Space charge shift) ×10^6 (mm^{-3})**
1.30	0.1	4.72	**9.01**	5.41	**10.33**
1.30	0.5	6.64	**12.68**	6.17	**11.79**
1.35	0.1	7.09	**13.54**	6.78	**12.96**
1.35	0.5	7.62	**14.55**	7.27	**13.89**
1.40	0.1	8.13	**15.53**	7.88	**15.06**
1.40	0.5	8.60	**16.45**	8.61	**16.45**
1.45	0.1	9.22	**17.61**	9.44	**18.04**
1.45	0.5	9.64	**18.41**	9.89	**18.78**
1.50	0.1	10.17	**19.43**	10.35	**19.99**
1.50	0.5	10.87	**20.77**	10.69	**20.43**

5. DISCUSSION

Number distribution of trapped electrons:

The number distribution of the trapped electrons with trap voltage is measured. The data is fitted to a polynomial function as shown in figure 3.

The number density from space charge shifts:

The clear detection signal of trapped electrons was acquired for filament currents of 1.3A, 1.35A, 1.4A, 1.45A and 1.5A for bias voltages 0.1V and 0.5V at 750G magnetic field and 4×10^{-8} torr pressure. When the filament current or bias voltage is varied, the trapping voltage also varies and it can be measured using cursor on CRO. The shift in trap voltage is measured from $\Delta U = U - U_0$, the total number of trapped electrons is calculated from equation 5 and the number density is calculated from equation 6. The estimated values are entered in table 1.

The number density from resonance absorption signal:

The scanning time axis was transformed to the corresponding frequency axis using the relation between axial frequency and trap voltage $\left(f_z = \frac{1}{2\pi}\sqrt{\frac{4qU}{md^2}}\right)$ and peak signal amplitude axis was transformed to relative electron response signal height as shown in figure 4 (i), (ii) (K T Satyajith, et al., 2009; Soumen Bhattacharya, et al., 2006). The experiment was performed at filament currents of 1.3A, 1.35A, 1.4A, 1.45A and 1.5A at bias voltages of 0.1V and 0.5V. The figure 4 (iii), (iv), (v) and (vi) show the different values of areas under detection signals of trapped electrons (number of trapped electrons) at filament currents of 1.5A and 1.3A at bias voltages of 0.1V and 0.5V, which are directly proportional to the number of trapped electrons and corresponding number density of trapped electrons are as in table 2.

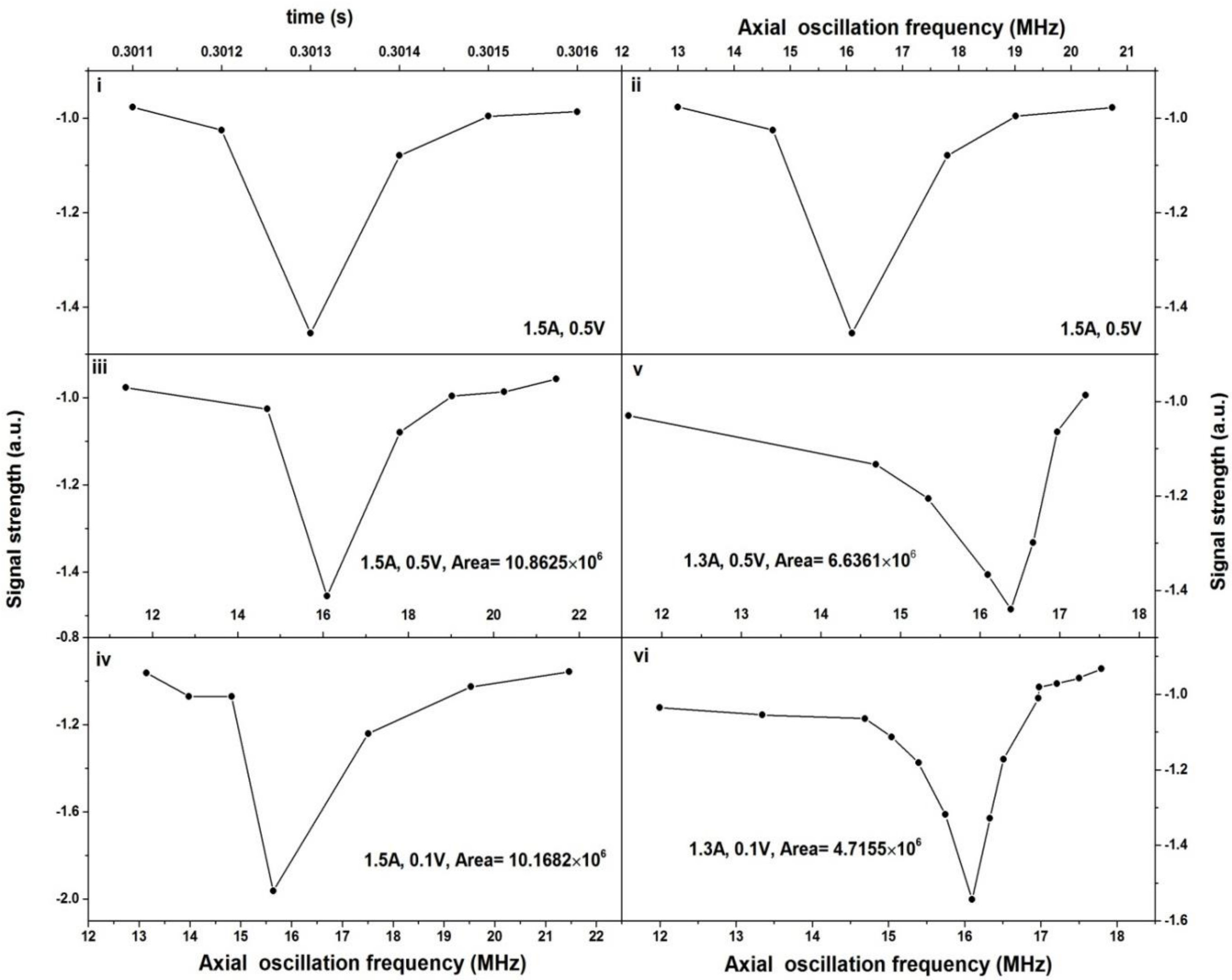

Figure 4 (i), (ii) The scanning time axis is transformed to frequency axis and peak signal amplitude axis is transformed to relative electron response signal height; (iii), (iv), (v) and (vi) Axial oscillation frequencies versus relative electron response signal heights show different areas (or the number of trapped electrons) for different currents and bias voltages

6. CONCLUSION

The number distribution of the trapped electrons with trap voltage is measured. The data is fitted to a polynomial function and reveals the distribution of number of the trapped electrons from 50,000 to 3, 00,000 for variation of trap voltage from 2 V to 9 V. The number density estimated from space charge shift and analysis of shape of resonance absorption signals at lower bias voltages and filament currents show more fluctuation. This is due to the uncertainty in the volume of trapped electron cloud, as we have assumed it to be a constant for all bias voltages and filament currents. The number density of trapped electrons estimated from both space charge shifts and analysis of shape of resonance absorption methods are comparable from $\sim 10 \times 10^6 mm^{-3}$ to $\sim 20 \times 10^6 mm^{-3}$. The estimated values of number of trapped electrons in our trap, agree well with the values obtained by others in traps of comparable dimensions.

SUMMARY OF RESEARCH

1. The data of the number distribution of the trapped electrons with trap voltage is fitted to a polynomial function and the number of the trapped electrons varies from 50,000 to 3,00,000 when the trap voltage is varied from 2 V to 9 V.
2. The number density of trapped electrons estimated from both space charge shifts and analysis of shape of resonance absorption methods are comparable.

3.The estimated values of number of trapped electrons in our trap as entered in table 2, agree well with the values obtained by others in traps of comparable dimensions.

FUTURE ISSUES

I believe that many scientists in ion traps have to pay attention to minimize space charge effects and other anhormonicities in the traps. More accurate methods for the determination of number of trapped particles should be developed in future.

Disclosure Statement

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