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Vibrations dissipative plate mechanical systems with point communications

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ABSTRACT

We investigated the natural oscillations of dissipative inhomogeneous plate mechanical systems with point connections. Based on the principle of virtual displacements, we equate to zero the sum of all active work force, including the force of inertia obtain equations vibrations of mechanical systems. Frequency equation is solved numerically by the method of Muller. According to the result of numerical analysis established nonmonotonic dependence damping coefficients of the system parameters.

Keywords: plate, viscoelastic properties, natural frequencies, damping ratios, structurally homogeneous and structurally heterogeneous.

INTRODUCTION

Studies related to the definition of inherent characteristics of plates with attached masses discussed in [1-3]. In these studies to determine the actual shapes and vibration frequencies of the variational equation is made on the basis of the principle of Hamilton - Ostrogradskii. In [1] in the derivation of the frequency equation is used the approach taken A.S.Gershgorin [4]. Various fastening plate and concentration of the mass limits the scope of application of this approach. Taking into account the viscoelastic (dissipative) material properties of the plate and the elastic elements in papers [5-7]. Free oscillations of dissipative systems are damped. The amplitude of vibration modes decreases with time, so this process is not strictly periodic. But the frequency of the corresponding forms at the same time remain constant [7], and in this sense the dissipative system can be studied as a system that has its own vibrations. In this paper we consider the linear problem of natural vibrations of viscoelastic rectangular plates (or package of rectangular plates) having a connection point.

STATEMENT OF THE PROBLEM AND THEIR SOLUTIONS

We will consider the mechanical system consisting of N isotropic viscoelastic plates, occupying volume V_n and limited by surfaces $\Omega_n(n = 1, \dots, N)$. It is assumed that one linear dimension of each plate is much less than the other two. At each n on part of a surface of n-th plate uniform Ω_n^{gr} boundary conditions are set, on other free surface $\Omega_n^{sv} = \Omega_n/\Omega_n^{gr}$ in final number of points communications of kinematic and dynamic character are imposed: dot rigid, elastic and (or) viscoelastic hinged type of a support (rigid support can be jammed), the rigid elastic and (or) viscoelastic shock-absorbers connecting bodies (at$N > 1$), the concentrated masses $M_{qn}(q = 1, \dots, Q)$. The arrangement of communications and masses on surfaces is any Ω_n^{sv}. Generally dissipative properties of elements of system are various. Special case of such structurally non-uniform viscoelastic system is the system with elastic and viscoelastic elements. For the last case $N = N_y + N_n$, where N_y– – quantity of elastic elements of system, N_n- quantity of viscoelastic elements. For N = 1 the rack available. Required to determine the natural frequency of the viscoelastic system, as well as to evaluate its damping capacity.

Mathematically, the viscoelastic problem is as follows. Let all the n-th point of the body are subject to a harmonic law fluctuations, ie

$$U_{nj}(\bar{x}^n, t) = U_{nj}^0(\bar{x}^n)e^{-i\omega t},\ n = 1, \dots N, j = 1, \dots, J, \quad (1)$$

where $U_{nj}^0(\bar{x}^n)$- j-th component of the displacement vector of n-th body, J- number of components of the displacement vector,$\bar{x}^n = (x_1^n, x_2^n, x_3^n)$- the radius vector of the n-th body, $\omega = \omega_R + i\omega_I$- desired complex frequency of the system, and ω_R - the natural frequency, ω_I– damping coefficient $(\omega_I < 0)$. As everyone a component of a vector of movements already has an index n, the last for designation a component radius vector isn't used further.

For rectangular plates $J = 1$ and

$$U_{n1}^0(x_1, x_2) = U_n^0(x, y), U_{n2}^0(x, y) = V_n^0(x, y),\ U_{n3}^0(x, y) = W_n^0(x, y),$$

where x, y–coordinate. Proceeding from the principle of possible movements, we will equate to zero sum of works of all active forces, including inertia forces on possible movements $\delta U_{nj}(\bar{x}, t)$:

$$\delta A_\sigma + \delta A_a + \delta A_m = 0, \quad (2)$$

where $\delta A_\sigma, \delta A_a, \delta A_m$ – virtual works of internal forces of bodies of springs, and also inertia forces taking into account the concentrated masses. These works can be presented the following ratios:

$$\delta A_\sigma = -\sum_{n=1}^{N} \sigma_{mk}^n\, \delta\varepsilon_{mk}^n dV, \delta A_a = -\sum_{n=1}^{N}\sum_{l=1}^{L_n} \sigma_l^n\, \delta\varepsilon_l^n - \sum_{n=1}^{N}\sum_{l^/=1}^{L_n^/} \sigma_{l^/}^n\, \delta\varepsilon_{l^/}^n,$$

$$\delta A_m = -\sum_{n=1}^{N} \rho_n \int\limits_{V_n} \left(\sum_{j=1}^{J} \ddot{U}_{nj}\,(\bar{x}, t)\delta U_{nj} \right) dV -$$

$$-\sum_{n=1}^{N}\sum_{q=1}^{Q_n} M_{qn}\sum_{j=1}^{J}\ddot{U}_{nj}\left(\bar{x}_n^q,t\right)\delta U_{nj}, \quad (3)$$

where ρ_n, V_n – density and volume of the n-th body, M_{qn} - q-th accessions the mass of n-th bodies with coordinates $\bar{x}_n^q = \left(x_{n1}^q, x_{n2}^q, 3\right), L_n$– number of springs (shock-absorbers) between n-th and (n+1) – th bodies, Q_n- number of the concentrated masses on n-th body, $L_n^{/}$ - number of elastic (viscoelastic) support on n-th body, $\sigma_{mk}^n, \varepsilon_{mk}^n, \sigma_l^n, \varepsilon_l^n, \sigma_{l/}^n, \varepsilon_{l/}^n$- components of tensors of tension and deformations according of n-th body, l-st of a spring (shock-absorber) and $l^{/}$– th elastic (viscoelastic) support.

We will determine physical ratios for n-th viscoelastic body of system by equality [6]

$$\sigma_{mk}^n(t) = \frac{\tilde{E}_n}{1+v_n}\left[\frac{v_n}{1-2v_n}\Theta^n(t)\delta_{mk} + \varepsilon_{mk}^n(t)\right], \quad (4)$$

where $\tilde{E}_n$,- Volterra's having the following appearance the operators:

$$\left(\tilde{E}_n\varphi\right)(t) = E_n\left[\varphi(t) - \int_0^t R^n(t-\tau)\varphi(\tau)d\tau\right], \quad (5)$$

here Θ^n- a dilatation, δ_{mk}- Kronecker's symbol, E_n- the instant module of elasticity, and R^n– a relaxation kernel. Poisson's coefficient $\tilde{v}_n = v_n = const$ in an offered problem definition is accepted to constants. It means that for structurally uniform viscoelastic system of a form of own fluctuations will be equal to own vectors of the corresponding elastic task [7,8].

Considering (1), time function in equality (5) will be with slowly $\varphi(t) = exp(-i\omega t)$ changing amplitude. Assuming a little integral $\int_0^\infty R(\tau)d\tau$, by means of a method of freezing we will replace a ratio (5) confidants:

$$\tilde{E}_n \cong E_n[1 - \Gamma_c(\omega_R) - is(\omega_R)]\varphi, \quad (6)$$

where

$$\begin{Bmatrix}\Gamma_c\\ \Gamma_s\end{Bmatrix} = \int_0^\infty R^n(\tau)\begin{Bmatrix}cos\omega_R\tau\\ sin\omega_R\tau\end{Bmatrix}d\tau.$$

Time allows to exclude it from the variation equation integrated members and, finally. In a symbolical look it can be presented in a look

$$\delta G\left(U_{nj}^0(\bar{X}), \omega^2\right) = 0. \quad (7)$$

We will write out concrete representation of functionality of G, for example, for a package of rectangular plates with dot communications:

$$G[W_n^0(x,y),\omega^2] = -\frac{1}{2}\sum_{n=1}^{N}\bar{D}_n\int_0^{a_n}\int_0^{b_n}\left[\left(\frac{\partial W_n^0}{\partial x^2} + \frac{\partial W_n^0}{\partial y^2}\right)^2 - \right.$$

$$\left. -2(1-v)\left(\frac{\partial^2 W_n^0}{\partial x^2}\frac{\partial^2 W_n^0}{\partial y^2} - \left(\frac{\partial^2 W_n^0}{\partial x\partial y}\right)^2\right)\right]dxdy -$$

$$-\frac{1}{2}\sum_{n=1}^{N-1}\sum_{l=1}^{L_n}\bar{D}_n\left[W_n^0(x_n^l,y_n^l) - W_{n+1}^0(x_n^l,y_n^l)\right]^2 -$$

$$-\frac{1}{2}\sum_{n=1}^{N}\sum_{l^{/}=1}^{L_n^{/}}\bar{C}_{l^{/}n}(W_n^0)^2(x_n^{l^{/}},y_n^{l^{/}})+\frac{\omega^2}{2}\sum_{n=1}^{N}\rho_n h_n\int_0^{a_n}\int_0^{b_n}(W_n^0)^2dxdy+$$

$$+\frac{\omega^2}{2}\sum_{n=1}^{N}\sum_{q=1}^{Q_n}M_{qn}(W_n^0)^2(x_n^q,y_n^q),$$

where h_n, a_n, b_n – thickness and the linear sizes of *n-th* plate, x_n^q, y_n^q- coordinates of *l-th* concentrated weight, x_n^l, y_n^l - coordinates of *l-st* spring (shock-absorber),$x_n^{l^{/}}, y_n^{l^{/}}$ - coordinates of *l^/^-th* elastic (viscoelastic) support. If *n*-th plate and *l-th* spring and *l^/^-th* support are viscoelastic, then $\bar{D}_n, \bar{C}_{ln}, \bar{C}_{l^{/}n}$are represented by the following formulas:

$$\bar{D}_n=D_nf_n(\omega_R),\ \bar{C}_{ln}=C_{ln}f_{ln}(\omega_R),\bar{C}_{l^{/}n}=C_{l^{/}n}f_{l^{/}n}(\omega_R),$$

where $f(\omega_R)=1-\Gamma_c(\omega_R)-i\Gamma_s(\omega_R)$- the complex function, the numerical coefficients which depend on the parameters of relaxation kernel corresponding viscoelastic elements,$D_n=\frac{Evh_n^3}{12(1-\nu_n^2)}$, $C_{ln}, C_{l^{/}n}$ - generalized instant rigidity according to of *n-th* plate, of *l-th* shock-absorber, of *l^/^-th* support. In an elastic case $\bar{D}_n=D_n$, $\bar{C}_{ln}=C_{ln}$, $\bar{C}_{l^{/}n}=C_{l^{/}n}$, where D_n, C_{ln}, $C_{l^{/}n}$- generalized rigidity according to of *n-th* plate, of *l-th* spring, of *l^/^-th*support.

The similar functionality can be written down for system of covers of rotation.

Components of a vector of movements $U_{nj}^0(\bar{x})$are required functions of the variation equation (7) and have to meet boundary conditions on surfaces Ω_n^{gr}, ie

$L_nU_{nj}^0(\bar{x})=0,\ \bar{x}\in\Omega_n^{\text{gr}}.$ (8)

It was necessary to impose rigid dot communications which don't make work at fluctuations on system. Terms of hard hinged support the *n-th* body in S_n point supports can be written as

$U_{nj}^0(\bar{x}_n^s)=0\quad(s=1,\dots,S_n;j=1,\dots,J),$ (9)

where $\bar{x}_n^s$– coordinates of s-th support of n-th body.

If the part of support has jamming, conditions will be added

$$\frac{\partial U_{nj}^0(\bar{x}_n^s)}{\partial\alpha_n^s}=0\ \ (s=1,\dots,S_n^\alpha;j=1,\dots,J)\quad(10)$$

where α_n^s– the direction of a single vector along which in a pointx_n^s the rigid jamming of a body is carried out.

Existence of rigid racks between n-th and (n+1) - th body at$N\geq 2$ is considered by ratios

$$U_{nj}^0(\bar{x}_n^r)-U_{n+1,j}^0(\bar{x}_n^r)=0\ \ (r=1,\dots,R_n;j=1,\dots,J).\quad(11)$$

Where $\bar{x}^r$ - the coordinate of r-th are resistant, R_n- number of racks between n-th and (n+1) - th bodies. In case of N=1 conditions (11) are absent.

Thus, on a vector of movements restrictions of type (8)-(11) are in addition imposed. On system dot communications we will consider imposing by means the method of Lagrange multipliers. Then the variational equation (10) will correspond in a look

$$\delta\left\{G\left(U_{nj}^0(\bar{x}),\omega^2\right)+\sum_{n=1}^{N}\sum_{s=1}^{S_n}\sum_{j=1}^{J}\lambda_{nj}^s\,U_{nj}^0(\bar{x}_n^s)+\sum_{n=1}^{N}\sum_{s=1}^{S_n^\alpha}\sum_{j=1}^{J}\kappa_{nj}^s\frac{\partial U_{nj}^0(\bar{x}_n^s)}{\partial\alpha_n^s}+\right.$$

$$\left.+\sum_{n=1}^{N-1}\sum_{r=1}^{R_n}\sum_{j=1}^{J}\mu_{nj}^r\left[U_{nj}^0(\bar{x}_n^r)-U_{n+1,j}^0(\bar{x}_n^r)\right]\right\}=0, \quad (12)$$

where $\lambda_{nj}^s,\kappa_{nj}^s,\mu_{nj}^r$ – Lagrange's multipliers.

It is necessary to find a range of complex own frequencies $\omega^k=\omega_R^k+i\omega_I^k$, where ω_R^k– frequencies, and ω_I^k- coefficients of damping own attenuations of fluctuations.

ALGORITHM OF REALIZATION OF A VARIATION METHOD AT THE SOLUTION OF A VISCOELASTIC TASK ON OWN FLUCTUATIONS

Approach the solution of the variational equation (12), as well as in case of an elastic task, we look for in the form of the approximating form made of fundamental functions, satisfying both to the equation, and the set geometrical boundary conditions on surfaces of Ω_n^{gr}each body. It is offered that functions $\Phi_{nj}^k(\bar{x})$for such bodies are known. Then approximating forms can be built in the form of final decomposition on these known functions:

$$U_{nj}^0(\bar{x})=\sum_{k=1}^{K}\gamma_{nj}^k\,\Phi_{nj}^k(\bar{x}) \quad . \qquad (13)$$

where γ_{nj}^k– required complex coefficients.

Previously Φ_{nj}^k it is possible to normalize. On the Ω_n^{gr} sum (13) meets regional conditions automatically, owing to a choice of the composed. Variation on the generalized coordinates $\lambda_{nj}^s,\ \kappa_{nj}^s,\ \mu_{nj}^r,\ \gamma_{nj}^r$, the equations (12) we will receive uniform system of the linear equations. Dimension of this system $J\cdot N^/\times J\cdot N^/$,, where

$$N^/=\sum_{n=1}^{N}(S_n+S_n^\alpha+R_n)+N\cdot K,J$$

- number component of a vector of movements U_{nj}^0.

Without providing concrete calculations, we will write down this system in a matrix look:

$$\left(A+\sum_{n=1}^{N_n}f_n(\omega_R)\,A_n^n+\sum_{n=1}^{N-1}\sum_{l=1}^{L_n}f_{ln}(\omega_R)\,A_{ln}^n+\right.$$

$$\left.+\sum_{n=1}^{N}\sum_{l^/=1}^{L_n}f_{l^/n}(\omega_R)\,A_{l^/n}^n-\omega^2B\right)\bar{\xi}=0, \qquad (14)$$

where $\bar{\xi}$– a vector column of the generalized coordinates $\lambda_{nj}^s,\ \kappa_{nj}^s,\ \mu_{nj}^s,\ \gamma_{nj}^s, N_n$ - number of viscoelastic bodies of system; B - symmetric singular matrix generalized mass of the system; $A_n^n, A_{ln}^n, A_{l^/n}^n$- the square matrixes of dimension consisting of zero $J\cdot N^{'}\times J\cdot N^{'}$, consisting of zeros, except the stiffness matrices of instantaneous n-th viscoelastic body, of l-th shock-absorber and of l/ -th viscoelastic support, respectively; A – a symmetric matrix (its submatrix of A^0 of dimension represents $J\cdot K\times J\cdot K$ the generalized total rigidity of elastic elements of system, and $A_H=A_b^T$submatrixes consider kinematic conditions of the rigid dot communications imposed on system); $f(\omega_R)=1-\Gamma_c(\omega_R)-\Gamma_s(\omega_R)$ - the complex function characterizing viscosity of a viscoelastic element (its coefficients depend on relaxation parameters).

Structurally matrixes A and B are similar described in work [6]. Generally they differ from each other parameters of kernel relaxation. If all elements of the viscoelastic rheological properties are the same, then$f_1(\omega_R)=f_2(\omega_R)=\ldots$,and so the second, third and fourth terms in equation (14) are replaced by a matrix of total instantaneous stiffness of viscoelastic elements (in the case of structural homogeneous viscoelastic system).

The degeneracy of the matrix B as a resilient problem is caused in the introduction of additional point connections (rigid supports and pillars). The transformed matrixes will have dimension $N^{"} \times N^{"}$ where $N^{"} = J \cdot N^{'} - 2J\sum_{n=1}^{N}(S_n + S_n^{\alpha} + R_n)$. Equating to define systems to zero, we will receive the frequency equation which, unlike a case of an elastic task, will be complex. The most effective way of the solution of the similar equations, apparently, is the method of Müller [7] which and here was used. Without opening frequency to define, and calculating on each step only its value for the fixed value ω, the specified method $N^{"}$ are own complex frequencies $\omega = \omega_R + i\omega_I$. Coefficients of damping allow to judge damping properties of considered system. In equipment for an assessment of speed of attenuation of oscillatory processes other characteristic, namely, logarithmic decrement of attenuation of fluctuations is used. It is connected with damping coefficient by the following formula [6]:

$$\delta = \frac{2\pi\omega_I}{\omega_R}$$

Systems of rectangular plates and with dot communications are considered. We will consider the design representing a package from two parallel square elastic plates with the shock-absorber and the attached weight. The relaxation kernel for the shock-absorber is chosen in a look

$$R(t) = Aexp(-\beta t)t^{\alpha-1},$$

where A, β, α - parameters of a kernel [9]. Two elastic plates identical on mechanical properties (*E*=28;*ρ*=4;*ν*=0,3) are connected in the center of one spring. Mass of a spring M_0 =0.05. Plates square (a=b=1), supported on a contour, thickness of the bottom plate h_1 =0.1, and of the top plate h_2 =0.046, on the bottom plate in the center dot weight is attached. Viscosity of the shock-absorber is accepted such that its deformation of creep at quasistatic process made a small share from the general (~ 12%). For this case kernel parameters following:$A = 0.01, \alpha = 0.1,\ \beta = 0.05$ [9]. Unlike an elastic task, dependence of two lowest frequencies and the corresponding coefficients of damping from the size of instant rigidity of the shock-absorber here was investigated. The last changed from 10^{-4} to 10^{-1}. On the right this range is limited by the size since at C=C_2 there is a change of the second form. On fig. 1 dependence of the first two frequencies ω_R^1, ω_R^2 and the corresponding coefficients of damping from $\omega_I^1, \omega_I^2$the size of instant rigidity of the shock-absorber C is shown. From the analysis of the figures that the dissipative properties of the system as a whole are determined not only the rheology of its elements, but depend strongly on the interaction of the vibrations of their own forms. This effect is reflected in the fact that under certain conditions (of which below), and to a certain value of the damping energy higher capacity (in this case, the second) shape dissipates less energy, less energy-intensive than the form. Then, since some value of instant rigidity of the shock-absorber (in this case $C^* = 5.4 \cdot 10^{-3}$), process of dissipation of energy by own forms is normalized and proceeds according to power hierarchy of forms.

Real illustration of this effect is existence of a point of intersection of schedules of damping coefficientsω_I^1 and ω_I^2 at $C = C^*$. And one more feature: in this point the difference of speeds of attenuation of two forms of fluctuations of a design changes a sign, if a spring viscoelastic (i.e. system structurally non-uniform). As well as in an elastic task [6],ω_R^1, ω_R^2 own frequencies which it is slightly less, ω_1, ω_2,than in this point approach. The analysis of tasks of this kind showed that the effect of interaction of own forms is observed only in structurally non-uniform systems (in this case with elastic and viscoelastic elements) and at noticeable rapprochement of own frequencies. Absence at least one of these conditions excludes effect manifestation. The physical explanation for observed effect should be looked for in the nature of dynamic redistribution of energy of system between two forms possessing properties described above. We will consider now, what distinctions exist between structurally uniform and non-uniform viscoelastic systems. Structurally uniform viscoelastic system (all elements viscoelastic with identical rheological properties) is characterized by that in a formula (14), first, there is no matrix A (submatrixes of A_H and $A_b$can be transferred to the following matrix) and, secondly, all functions are identical. Then the system of the equations (14) in a matrix look can be copied so:

$$[f(\omega_R)A^n - \omega^2 B]\bar{\xi} = 0, \qquad (15)$$

where A^n – a numerical matrix of the total instantaneous stiffness of viscoelastic elements of the system.After an exception linearly dependent the component from system also can be written down (15) transformed matrixes of the generalized instant $\tilde{A}^n$, $\tilde{B}$ in an initial form, i.e. special transformation of the generalized coordinates to lead these matrixes to a diagonal look. And it means that the mechanical system represents as though set of partial systems independent with one degree of freedom. In other words, own forms of such system are independent and they can be considered and calculated separately from each other.

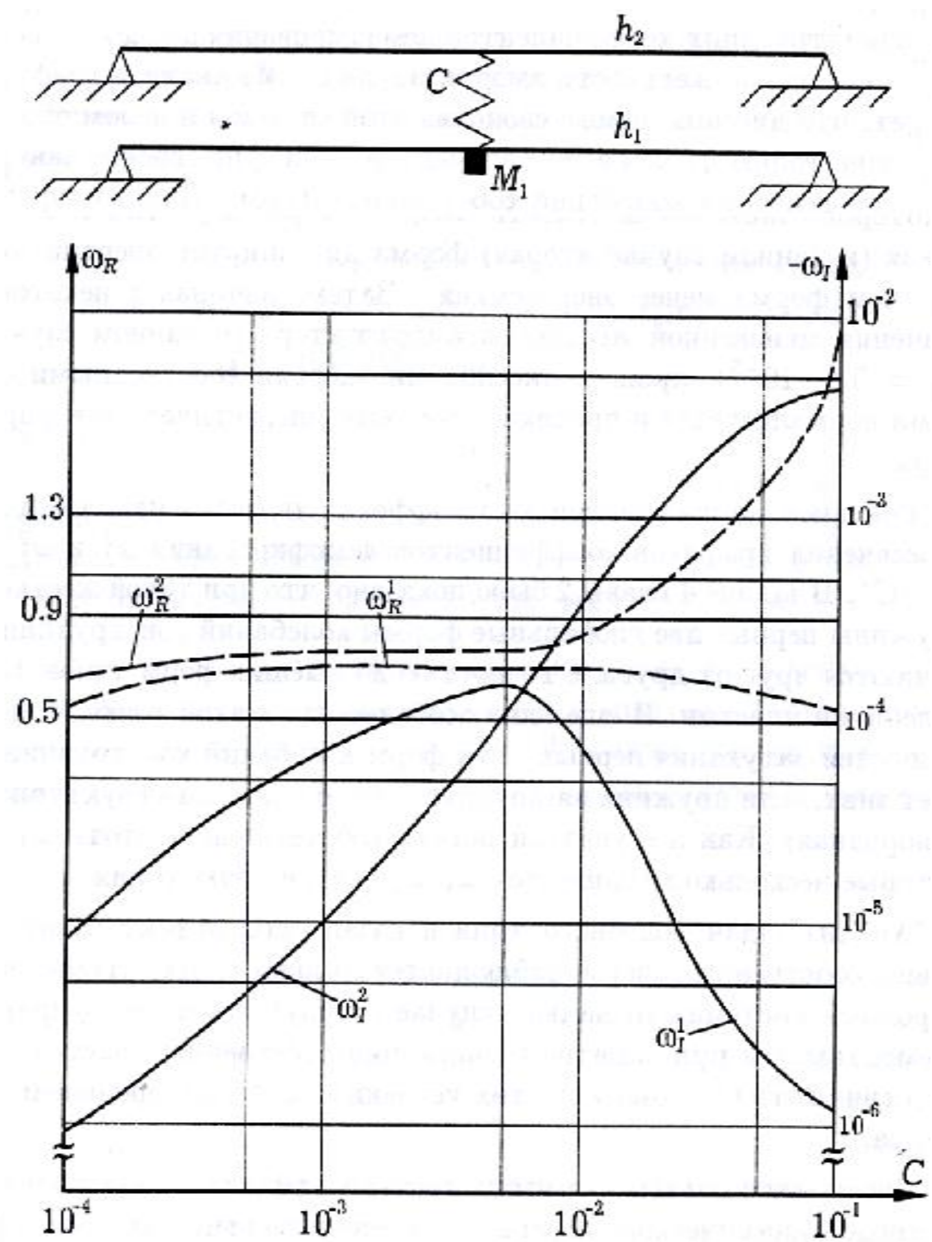

Figure 1 Dependence of frequencies and damping coefficients from rigidity of the shock-absorber

Other situation develops for structurally non-uniform viscoelastic system. In a formula (15) the matrix of the generalized of elastic elements A will increase and then (15) will look so:

$$[\bar{A}(\omega_R) - \omega^2 B]\bar{\xi} = 0,$$

where $\bar{A}(\omega_R) = A + f(\omega_R)$. Generally for two matrixes $\bar{A}(\omega_R)$and B (one of which functional), after a preliminary exception linearly dependent a component, it is impossible to pick up at the same time the nondegenerate transformation of coordinates bringing them to a canonical form. And it means that own forms of such mechanical system can't be considered separately from each other, i.e. they are dependent. Therefore, at free fluctuations between forms there is an exchange of energy. Especially strongly it is shown if forms have close own frequencies. In a point of intersection of schedules of coefficients of damping and both forms equally disseminate energy, though are excellent from each other (to within a phase). To C=C* point * there is energy "transfer". Consequently, for free oscillations between forms of energy is exchanged. This is especially true if the forms have similar natural frequencies. At the point of intersection of the graphs of damping factors ω_I^1 and ω_I^2, both forms are equally dissipate energy, although different from each other (up to a phase). To the point $C = C^*$, the "pumping" of energy from the second form to the first, so the latter most intensively dissipates energy.After the point of intersection of the difference between the first natural frequency increases, the appropriate forms of interaction decreases and their dissipative properties take a regular character.The practical conclusion is that the damping capacity of the structure is mainly determined by the absolute value of the minimum damping factor (in this case the latest damped oscillations precisely this form);global (determining) the damping factor of the system is the first$\omega_I^1$to the intersection, and thenω_I^2. Optimal in the sense of decay mode vibrations of the structure will be at the $C = C^*$, when the damping factor of the global maximum.

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