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Block implicit algorithms for the solution of system of first order ordinary differential equations

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ABSTRACT
A linear multi-step method (LMM) with continuous coefficient is considered in this paper and applied to solve system of first order ordinary differential equations of the form $y' = f(x,y)$ and $z' = g(x,z)$. All the implicit algorithms derived were of the uniform order 8 and zero-stable which are combined as simultaneous numerical integrators to provide solutions to system of first order initial value problems over sub-intervals which do not overlaps. The superiority of the method is demonstrated on both system of linear and non linear first order ODEs numerically.

Keywords: Block Implicit Algorithms, system of first order Odes, Uniform Order, Continuous coefficients

1. INTRODUCTION

Most life and physical problems are modeled into system of first order ordinary differential equations of the form

$$y' = f(x,y) \text{ and } z' = g(x,z) \;, y(0) = \alpha \,, z(0) = \beta \qquad (1)$$

Many numerical techniques are available for the solution of initial value problems (ivps) and these techniques depend on many factors such as speed of convergence, computational expense, data-storage requirements, accuracy and stability of the methods. Chu and Hamilton [3] both suggested that the stability problems appear to be the most serious limitations of Block methods and also starting value for the method. The aim of this research paper is to make our block method zero stable, consistent, increase the rate of convergence and self starting schemes. Also attention has been devoted for the collocation method for solving system of first order ordinary differential equations of the form (1). In literature we have Runge-kutta method, Three term Taylor method and Runge kutta fehlberg method for system of first order ordinary differential equations. Most of these methods are single step methods and has computational burdens in their implementations.

Definition 1.1

A linear multistep method or linear k-step method can be represented in a standard form by an equation

$$\sum_{j=0}^{m+t-1} \alpha_j y_{n+j} = h \sum_{j=0}^{k} \beta_j f_{n+j} \qquad (2)$$

where $y_{n+j} = y(x_{n+j})$ and $f_{n+j} = f(x_{n+j}, y_{n+j})$, coefficients α_j and β_j are satisfy chosen constants subject to the conditions $\alpha_k = 1$ and $|\alpha_0| + |\beta_0| \neq 0$ (Lambert [4])

Definition 1.2

A LMM of (2) is said to satisfy the root conditions if all of the roots of the first characteristics polynomial have modulus less than or equal to unity and those of modulus unity are simple. The method (2) is said to be zero-stable if it satisfies the root condition. (Lambert [4])

Definition 1.3

A LMM (2) is explicit if $\beta_k = 0$ and implicit if $\beta_k \neq 0$, for an explicit method yields the curr Rent value y_{n+k} directly in terms of $y_{n+j}, f_{n+j}\; j = 0,1, \dots k-1$, which, at this stage of the computation have already been calculated. Also an implicit method, however will call for the solution at each stage of the computation of the equation $y_{n+k} = h\beta_k f(x_{n+k}, y_{n+k}) + g$,
where g is a known function of the previous calculated values of $y_{n+j}, f_{n+j}\; j = 0,1, .. k-1$ (Butcher 1987)

Definitions 1.4

A linear Multi-step method (2) is said to be Zero-stable if no root of the first characteristic polynomial $\rho(x) = \sum_{j=0}^{k} \alpha_j r^j$ has modulus greater than one and if every root with modulus one is simple.

2. THE DERIVATION OF THE METHOD

We use the interpolation and collocation procedures to characterize the LMM that is of interest to us by choosing the right number of interpolation points (t) and the right number of collocation points (m). These will lead us to $(t + m - 1)$ system of non linear equations involving $(t + m - 1)$ unknown coefficients to be determined. The Block discrete schemes are much easier to derived using maple 17 (Mathematical software) rather than using matrix inverse approach or purely algebraic approach.

Theorem 1.0

Let it be that $f(x, y(x))$ and $g(x, z(x))$ are continuous in interval $[a, b] \subset R$ then $f(x, y(x))$,$g(x, z(x))$ are Lipschitzian with regard to its second argument,

$$\| f(x, y(x)) - g(x, z(x))\| \leq \|y(x)(x) - z\|, \forall\, x \in [a, b], y(x), z(x) \in S \subset R$$

Then the solutions $y(x), z(x) \in R$ of (1.1) exist and unique in the strip

$S = \{[a, b] * \{y(x), z(x) < \infty\}\}$ Badmus [1]

3. SPECIFICATION OF THE METHOD

Our method is developed through power series solution as the bases function of the form

$$y(x) = \sum_{j=0}^{m+t-1} \varphi_j x^j \qquad (3)$$

The first derivative of (3.1) is of the form

$$y'(x) = \sum_{j=1}^{m+t-1} j\varphi_j x^{j-1} \qquad (4)$$

Specifically in this $k = 3, t = 1$ *and* $m = 8$ which yields the system of non linear equations of the form

$$\sum_{j=0}^{m+t-1} \varphi_j x^j = y_{n+i} \quad i = 0 \qquad (5)$$

$$\sum_{j=1}^{m+t-1} j\varphi_j x^{j-1} = f_{n+i} \quad i = \left(0, \frac{1}{2}, \frac{3}{4}, 1, \frac{3}{2}, 2, \frac{5}{2}, 3\right) \qquad (6)$$

The continuous formulation of this method will be of the form

$$y(x) = \propto_n y_n + h\Big[\beta_n f_n + \beta_{n+\frac{1}{2}} f_{n+\frac{1}{2}} + \beta_{n+\frac{3}{4}} f_{n+\frac{3}{4}} + \beta_{n+1} f_{n+1} + \beta_{n+\frac{3}{2}} f_{n+\frac{3}{2}} + \beta_{n+2} f_{n+2} + \beta_{n+\frac{5}{2}} f_{n+\frac{5}{2}} + \beta_{n+3} f_{n+3}\Big] \qquad (7)$$

where $\propto (x)$ and $\beta(x)$ are obtained as a continuous coefficients of the method. Using Maple 17 mathematical software for equation (5) and (6) and determines values of

$\varphi_j, j = \left(0, \frac{1}{2}, \frac{3}{4}, 1, \frac{3}{2}, 2, \frac{5}{2}, 3\right)$, and the continuous formula is obtained as

$$y(x) = y_n + \left[\xi - \frac{1683}{540h}\xi^2 + \frac{420}{81h^2}\xi^3 - \frac{5453}{1080h^3}\xi^4 + \frac{399}{135h^4}\xi^5 - \frac{413}{405h^5}\xi^6 + \frac{180}{945h^6}\xi^7 - \frac{2}{135h^7}\xi^8\right] f_n + \left[\frac{270}{15h}\xi^2 - \frac{2286}{45h^2}\xi^3 + \frac{957}{15h^3}\xi^4 - \frac{650}{15h^4}\xi^5 + \frac{740}{45h^5}\xi^6 - \frac{344}{105h^6}\xi^7 + \frac{4}{15h^7}\xi^8\right] f_{n+\frac{1}{2}}$$

$$+ \left[-\frac{92160}{2835h}\xi^2 + \frac{129024}{1215h^2}\xi^3 - \frac{59392}{405h^3}\xi^4 + \frac{43008}{405h^4}\xi^5 - \frac{51200}{1215h^5}\xi^6 + \frac{24576}{2835h^6}\xi^7 - \frac{2048}{2835h^7}\xi^8\right] f_{n+\frac{3}{4}} + \left[\frac{270}{12h}\xi^2 - \frac{1413}{18h^2}\xi^3 + \frac{2787}{24h^3}\xi^4 - \frac{1333}{15h^4}\xi^5 + \frac{331}{9h^5}\xi^6 - \frac{164}{21h^6}\xi^7 + \frac{2}{3h^7}\xi^8\right] f_{n+1}$$

$$+ \left[-\frac{180}{27h}\xi^2 + \frac{2004}{81h^2}\xi^3 - \frac{1066}{27h^3}\xi^4 + \frac{4428}{135h^4}\xi^5 - \frac{1184}{81h^5}\xi^6 + \frac{624}{189h^6}\xi^7 - \frac{8}{27h^7}\xi^8\right] f_{n+\frac{3}{2}} + \left[\frac{135}{60h}\xi^2 - \frac{774}{90h^2}\xi^3 + \frac{1713}{120h^3}\xi^4 - \frac{187}{15h^4}\xi^5 + \frac{265}{45h^5}\xi^6 - \frac{148}{105h^6}\xi^7\right.$$

$$+\frac{2}{15h^7}\xi^8\Big]f_{n+2}+\Big[-\frac{54}{105h}\xi^2+\frac{90}{45h^2}\xi^3-\frac{51}{15h^3}\xi^4+\frac{46}{15h^4}\xi^5-\frac{68}{45h^5}\xi^6+\frac{40}{105h^6}\xi^7$$

$$-\frac{4}{105h^7}\xi^8\Big]f_{n+\frac{5}{2}}+\Big[\frac{90}{1620h}\xi^2-\frac{531}{2430h^2}\xi^3+\frac{1223}{3240h^3}\xi^4-\frac{141}{405h^4}\xi^5+\frac{215}{1215h^5}\xi^6-\frac{132}{2835h^6}\xi^7$$

$$+\frac{2}{405h^7}\xi^8\Big]f_{n+3} \quad (8)$$

Also y_{n+j}, f_{n+j} in equations (5),(6) and (7) are replaced by z_{n+j} and g_{n+j} respectively which provides Continuous formulations as

$$z(x)=z_n+\Big[\xi-\frac{1683}{540h}\xi^2+\frac{420}{81h^2}\xi^3-\frac{5453}{1080h^3}\xi^4+\frac{399}{135h^4}\xi^5-\frac{413}{405h^5}\xi^6+\frac{180}{945h^6}\xi^7$$

$$-\frac{2}{135h^7}\xi^8\Big]g_n+\Big[\frac{270}{15h}\xi^2-\frac{2286}{45h^2}\xi^3+\frac{957}{15h^3}\xi^4-\frac{650}{15h^4}\xi^5+\frac{740}{45h^5}\xi^6-\frac{344}{105h^6}\xi^7+\frac{4}{15h^7}\xi^8\Big]g_{n+\frac{1}{2}}$$

$$+\Big[-\frac{92160}{2835h}\xi^2+\frac{129024}{1215h^2}\xi^3-\frac{59392}{405h^3}\xi^4+\frac{43008}{405h^4}\xi^5-\frac{51200}{1215h^5}\xi^6+\frac{24576}{2835h^6}\xi^7$$

$$-\frac{2048}{2835h^7}\xi^8\Big]g_{n+\frac{3}{4}}+\Big[\frac{270}{12h}\xi^2-\frac{1413}{18h^2}\xi^3+\frac{2787}{24h^3}\xi^4-\frac{1333}{15h^4}\xi^5+\frac{331}{9h^5}\xi^6-\frac{164}{21h^6}\xi^7$$

$$+\frac{2}{3h^7}\xi^8\Big]g_{n+1}+\Big[-\frac{180}{27h}\xi^2+\frac{2004}{81h^2}\xi^3-\frac{1066}{27h^3}\xi^4+\frac{4428}{135h^4}\xi^5-\frac{1184}{81h^5}\xi^6+\frac{624}{189h^6}\xi^7$$

$$-\frac{8}{27h^7}\xi^8\Big]g_{n+\frac{3}{2}}+\Big[\frac{135}{60h}\xi^2-\frac{774}{90h^2}\xi^3+\frac{1713}{120h^3}\xi^4-\frac{187}{15h^4}\xi^5+\frac{265}{45h^5}\xi^6-\frac{148}{105h^6}\xi^7$$

$$+\frac{2}{15h^7}\xi^8\Big]g_{n+2}+\Big[-\frac{54}{105h}\xi^2+\frac{90}{45h^2}\xi^3-\frac{51}{15h^3}\xi^4+\frac{46}{15h^4}\xi^5-\frac{68}{45h^5}\xi^6+\frac{40}{105h^6}\xi^7$$

$$-\frac{4}{105h^7}\xi^8\Big]g_{n+\frac{5}{2}}+\Big[\frac{90}{1620h}\xi^2-\frac{531}{2430h^2}\xi^3+\frac{1223}{3240h^3}\xi^4-\frac{141}{405h^4}\xi^5+\frac{215}{1215h^5}\xi^6-\frac{132}{2835h^6}\xi^7$$

$$+\frac{2}{405h^7}\xi^8\Big]g_{n+3}$$

(9)

Evaluating equation (8) at $x=x_{n+j}, j=\frac{1}{2},\frac{3}{4},1,\frac{3}{2},2,\frac{5}{2},3$, which gives the following discrete schemes as our Block method

$$y_{n+\frac{1}{2}}-y_n=\frac{11909}{90720}hf_n+\frac{20477}{20160}hf_{n+\frac{1}{2}}-\frac{2200}{1701}hf_{n+\frac{3}{4}}+\frac{16319}{20160}hf_{n+1}-\frac{19997}{90720}hf_{n+\frac{3}{2}}$$

$$+\frac{361}{5040}hf_{n+2}-\frac{323}{20160}hf_{n+\frac{5}{2}}+\frac{929}{544320}hf_{n+3}$$

$$y_{n+\frac{3}{4}}-y_n=\frac{149819}{1146880}hf_n+\frac{638469}{573440}hf_{n+\frac{1}{2}}-\frac{1233}{1120}hf_{n+\frac{3}{4}}+\frac{875313}{1146880}hf_{n+1}-\frac{60673}{286720}hf_{n+\frac{3}{2}}$$

$$+\frac{15849}{229376}hf_{n+2}-\frac{1269}{81920}hf_{n+\frac{5}{2}}+\frac{379}{229376}hf_{n+3}$$

$$y_{n+1}-y_n=\frac{2969}{22680}hf_n+\frac{347}{315}hf_{n+\frac{1}{2}}-\frac{8192}{8505}hf_{n+\frac{3}{4}}+\frac{2251}{2520}hf_{n+1}-\frac{622}{2835}hf_{n+\frac{3}{2}}+\frac{179}{2520}hf_{n+2}$$

$$-\frac{1}{63}hf_{n+\frac{5}{2}}+\frac{23}{13608}hf_{n+3}$$

$$y_{n+\frac{3}{2}}-y_n=\frac{29}{224}hf_n+\frac{513}{448}hf_{n+\frac{1}{2}}-\frac{8}{7}hf_{n+\frac{3}{4}}+\frac{2943}{2240}hf_{n+1}+\frac{19}{1120}hf_{n+\frac{3}{2}}+\frac{27}{560}hf_{n+2}$$

$$-\frac{27}{2240}hf_{n+\frac{5}{2}}+\frac{3}{2240}hf_{n+3}$$

$$y_{n+2}-y_n=\frac{373}{2835}hf_n+\frac{344}{315}hf_{n+\frac{1}{2}}-\frac{8192}{8505}hf_{n+\frac{3}{4}}+\frac{344}{315}hf_{n+1}+\frac{1136}{2835}hf_{n+\frac{3}{2}}+\frac{17}{63}hf_{n+2}$$

$$-\frac{8}{315}hf_{n+\frac{5}{2}}+\frac{4}{1701}hf_{n+3}$$

$$y_{n+\frac{5}{2}}-y_n=\frac{2305}{18144}hf_n+\frac{4825}{4032}hf_{n+\frac{1}{2}}-\frac{2200}{1701}hf_{n+\frac{3}{4}}+\frac{5875}{4032}hf_{n+1}+\frac{2375}{18144}hf_{n+\frac{3}{2}}+\frac{725}{1008}hf_{n+2}$$

$$+\frac{95}{576}hf_{n+\frac{5}{2}}-\frac{275}{108864}hf_{n+3}$$

$$y_{n+3}-y_n=\frac{41}{280}hf_n+\frac{27}{35}hf_{n+\frac{1}{2}}+\frac{27}{280}hf_{n+1}+\frac{34}{35}hf_{n+\frac{3}{2}}+\frac{27}{280}hf_{n+2}+\frac{27}{35}hf_{n+\frac{5}{2}}$$

$$+\frac{41}{280}hf_{n+3}$$

(10)

Also evaluating (9) at $x=x_{n+j}, j=\frac{1}{2},\frac{3}{4},1,\frac{3}{2},2,\frac{5}{2},3$, which gives the following discrete schemes as our second Block method

$$z_{n+\frac{1}{2}}-z_n=\frac{11909}{90720}hg_n+\frac{20477}{20160}hg_{n+\frac{1}{2}}-\frac{2200}{1701}hg_{n+\frac{3}{4}}+\frac{16319}{20160}hg_{n+1}-\frac{19997}{90720}hg_{n+\frac{3}{2}}$$

$$+\frac{361}{5040}hg_{n+2}-\frac{323}{20160}hg_{n+\frac{5}{2}}+\frac{929}{544320}hg_{n+3}$$

$$z_{n+\frac{3}{4}}-z_n=\frac{149819}{1146880}hg_n+\frac{638469}{573440}hg_{n+\frac{1}{2}}-\frac{1233}{1120}hg_{n+\frac{3}{4}}+\frac{875313}{1146880}hg_{n+1}-\frac{60673}{286720}hg_{n+\frac{3}{2}}$$

$$+\frac{15849}{229376}hg_{n+2}-\frac{1269}{81920}hg_{n+\frac{5}{2}}+\frac{379}{229376}hg_{n+3}$$

$$z_{n+1}-z_n=\frac{2969}{22680}hg_n+\frac{347}{315}hg_{n+\frac{1}{2}}-\frac{8192}{8505}hg_{n+\frac{3}{4}}+\frac{2251}{2520}hg_{n+1}-\frac{622}{2835}hg_{n+\frac{3}{2}}+\frac{179}{2520}hg_{n+2}$$

$$-\frac{1}{63}hg_{n+\frac{5}{2}}+\frac{23}{13608}hg_{n+3}$$

$$z_{n+\frac{3}{2}} - z_n = \frac{29}{224}hg_n + \frac{513}{448}hg_{n+\frac{1}{2}} - \frac{8}{7}hg_{n+\frac{3}{4}} + \frac{2943}{2240}hg_{n+1} + \frac{19}{1120}hg_{n+\frac{3}{2}} + \frac{27}{560}hg_{n+2}$$

$$-\frac{27}{2240}hg_{n+\frac{5}{2}} + \frac{3}{2240}hg_{n+3}$$

$$z_{n+2} - z_n = \frac{373}{2835}hg_n + \frac{344}{315}hg_{n+\frac{1}{2}} - \frac{8192}{8505}hg_{n+\frac{3}{4}} + \frac{344}{315}hg_{n+1} + \frac{1136}{2835}hg_{n+\frac{3}{2}} + \frac{17}{63}hg_{n+2}$$

$$-\frac{8}{315}hg_{n+\frac{5}{2}} + \frac{4}{1701}hg_{n+3}$$

$$z_{n+\frac{5}{2}} - z_n = \frac{2305}{18144}hg_n + \frac{4825}{4032}hg_{n+\frac{1}{2}} - \frac{2200}{1701}hg_{n+\frac{3}{4}} + \frac{5875}{4032}hg_{n+1} + \frac{2375}{18144}hg_{n+\frac{3}{2}}$$

$$+\frac{725}{1008}hg_{n+2} + \frac{95}{576}hg_{n+\frac{5}{2}} - \frac{275}{108864}hg_{n+3}$$

$$z_{n+3} - z_n = \frac{41}{280}hg_n + \frac{27}{35}hg_{n+\frac{1}{2}} + \frac{27}{280}hg_{n+1} + \frac{34}{35}hg_{n+\frac{3}{2}} + \frac{27}{280}hg_{n+2} + \frac{27}{35}hg_{n+\frac{5}{2}}$$

$$+\frac{41}{280}hg_{n+3} \qquad (11)$$

The Block methods of (10) and (11) have the same order and error constants as follows.

SCHEMES	ORDERS	ERROR CONSTANTS
$y_{n+\frac{1}{2}} = z_{n+\frac{1}{2}}$	8	$\frac{-44749}{14863564800}$
$y_{n+\frac{3}{4}} = z_{n+\frac{3}{4}}$	8	$\frac{-137601}{4697204800}$
y_{n+1} z_{n+1}	8	$\frac{-173}{58060800}$
$y_{n+\frac{3}{2}} = z_{n+\frac{3}{2}}$	8	$\frac{-477}{183500800}$
$y_{n+2} = z_{n+2}$	8	$\frac{-7}{2073600}$
$y_{n+\frac{5}{2}} = z_{n+\frac{5}{2}}$	8	$\frac{-725}{594542592}$
$y_{n+3} = z_{n+3}$	8	$\frac{-9}{716800}$

4. ANALYSIS AND IMPLEMENTATION OF THE METHODS

In order to analyze the methods for zero-stability, following Yahaya [5], we write equations (10) and (11) as

$A^0 Y_{\mu+i} = A^1 Y_\mu + h[\boldsymbol{B}^0 F_{\mu+i} + \boldsymbol{B}^1 F_\mu]$ and $A^0 Z_{\mu+i} = A^1 Z_\mu + h[\boldsymbol{B}^0 G_{\mu+i} + \boldsymbol{B}^1 G_\mu]$ respectively,

where $Y_{\mu+i} = y_{n+j}\ , j = \left(\frac{1}{2}, \frac{3}{4}, 1, \frac{3}{2}, 2, \frac{5}{2}, 3\right)$

$Y_\mu = y_{n-j}\ , j = \left(\frac{1}{2}, \frac{3}{4}, 1, \frac{3}{2}, 2, \frac{5}{2}, 3\right)$

$F_{\mu+i} = f_{n+j}\ , j = \left(\frac{1}{2}, \frac{3}{4}, 1, \frac{3}{2}, 2, \frac{5}{2}, 3\right)$

$F_\mu = f_{n-j}\ , j = \left(\frac{1}{2}, \frac{3}{4}, 1, \frac{3}{2}, 2, \frac{5}{2}, 3\right)$

Similarly,

$Z_{\mu+i} = z_{n+j}\ , j = \left(\frac{1}{2}, \frac{3}{4}, 1, \frac{3}{2}, 2, \frac{5}{2}, 3\right)$

$$Z_\mu = y_{n-j}\ ,j = \left(\tfrac{1}{2},\tfrac{3}{4},1,\tfrac{3}{2},2,\tfrac{5}{2},3\right)$$

$$G_{\mu+i} = f_{n+j}\ ,j = \left(\tfrac{1}{2},\tfrac{3}{4},1,\tfrac{3}{2},2,\tfrac{5}{2},3\right)$$

$$G_\mu = f_{n-j}\ ,j = \left(\tfrac{1}{2},\tfrac{3}{4},1,\tfrac{3}{2},2,\tfrac{5}{2},3\right)$$

(12)

Arranging (12) in Matrix form , we have

$$\begin{bmatrix}1&0&0&0&0&0&0\\0&1&0&0&0&0&0\\0&0&1&0&0&0&0\\0&0&0&1&0&0&0\\0&0&0&0&1&0&0\\0&0&0&0&0&1&0\\0&0&0&0&0&0&1\end{bmatrix}\begin{bmatrix}y_{n+\frac{1}{2}}\\y_{n+\frac{3}{4}}\\y_{n+1}\\y_{n+\frac{3}{2}}\\y_{n+2}\\y_{n+\frac{5}{2}}\\y_{n+3}\end{bmatrix}=\left[\begin{pmatrix}0&0&0&0&0&0&1\\0&0&0&0&0&0&1\\0&0&0&0&0&0&1\\0&0&0&0&0&0&1\\0&0&0&0&0&0&1\\0&0&0&0&0&0&1\\0&0&0&0&0&0&1\end{pmatrix}\right]\begin{bmatrix}y_{n-\frac{5}{2}}\\y_{n-\frac{9}{4}}\\y_{n-2}\\y_{n-\frac{3}{2}}\\y_{n-1}\\y_{n-\frac{1}{2}}\\y_n\end{bmatrix}$$

$$+\begin{bmatrix}\frac{20477}{20160}&-\frac{2200}{1701}&\frac{16319}{20160}&-\frac{19997}{90720}&\frac{361}{5040}&-\frac{323}{20160}&\frac{929}{544320}\\\frac{638469}{573440}&-\frac{1233}{1120}&\frac{875313}{1146880}&-\frac{60673}{286720}&\frac{15849}{229376}&-\frac{1269}{81920}&\frac{379}{229376}\\\frac{347}{315}&-\frac{8192}{8505}&\frac{2251}{2520}&-\frac{622}{2835}&\frac{179}{2520}&-\frac{1}{63}&\frac{23}{13608}\\\frac{513}{448}&-\frac{8}{7}&\frac{2943}{2240}&\frac{19}{1120}&\frac{27}{560}&-\frac{27}{2240}&\frac{3}{2240}\\\frac{344}{315}&-\frac{8192}{8505}&\frac{344}{315}&\frac{1136}{2835}&\frac{17}{63}&-\frac{8}{315}&\frac{4}{1701}\\\frac{4825}{4032}&-\frac{2200}{1701}&\frac{5875}{4032}&\frac{2375}{18144}&\frac{725}{1008}&\frac{95}{576}&-\frac{275}{108864}\\\frac{27}{35}&0&\frac{27}{280}&\frac{34}{35}&\frac{27}{280}&\frac{27}{35}&\frac{41}{280}\end{bmatrix}\begin{bmatrix}f_{n+\frac{1}{2}}\\f_{n+\frac{3}{4}}\\f_{n+1}\\f_{n+\frac{3}{2}}\\f_{n+2}\\f_{n+\frac{5}{2}}\\f_{n+3}\end{bmatrix}$$

$$+\begin{bmatrix}0&0&0&0&0&0&\frac{11909}{90720}\\0&0&0&0&0&0&\frac{149819}{1146880}\\0&0&0&0&0&0&\frac{2969}{22680}\\0&0&0&0&0&0&\frac{29}{224}\\0&0&0&0&0&0&\frac{373}{2835}\\0&0&0&0&0&0&\frac{2305}{181144}\\0&0&0&0&0&0&\frac{41}{280}\end{bmatrix}\begin{bmatrix}f_{n-\frac{5}{2}}\\f_{n-\frac{9}{4}}\\f_{n-2}\\f_{n-\frac{3}{2}}\\f_{n+2}\\f_{n-1}\\f_n\end{bmatrix}$$

(13)

Thus obtain the normalized form of (13) by multiply through by the inverse of $A^{(0)}$ and the first characteristics polynomial of the normalized matrix will be expressed as

$$\rho(R) = det\left[RA^{(0)} - A^{(1)}\right] = det\left[R\begin{pmatrix}1&0&0&0&0&0&0\\0&1&0&0&0&0&0\\0&0&1&0&0&0&0\\0&0&0&1&0&0&0\\0&0&0&0&1&0&0\\0&0&0&0&0&1&0\\0&0&0&0&0&0&1\end{pmatrix}-\begin{pmatrix}0&0&0&0&0&0&1\\0&0&0&0&0&0&1\\0&0&0&0&0&0&1\\0&0&0&0&0&0&1\\0&0&0&0&0&0&1\\0&0&0&0&0&0&1\\0&0&0&0&0&0&1\end{pmatrix}\right] = 0$$

$$det\left[R\begin{pmatrix} R & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & R & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & R & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & R & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & R & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & R & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & R-1 \end{pmatrix}\right] = R^6(R-1) = 0$$

which implies that $R_1 = R_2 = R_3 = R_4 = R_5 = R_6 = 0$ and $R_7 = 1$. From definition (1.4) and block method (13) is zero stable and also consistent as its Order $P = [8,8,8,8,8,8,8]^T > 1$, thus it is convergent.

5. IMPLEMENTATION STRATEGIES

The method (10) and (11) are simultaneously applied on the given problems at $n = 0$, gives the following set of solutions at $y_{\frac{1}{2}}, y_{\frac{3}{4}}, y_1, y_{\frac{3}{2}}, y_2, y_{\frac{5}{2}}, y_3, z_{\frac{1}{2}}, z_{\frac{3}{4}}, z_1, z_{\frac{3}{2}}, z_2, z_{\frac{5}{2}}$ *and* z_3 at once. Further integration is done at $n = 1,2,3, \dots$.

6. NUMERICAL EXPERIMENTS

Example 1

$y' = z - x \qquad y(0) = 1$

$z' = y + x \qquad z(0) = 1 \ , h = 0.1$

Exact solution: $y(x) = -e^{-x} + e^x + 1 - x$ and

$z(x) = e^{-x} + e^x - 1 + x$

Table 1 Approximate solution of Example 1 of $y(x)$

$y(x)$	Theoretical solution of $y(x)$	New Block method	Absolute error
0.1	1.1100333500039690	1.1100333500039690	0.00
0.2	1.202672005082190	1.202672005082180	1.0 x 10^{-14}
0.3	1.3090405868942280	1.309040586894300	8.0 x 10^{-14}
0.4	1.421504651605630	1.421504651605640	1.0 x 10^{-14}
0.5	1.5421906101987500	1.5421906101987510	0.00
0.6	1.673307164296480	1.673307164296510	3.0 x 10^{-14}
0.7	1.817167403679070	1.817167403679100	3.0 x 10^{-14}
0.8	1.976211964375250	1.976211964375300	5.0 x 10^{-14}
0.9	2.153033451416350	2.153033451416410	6.0 x 10^{-14}
1.0	2.350402387287610	2.350402387287680	7.0 x 10^{-14}

Table 2 Approximate solution of Example 1 of $z(x)$

$z(x)$	Theoretical solution of $z(x)$	New Block method	Absolute error
0.1	1.110008336111610	1.110008336111610	0.00

0.2	1.240133511238150	1.240133511238150	0.00
0.3	1.390677028257720	1.390677028257730	1.0 x 10^{-14}
0.4	1.562144743676910	1.562144743676920	1.0 x 10^{-14}
0.5	1.755251930412760	1.755251930412750	1.0 x 10^{-14}
0.6	1.970930436484540	1.970930436484560	2.0 x 10^{-14}
0.7	2.210338011261890	2.210338011261900	1.0 x 10^{-14}
0.8	2.47486989260960	2.474869892609700	1.0 x 10^{-13}
0.9	2.766172770897550	2.766172770897590	4.0 x 10^{-14}
1.0	3.086161269630490	3.086161269630520	3.0 x 10^{-14}

Example 2

$y' = xy \qquad y(0) = 1$

$z' = -xz \qquad z(0) = 1\ , h = 0.1$

Exact solution: $y(x) = -e^{\frac{1}{2}x^2}$ and

$z(x) = e^{-\frac{1}{2}x^2}$

Table 3 Approximate solution of Example 2 of $y(x)$

$y(x)$	Theoretical solution of $y(x)$	New Block method	Absolute error
0.1	1.005012520859400	1.005012520859770	3.7 x 10^{-13}
0.2	1.020201340026760	1.020201340027190	4.3 x 10^{-13}
0.3	1.046027859908720	1.046027859910610	1.89 x 10^{-12}
0.4	1.083287067674960	1.083287067678760	3.8 x 10^{-12}
0.5	1.133148453066830	1.133148453072620	5.79 x 10^{-12}
0.6	1.197217363121810	1.197217363131810	1.00 x 10^{-11}
0.7	1.277621313204890	1.277621313220700	1.581 x 10^{-11}
0.8	1.377127764335960	1.377127764359310	2.335 x 10^{-11}
0.9	1.499302500056770	1.499302500092530	3.576 x 10^{-11}
1.0	1.648721270700130	1.648721270753840	5.371 x 10^{-11}

Table 4 Approximate solution of Example 2 of $z(x)$

$z(x)$	Theoretical solution of $z(x)$	New Block method	Absolute error
0.1	0.995012479192682	0.995012479192341	3.41 x 10^{-13}
0.2	0.980198673306755	0.980198673306359	3.96 x 10^{-13}
0.3	0.955997481833100	0.955997481831416	1.684 x 10^{-12}
0.4	0.923116346386636	0.923116346383736	2.9 x 10^{-12}
0.5	0.882496902584595	0.882496902581041	3.554 x 10^{-12}
0.6	0.835270211411272	0.835270211406296	4.97 x 10^{-12}
0.7	0.782704538241868	0.782704538235973	5.895 x 10^{-12}
0.8	0.726149037073691	0.726149037067731	5.96 x 10^{-12}
0.9	0.666976810858474	0.666976810852084	6.39 x 10^{-12}

1.0	0.606530659712633	0.606530659706490	6.143 x 10^{-12}

Example 3

$y^{'} = z + x \qquad y(0) = 2$

$z^{'} = x - y^2 \qquad z(0) = 1$

Exact solution: No solution

Table 5 Approximate solution of Example 3 of $y(x)$ with Approximate Error

$y(x)$	New Block method at $h = 0.1$	New Block method at $h = 0.05$	Approximate Absolute error
0.1	2.084544061697350	2.084544061689820	7.53 x 10^{-12}
0.2	2.136739480880810	2.136739480881470	6.6 x 10^{-13}
0.3	2.155395749576290	2.155395749542630	3.366 x 10^{-11}
0.4	2.140714535097050	2.140714535138780	4.173 x 10^{-11}
0.5	2.094318380399620	2.094318380518020	1.184 x 10^{-10}
0.6	2.019154774276570	2.019154774426750	1.5018 x 10^{-10}
0.7	1.919291571122690	1.919291571398360	2.7567 x 10^{-10}
0.8	1.799634629998490	1.79963463034926	3.5077 x 10^{-10}
0.9	1.665606887640460	1.6656066887978680	1.9884259 x 10^{-7}
1.0	1.522828761758040	1.522828762146360	3.8832 x 10^{-10}

Table 6 Approximate solution of Example 3 of $z(x)$ with Approximate Error

$z(x)$	New Block method at $h = 0.1$	New Block method at $h = 0.05$	Absolute error
0.1	0.586785292449664	0.586785292554199	1.04535 x 10^{-10}
0.2	0.155113259833000	0.155113259952395	1.19395 x 10^{-10}
0.3	-0.281656618354304	-0.2816566178845021	4.6980 x 10^{-10}
0.4	-0.709247049308485	-0.709247048622105	6.86380 x 10^{-10}
0.5	-1.113728704010540	-1.1137287034265	5.84040 x 10^{-10}
0.6	-1.482715635577140	-1.482715634912680	6.64460 x 10^{-10}
0.7	-1.806317529004200	-1.806317528465990	5.38210 x 10^{-10}
0.8	-2.077728976093890	-2.077728975965900	1.27990 x 10^{-10}
0.9	-2.2934105366354320	-2.293410536636610	1.180 x 10^{-12}
1.0	-2.452891660194900	-2.452891660340330	1.45430 x 10^{-10}

7. DISCUSSION OF RESULTS

The new method of equations (10) and (11) when simultaneously implemented on examples 1 and 2 performed accurately with the exact solutions. In example 3 which is non linear problem without theoretical solution, the approximate Errors between h and $\frac{h}{2}$ is very small justifying that the method is accurate.

8. CONCLUSION

We want to conclude that this paper demonstrated a successful implementation of linear multi-step block methods for the solution of system of first order differential equations. The results for $y(x_{n+j})$ and $z(x_{n+j})$ $j = (0,1,2,..,10)$ were obtained in block forms which speed up the computational processes, less burden in the implementations and also increase the rate of convergence of the solutions.

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