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To a question about active vibro protection of the system having final number of degrees of freedom

Teshaev MKH, Yuliyev OO, Rabbamayeva M

Bukhara engineering - technological institute, Republic of Uzbekistan

Corresponding author:
Bukhara engineering - technological institute, Republic of Uzbekistan
Ph.: +99865-2236197 (work), +99893-681-0431 (cell)
E-mail: muhsin_5@mail.ru, muhsin.teshayev@rambler.ru

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ABSTRACT
The problem of clearing of fluctuations of the system having *n* of degrees of freedom is considered. Unlike considered before the works devoted to this direction, here for the first time vibro protection it will be carried out for the account of attraction of additional energy sources. For this purpose to vibro protection to systems methods of dynamics of systems with servo constraints are applied.

Key words:
Vibro protection, clearing, a viscoelastic support, fluctuation, servo constraint, clearing from servo constraint, equations of motion, (A) - moving, reaction of servo constraint, the characteristic equation, stability.

1. INTRODUCTION

Problems active vibro protection plants were considered in works [1,2]. However, in these works these problems are considered from purely engineering point of view. In the given work for the first time to vibro protection to systems substantive provisions of the theory of systems with servo constraints, introduced by A. Beghen [3], and developed by A.G. Azizov [4,5] are applied. Applying a constructive method of definition of structure of forces of responses servo constraints (method of A.G. Azizov), are defined laws of shaping of operating actions [6-9], are defined conditions at which the system stability under the ratio vibro protecting conditions is ensured.

Compiling of the equations of motion and the definition of forces of responses servo constraints

Let the mechanical system, consists from N the weights connected by among themselves deformable elements. We will be limited to consideration of such fluctuations which are described by the linear differential equations. These equations of motions are linear, it is necessary, that deviations of system from balance position are small enough (that is provided by littlest of initial indignations). As it is required to extinguish fluctuations of system concerning zero initial positions, we assume that motion of system is constrained by servo-constraints /2/:

$$q_1 = 0, \quad q_2 = 0, \quad \ldots, q_n = 0 \tag{1}$$

where - $q_1, q_2, \ldots, q_n$ - the generalized co-ordinates by which motion of system is described. As the system makes vibrational motions concerning position (1) along with parities (1) parities: /3/

$$q_1 = \eta_1, q_2 = \eta_2, \ldots, q_n = \eta_n, \tag{2}$$

where $\eta_1, \eta_2, \ldots, \eta_n$ - the independent parameters, characterizing clearing of system from servo-constraints (1) take place also. Moving, on which servo-constraints (1) works do not make reaction /2/, look like

$$\delta q_1 = 0, \ \delta q_2 = 0, \ldots, \delta q_n = 0$$

Or with the account of parities (2):

$$\delta \eta_1 = 0, \ \delta \eta_2 = 0, \ldots, \delta \eta_n = 0 \tag{3}$$

In this case the equations of motion of system with multipliers can be written down in a kind:

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_i}\right) - \frac{\partial T}{\partial q_i} = \frac{\partial \Pi}{\partial q_i} + Q_i + \lambda_i, \tag{4}$$

where T, Π - kinetic and potential energy of system; Q_i - the generalized force corresponding to the generalized co-ordinate q_i ; λ_i - forces of reaction of servo-constraints.

Putting in (4) expressions kinetic and potential energies:

$$T = \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij}\dot{q}_i\dot{q}_j \quad ; \quad \Pi = \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij}q_i q_j$$

and, after calculation $\frac{\partial T}{\partial \dot{q}_i}, \frac{\partial T}{\partial q_i}$, considering parities (2), we will receive:

$$\begin{cases} a_{11}\ddot{\eta}_1 + a_{12}\eta_2'' + \ldots + a_{1n}\eta_n'' + c_{11}\eta_1 + c_{12}\eta_2 + \ldots + c_{1n}\eta_n = Q_1 + \lambda_1 \\ a_{21}\eta_1'' + a_{22}\eta_2'' + \ldots + a_{2n}\eta_n'' + c_{21}\eta_1 + c_{22}\eta_2 + \ldots + c_{2n}\eta_n = Q_2 + \lambda_2 \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ a_{n1}\eta_1'' + a_{n2}\eta_2'' + \ldots + a_{nn}\eta_n'' + c_{n1}\eta_1 + c_{n2}\eta_2 + \ldots + c_{nn}\eta_n = Q_n + \lambda_n \end{cases} \quad (5)$$

The equations (5) are the differential equations of the compelled fluctuations of system with final number of degrees of freedom. If using a constructive method of definition of structure of forces of reactions servo constraints /3/, reactions of servo constraints $\lambda_1, \lambda_2, \ldots, \lambda_n$ to form under laws:

$$\begin{aligned} \lambda_1 &= -Q_1 - k_1\dot{\eta}_1 \\ \lambda_2 &= -Q_2 - k_2\dot{\eta}_2 \\ &\ldots\ldots\ldots\ldots\ldots\ldots \\ \lambda_n &= -Q_n - k_n\dot{\eta}_n \end{aligned}$$

where $k_1, k_2, \ldots, k_n$ - some constant factors, the equations of the compelled fluctuations of system (5) will look like:

$$\begin{cases} a_{11}\ddot{\eta}_1 + a_{12}\eta_2'' + \ldots + a_{1n}\eta_n'' + k_1\dot{\eta}_1 + c_{11}\eta_1 + c_{12}\eta_2 + \ldots + c_{1n}\eta_n = 0 \\ a_{21}\eta_1'' + a_{22}\eta_2'' + \ldots + a_{2n}\eta_n'' + k_2\dot{\eta}_2 + c_{21}\eta_1 + c_{22}\eta_2 + \ldots + c_{2n}\eta_n = 0 \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ a_{n1}\eta_1'' + a_{n2}\eta_2'' + \ldots + a_{nn}\eta_n'' + k_n\dot{\eta}_n + c_{n1}\eta_1 + c_{n2}\eta_2 + \ldots + c_{nn}\eta_n = 0 \end{cases} \quad (6)$$

Stability research. The system (6) has the private decision

$$\eta_1 = 0, \eta_2 = 0, \ldots, \eta_n = 0 \quad (7)$$

corresponding the position (1).

We investigate stability of a condition (1) or (7). For this purpose we will make a characteristic determinant:

$$\begin{vmatrix} \grave{a}_{11}p^2+k_1p+c_{11} & \grave{a}_{12}p^2+c_{12} & ..... & \grave{a}_{1n}p^2+c_{1n} \\ \grave{a}_{21}p^2+c_{21} & \grave{a}_{22}p^2+k_2p+c_{22} & ..... & \grave{a}_{2n}p^2+c_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ \grave{a}_{n1}p^2+c_{n1} & \grave{a}_{n2}p^2+c_{n2} & \ldots & \grave{a}_{nn}p^2+k_np+c_{nn} \end{vmatrix} = 0$$

and revealing a determinant we will receive the characteristic equation of system (7):

$$b_1 p^{2n} + b_1 p^{2n-1} + \ldots + b_{2n-1} p + b_{2n} = 0 \qquad (8)$$

here $b_0, b_{1,} \ldots b_{2n}$ - the positive constants, depending on $a_{11}, a_{22}, \ldots, a_{nn}$, c_{11}, c_{22}, …c_{nn}, $K_1, K_2, \ldots, K_n$.

According to Lyapunov's theorem about asymptotically stability /10/, a necessary and sufficient condition asymptotically stability of system (6) are negativity of the real parts of all roots of the characteristic equation (8). We will construct of factors $b_0, b_{1,} \ldots b_{2n}$ the equation (8) following matrix:

$$\begin{Vmatrix} b_1 b_3 b_5 \ldots .0 \\ b_0 b_2 b_4 \ldots .0 \\ ------ \\ 0 \;\; 0 \;\; 0 \ldots\ldots b_{2n} \end{Vmatrix} \qquad (9)$$

According to Hurvitz's theorem /10 /, that all roots of the equation (8) has to be negative real parts, it is necessary and enough, that all main diagonal minors

$$\Delta_1 = b_1, \Delta_2 = \begin{vmatrix} b_1 & b_3 \\ b_0 & b_2 \end{vmatrix}, \ldots, \Delta_{2n} = b_{2n} \cdot \Delta_{2n-1}$$

must be positive:

$$\Delta_1 \rangle 0, \quad \Delta_2 \rangle 0, \ldots, \Delta_{2n} \rangle 0. \qquad (10)$$

As positive factors $b_0, b_{1,} \ldots b_{2n}$ is depend from constants c_{11}, $c_{22}, \ldots, c_{nn}$, $a_{11}, a_{22}, \ldots, a_{nn}$, K_1, $K_2, \ldots, K_n$, to n that performance of conditions (10) will depend on a choice of constants $k_1, k_2, \ldots, k_n$.

As illustrations we shall consider the system, which has six degrees of the freedom.

Let's consider a body in weight *m*, suspended to the basis on shock-absorbers. Let on a body are imposed servo constraints [12,13]:

$$\left.\begin{matrix} q_1 = 0, & q_2 = 0, & q_3 = 0, \\ \varphi_1 = 0, & \varphi_2 = 0, & \varphi_3 = 0 \end{matrix}\right\} \qquad (11)$$

where $q_1, q_2, q_3, \varphi_1., \varphi_2., \varphi_3$ -generalized coordinates.

It is known [4-6], that along with (11) parities also take place:

$$\left.\begin{array}{lll} q_1 = \xi_1, & q_2 = \xi_2, & q_3 = \xi_3, \\ \varphi_1 = \xi_4, & \varphi_2 = \xi_5, & \varphi_3 = \xi_6 \end{array}\right\} \quad (12)$$

where $\zeta_{1,} \zeta_2, \ldots, \zeta_6$ - the independent parameters, characterizing continuous clearing of system from servo constraints (11).

Moving, on which servo constraints work do not make reaction [3], look like:

$$\left.\begin{array}{lll} \delta\xi_1 = 0, & \delta\xi_2 = 0, & \delta\xi_3 = 0, \\ \delta\xi_4 = 0, & \delta\xi_5 = 0, & \delta\xi_6 = 0 \end{array}\right\} \quad (13)$$

If to neglect weight and damping of the shock-absorbers, and body displacement to consider small enough, motion of such one-mass system can be described by six the differential equations of the second order [11]:

1) $$ò\,\ddot{\xi}_1 + \sum_{i=1}^{4} C_{xi}\xi_1 + \sum_{i=1}^{4} C_{xi}\xi_3 \cdot \xi_5 - \sum_{i=1}^{4} C_{xi}\xi_2 \cdot \xi_6 = Q_1 \cdot \cos(\Omega t + \psi_1) + \lambda_1 .$$

2) $$ò\,\ddot{\xi}_2 + \sum_{i=1}^{4} C_{yi}\xi_2 - \sum_{i=1}^{4} C_{yi} \cdot \xi_3 \cdot \xi_4 + \sum_{i=1}^{4} C_{yi}\xi_1 \cdot \xi_6 = Q_2 \cdot \cos(\Omega t + \psi_2) + \lambda_2 .$$

3) $$ò\,\ddot{\xi}_3 + \sum_{i=1}^{4} C_{zi}\xi_3 + \sum_{i=1}^{4} C_{zi}\xi_2 \cdot \xi_4 - \sum C_{zi}\xi_1 \cdot \xi_5 = Q_3 \cdot \cos(\Omega t + \psi_3) + \lambda_3 .$$

4) $$I_x\ddot{\xi}_4 - I_{xy} \cdot \ddot{\xi}_5 - I_{zx}\ddot{\xi}_6 - \sum_{i=1}^{4} C_{yi} \cdot \xi_3 \cdot \xi_2 + \sum_{i=1}^{4} C_{zi}\xi_2 \cdot \xi_3 + \sum_{i=1}^{4}(C_{zi} \cdot y\xi_2{}^2 - C_{yi}\xi_3{}^2) \cdot$$
$$\cdot\, \xi_4 - \sum_{i=1}^{4} C_{zi}\xi_1\xi_2 \cdot \xi_5 - \sum_{i=1}^{4} C_{yi}\xi_1\xi_2 \cdot \xi_6 = M_1 \cdot \cos(\Omega t + \psi_4) + \lambda_4$$

5) $$I_y\ddot{\xi}_5 - I_{xy} \cdot \ddot{\xi}_4 - I_{yz}\ddot{\xi}_6 + \sum_{i=1}^{4} C_{xi}\xi_3 \cdot \xi_1 - \sum_{i=1}^{4} C_{zi}\xi_1 \cdot \xi_3 - \sum_{i=1}^{4} C_{zi}\xi_1\xi_2 \cdot$$
$$\cdot\, \xi_4 + \sum_{i=1}^{4}(C_{xi}\xi_3{}^2 + C_{zi}\xi_1{}^2) \cdot \xi_5 - \sum_{i=1}^{4} C_{xi}\xi_2 \cdot \xi_6 = M_2 \cdot \cos(\Omega t + \psi_5) + \lambda_5$$

6) $$I_z \cdot \ddot{\xi}_6 - I_{xy}\ddot{\xi}_4 - I_{yz}\ddot{\xi}_5 - \sum_{i=1}^{4} C_{xi}\xi_2 \cdot \xi_1 + \sum_{i=1}^{4} C_{yi}\xi_1 \cdot \xi_2 - \sum_{i=1}^{4} C_{yi}xy\xi_1\xi_2 \cdot$$
$$\cdot\, \xi_4 - \sum_{i=1}^{4} C_{xi}\xi_2\xi_3 \cdot \xi_5 + \sum_{i=1}^{4}(C_{xi}\xi_2{}^2 + C_{yi}\xi_1{}^2)\xi_6 = M_3 \cdot \cos(\Omega t + \psi_3) + \lambda_6 \quad (14)$$

In the equations (14), representing forces and the moments, for convenience of using for the generalized co-ordinates are accepted: q_1, q_2, q_3 - linear motions on axes x, y, z; $\varphi_1, \varphi_2, \varphi_3$ - for angular turns round the axes which beginning coincides with the block centre of gravity. The main axes of inertia vibro protecting the block are directed on the same axes. Other parameters entering into the equations (14):

m - weight of vibro protecting the block; $\overline{C}_x, \overline{C}_y, \overline{C}_z$ - operational factors of rigidity; $I_x, I_y, I_z I_{xy}, I_{xz}, I_{yz}$ - the moments of inertia of vibro protecting the block; $Q_1, Q_2, Q_3, M_1, M_2, M_3$ - components of external forces and the moments, operating on vibro protecting the block on corresponding axes; $\lambda_1, \lambda_2, \ldots, \lambda_6$ - reactions of servo constraints.

From expressions (14) it is visible that at block installation on four shock-absorbers the system possesses six degrees of freedom. Here as the generalized system the co-ordinates, characterizing motions of object concerning the basis are chosen. It is convenient, as for active vibro protection relative motions coincide with absolute, and for passive vibro protection, considering known position of dynamics of relative motion; it is possible to consider basis motion, entering forces of inertia in portable motion. Forces of Koriolis by consideration of small fluctuations can be rejected, as they are proportional to products of small portable angular speeds for small speeds in relative motion of system, are equally suitable both for active, and for passive vibro protection systems. Using a constructive method of search of structure of forces of reactions of servo constraints (Method of A.G. Azizov) [4;5], if reaction of servo constraints to form under laws [6-9; 12-14]:

$$\lambda_1 = -Q_1 \cdot \cos(\Omega t + \psi_1) - k_{11}\dot{\xi}_1 - k_{12}\xi_1, \quad \lambda_2 = -Q_2 \cdot \cos(\Omega t + \psi_2) - k_{21}\dot{\xi}_1 - k_{22}\xi_2,$$

$$\lambda_3 = -Q_3 \cdot \cos(\Omega t + \psi_3) - k_{31}\dot{\xi}_3 - k_{32}\xi_3, \quad \lambda_4 = -M_i \cos(\Omega t + \psi_4) - k_{41}\dot{\xi}_4 - k_{42}\xi_4,$$

$$\lambda_5 = -M_2 \cos(\Omega t + \psi_5) - k_{51}\dot{\xi}_5 - k_{52}\xi_5, \quad \lambda_6 = -M_3 \cos(\Omega t + \psi_6) - k_{61}\dot{\xi}_6 - k_{62}\xi_6 \qquad (15)$$

where $k_{11}, k_{12}, \ldots, k_{61}, k_{62}$ - some constants, substituting (15) in (14), we will receive the equations of the indignant motions:

$$\begin{cases} m\ddot{\xi}_1 + k_{11}\dot{\xi}_1 + k_{12}\xi_1 = 0 \\ m\ddot{\xi}_2 + k_{21}\dot{\xi}_2 + k_{22}\xi_2 = 0 \\ m\ddot{\xi}_3 + k_{31}\dot{\xi}_3 + k_{32}\xi_3 = 0 \\ I_x \cdot \ddot{\xi}_4 + k_{41}\dot{\xi}_4 + k_{42}\xi_4 = 0 \\ I_y \cdot \ddot{\xi}_5 + k_{51}\dot{\xi}_5 + k_{52}\xi_5 = 0 \\ I_z \cdot \ddot{\xi}_6 + k_{61}\dot{\xi}_6 + k_{62}\xi_6 = 0 \end{cases} \qquad (16)$$

System (16) has the private decision

$$\xi_1 = 0, \quad \xi_2 = 0, \quad \xi_3 = 0,$$
$$\xi_4 = 0, \quad \xi_5 = 0, \quad \xi_6 = 0, \qquad (8)$$

corresponding the parities (1).

As it is known, for stability of the decision (8) it is necessary and enough that all roots of the characteristic equation had negative material parts [10]. As factors of the equation constant numbers last conditions can be received using Hurvits's criterion [10]. We investigate at what values of constants $k_{11}, k_{21}, \ldots, k_{61}$ stability of the zero decision (8) systems (7) is provided. For this purpose we will make a characteristic determinant of system (7):

$$\begin{vmatrix} m\lambda^2+\kappa_{11}\lambda+\kappa_{12} & 0 & 0 & 0 & 0 & 0 \\ 0 & m\lambda^2+\kappa_{21}\lambda+\kappa_{22} & 0 & 0 & 0 & 0 \\ 0 & 0 & m\lambda^2+\kappa_{31}\lambda+\kappa_{32} & 0 & 0 & 0 \\ 0 & 0 & 0 & I_x\lambda^2+\kappa_{41}\lambda+\kappa_{42} & 0 & 0 \\ 0 & 0 & 0 & 0 & I_y\lambda^2+\kappa_{51}\lambda+\kappa_{52} & 0 \\ 0 & 0 & 0 & 0 & 0 & I_z\lambda^2+\kappa_{61}\lambda+\kappa_{62} \end{vmatrix}$$

The characteristic equation of system looks like:

$$(m\lambda^2+\kappa_{11}\lambda+\kappa_{12})(m\lambda^2+\kappa_{21}\lambda+\kappa_{22})(m\lambda^2+\kappa_{31}\lambda+\kappa_{32})$$
$$(I_x\lambda^2+\kappa_{41}\lambda+\kappa_{42})(I_y\lambda^2+\kappa_{51}\lambda+\kappa_{52})(I_z\lambda^2+\kappa_{61}\lambda+\kappa_{62})=0 \qquad (9)$$

which it is led to a kind:

$$m\lambda^2+\kappa_{11}\lambda+\kappa_{12}=0$$
$$m\lambda^2+\kappa_{21}\lambda+\kappa_{22}=0$$
$$m\lambda^2+\kappa_{31}\lambda+\kappa_{32}=0$$
$$I_x\lambda^2+\kappa_{41}\lambda+\kappa_{42}=0$$
$$I_y\lambda^2+\kappa_{51}\lambda+\kappa_{52}=0$$
$$I_z\lambda^2+\kappa_{61}\lambda+\kappa_{62}=0 \qquad (9a)$$

Roots of the characteristic equation (9) or (9a)

$$\lambda_i=\frac{-k_{i1}\pm(k_{i1}^2-4\cdot m\cdot k_{i2})^{1/2}}{2\cdot m},$$

$$\lambda_{3+i}=\frac{-k_{3+i,1}\pm(k_{3+i,1}^2-4\cdot I\cdot k_{3+i,2})^{1/2}}{2\cdot I}, \quad (i=1;2;3)$$

We will choose such, that they had negative material parts, i.e. performance of conditions is required:

$$k_{i1}>0 \quad , \quad k_{3+i,1}>0 \quad , \quad i=1,2,3 \qquad (10)$$

2. CONCLUSION

The received results show that, for a protection of mechanical systems from vibrations, except application of correcting actions to them will be necessary a corresponding choice of some magnitudes (factors) as well. The results of work can be useful to engineers and the designers who are engaged in working out of adjustable systems (devices).

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