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Influence of change of moment of inertia and mass-loss of pulsar on variation of pulse period in magnetic dipole model

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ABSTRACT

The theoretical formulas for the influence of the change of moment of inertia and mass-loss on the variation of the pulse period in the magnetic dipole model are given. The numerical solutions of slow down of period due to mass-loss are calculated by using the given formulas for three pulsars: PSR0355+54, PSR1930+22 and PSR1800-21. The numerical results are listed in Table 1. Discussion and conclusion are drawn.

Key words: Pulsars- variation of moment of inertia and mass-loss-i-- pulse period----influences

1. INTRODUCTION

Some authors studied the variation of Pulse Period arisen from the change of moment of inertia. However, they always use the method of the angular momentum conservation (IΩ= const). The angular momentum is not conservative, if when we consider mass-loss of a star. Hence the change of angular momentum arisen from variation of mass-loss is not suitable to the research by using the angular momentum conservation. In the formula of the magnetic dipole model of pulsars the moment of inertia is not variable but when we consider the mass-loss of pulsar, the moment of inertia may be changed. Hence it is necessary to give the formula for the magnetic dipole model suited to the variation of moment of inertia. This is an important work in this paper.

There are some authors for researching the influence of mass-loss of a star on the rotation .However, there is a few author for researching the influence of mass-loss of pulsar on the rotation. Esposito and Harrison (1975) researched the influence of mass-loss on the orbital period for the Taylor binary pulsar, but they did not research the influence of mass-loss through the change of moment of inertia on variation of the period. This paper studies some works in this aspect.

2. THE FORMULAE FOR THE INFLUENCE OF CHANGE OF MOMENT OF INERTIA ON THE VARIATION OF THE PERIOD OF PULSAR

The pulsar radiating power W is transformed from the rotational energy at a rate $\frac{dE}{dt}$,

$$W = -\frac{dE}{dt} \quad \text{or} \quad W + \frac{dE}{dt} = 0 \qquad (1)$$

According to the theory of magnetic model (Ostriker &Gunn,1969,and Shapiro & Teukolsky 1983)

$$\frac{dE}{dt} = -\frac{2}{3c^3}(M\sin\alpha)^2\Omega^4 = -\frac{32}{3c^3}\frac{\pi^4\mu^2}{P^4}, \quad \Omega = \frac{2\pi}{P} \qquad (2)$$

Substituting the equation (2) into the equation (1), we can obtain the radiating power W

$$W = \frac{32\pi^4\mu^2}{3c^3P^4} \qquad (3)$$

Where P is the period of pulsar, and μ is the projection of the magnetic dipole moment M on the direction perpendicular to the rotational axis. α denotes magnetic inclination

$$\mu^2 = (M\sin\alpha)^2 = (M_0\sin\alpha)^2 e^{-\xi t} = \mu_0{}^2 e^{-\xi t}. \qquad (4)$$

ξ is the coefficient of magnetic decay,

The energy carried away by radiation from the rotational energy of pulsar can be written

$$E = \frac{1}{2}I\Omega^2 = 2\pi^2 I/P^2 . \qquad (5)$$

Here I is moment of inertia. If we consider the variation of moment of inertia with time, then

$$\frac{dE}{dt} = 2\pi^2[\frac{1}{P^2}\frac{dI}{dt} - \frac{2I(t)}{P^3}\frac{dP}{dt}]. \qquad (6)$$

Substituting the formula (3) and (6) into the formula (1), we obtain the Bernoulli equation for n=1

$$\frac{dP}{dt} - \frac{1}{2}(\frac{1}{I}\frac{dI}{dt})P = \frac{8\pi^2\mu^2(t)}{3c^3I(t)}P^{-1} \qquad (7)$$

We can transform Bernoulli equation into the first order linear differential equation. Both sides of the equation $(7)_1$ are multiplied by 2P. i.e..

$$2P\frac{dP}{dt} - \frac{\dot{I}}{I}P^2 = \frac{16\pi^2\mu^2}{3c^3I(t)}. \qquad (7)$$

The equation (7) may be written as the form of the first order linear differential equation

$$\frac{dP^2}{dt} - \frac{\dot{I}}{I} P^2 = Q(t)\,, \qquad (8)$$

We define $N = -\frac{\dot{I}}{I}$

According to the first linear differential equation (6), N is the function of time t or it is a constant value. In this paper N is a constant value as shown in the expressions (22). Integrating (9), one yields

$$I = I_0\, e^{-Nt}\,, \qquad (10)$$

$$Q(t) = \frac{16\pi^2 \mu(t)^2}{3c^3 I(t)} = \frac{16\pi^2 \mu^2{}_0}{3c^3 I_0} e^{-(\xi - N)t}\,,$$

The equation (8) may be written as

$$\frac{dP^2}{dt} + NP^2 = Q(t). \qquad (11)$$

Integrating the equation (11), one yields

$$P(t)^2 = e^{-\int Ndt}[\int Q(t) e^{\int Ndt} dt + C].$$

Substituting (10) into the above integral expression, we obtain

$$P(t)^2 = e^{-Nt}[C + (\frac{16\pi^2 \mu_0{}^2}{3c^3 I_0}) \int e^{-(\xi - 2N)t} dt].$$

When we take t = 0, $P^2(t) = P^2(0)$, $\therefore C = P(0)^2$. i. e

$$P(t)^2 = e^{-kt}[P(0)^2 + (\frac{16\pi^2 \mu_0{}^2}{3c^3 I_0}) \int_0^t e^{-(\xi - 2N)t} dt].$$

Integrating the above expression, we obtain

$$P(t)^2 = e^{-kt}\left\{P(0)^2 - \frac{16\pi^2}{3c^3(\xi + 2N)} (\frac{I_0}{\mu_0{}^2})^{-1} [e^{-(\xi - 2N)t} - 1]\right\}. \qquad (12)$$

When we only consider magnetic decay and do not consider the variation of moment of inertia, i. e

$N = 0,,.\ \xi \lhd 0,$

$$P(t)^2 = P(0)^2 - \frac{8\pi^2}{3c^3 \xi} (\frac{I_0}{\mu_0{}^2})^{-1} (e^{-\xi t} - 1). \qquad (13)$$

When we only consider the variation of moment of inertia and do not consider the magnetic decay, i. e. $N \neq 0, \xi = 0,$ .then

$$P(t)^2 = e^{-kt}[P(0)^2 = \frac{8\pi^2}{3c^3 N} (\frac{I_0}{\mu_0{}^2})^{-1} (e^{2Nt} - 1)]. \qquad (14)$$

In the equation (7) I = I_0, $\frac{dI}{dt} = 0,$ P = P(0), $\dot{P} = \dot{P}(0),$ as $t = 0$ we obtain

$$\frac{8\pi^2\mu^2{}_0}{3c^3 I_0} = P(0)\dot{P}(0) \qquad (15)$$

Substituting (15) into equations (13) and (14), the equations (13) and (14) can be written as

$$P(t)^2 = P(0)^2 - 2P(0)\dot{P}(0)(e^{-\xi t} - 1)/\xi \qquad (16)$$

$$P(t)^2 = e^{-Nt}\{P(0)^2 = P(0)\dot{P}(0)[e^{2Nt} - 1]\}/N \qquad (17)$$

Hence, we can estimate the variable rate of pulse period per century

$$\delta P = [P(t) - P(t_0)][({}^{s}\!/_{cent}) . \qquad (18)$$

Here P(0) is the initial value as t = 0.

3. THE INFLUENCE OF MASS-LOSS OF PULSAR ON THE VARIATION OF ITS PERIOD

Esposito and Harrison (1975) wrote the luminosity of pulsar $B = 4\pi mR^2(\frac{dP}{dt})/5P^2$ which equal to the rate of mass-loss-$c^2\frac{dm}{dt}$. This is obtained from moment of inertia as a constant. In the present paper moment of inertia, I, is variable with time. We use the formula: $I = \frac{2}{5}mR^2$, where R is radius of pulsar which is assumed to be constant.

Because mass is variable with time .Hence

$$\frac{dI}{dt} = \frac{2}{5}R^2\frac{dm}{dt}, \quad I = \frac{2}{5}mR^2 \quad , \frac{\dot{I}}{I} = \frac{\dot{m}}{m} \qquad (19)$$

Substituting these into the expression (5), and letting it equals $-c^2\frac{dm}{dt}$, and then, we have

$$c^2\frac{dm}{dt} - \frac{4}{5}\pi^2(\frac{R^2}{P^2})\frac{dm}{dt} = -\frac{8}{5}\pi^2\frac{mR^2}{P^3}\frac{dP}{dt}$$

$$\therefore \frac{1}{m}\frac{dm}{dt} = -\frac{8\pi^2R^2}{5P^3}\frac{dP}{dt}(c^2 - \frac{4\pi^2R^2}{5P^2})^{-1} = -\frac{8\pi^2R^2}{5c^2P^3}\frac{dP}{dt}(1 + \frac{4\pi^2R^2}{5c^2P^2} -) \qquad (20)$$

The second term of right hand side may be neglected because ${}^{1}\!/_{c^4}$ is very small. According to the expression（9）and (19)-(20) we can write

$$N = -\frac{\dot{I}}{I} = -\frac{1}{m}\frac{dm}{dt} = \frac{8}{5}\frac{\pi^2R^2}{c^2P^3}\frac{dP}{dt}. \qquad (21)=$$

Based on the formula (9), N is a constant, but in the right hand side of the above formula $\frac{dP(t)}{dt}$ and $P(t)^3$ is variable with time..

In the following we may prove $\frac{dP(t)}{dt} = \frac{dP(0)}{dt} = const$ and $P(t)^3 = P0)^3$ **= const. We may use Taylor series to expand** $\frac{dP(t)}{dt}$ **and** $P(t)^3$ **as follows**

$$\frac{dP(t)}{dt} = \dot{P}(t) = \dot{P}(0) + \ddot{P}(0)(t - t_0) + ...$$
$$P(t)^3 = P0)^3 + 3P(0)^2 \dot{P}(0)(t - t_0) + ...$$

For a lot of pulsars $P(0) \sim 0.1, \dot{P}(0) \sim 10^{-15}, \ddot{P}(0) \sim 10^{-24}$ we take $t - t_0 = 100 year(Century) \sim 10^9 (s)$

$$\ddot{P}(0)(t - t_0) \sim 10^{-15}$$
$$3P(o)^2 \dot{P}(0)(t - t_0) \sim 10^{-6}$$

The terms $\ddot{P}(0)(t - t_0)$ and $3P(0)^2 \dot{P}(0)$ so small that may be neglected. Hence $\frac{dP(t)}{dt} = \frac{dP(0)}{dt}$ =const and

$P(t)^3 = P(0)^3$ =constant. Therefore, in the formula (16) N is a constant. The exponential form (8) is hold well

$$N = -\frac{\dot{I}}{I} = -\frac{1}{m}\frac{dm}{dt} = \frac{8}{5}\frac{\pi^2 R^2}{c^2 P(0)^3} \quad \frac{dP(00}{dt} = 2.5266 \times 10^{-8} \frac{\dot{P}(0)}{P(0)} = Cons\tan t. \quad (22)$$

4. NUMERICAL RESULTS

We only consider the case of the variation of moment of inertia due to mass-loss and do not consider magnetic decay. We use the formulas (17)- (18) and (22) to calculate the variation of pulse period due to mass-loss for PSR0355+54, PSR1930+22 and PSR1800-21. For these pulsars we assume that $R = 12 \times 10^5 km$. The data of P and $\frac{dP}{dt}$ of PSR0355+54 are adopted from Table of pulsar parameters given by Manchester & Taylor (1977). The data of P and $\frac{dP}{dt}$ of PSR1930+22 and PSR1800-21 are adopted from given by Arzoumanian et al (1994). Substituting the above data into the formula (17) and (18), we obtain the numerical results for the rates of spin down of pulse periods due to mass-loss for three pulsars are listed in Table 1.

Table 1 Numerical results of three pulsars

Pulsars	P (0) (s)	$dP(0)/dt$ $(10^{-15} s/s)$	$N = -\frac{1}{I}\frac{dI}{dt} = -\frac{1}{m}\frac{dm}{dt} (10^{-8}/s)$	$P(t)$ $(s/cent)$	δP $(s/cent)$
PSR0355+54	0.1563	4.39	$.90 \times 10^{-2}$	0.1576	0.0013
PSR1930+22	0.1444	63	5.28×10^{-1}	0.1463	0.0019
PSR1800-21	0.1336	134.229	1.4225	0.1376	0.0040

5. DISCUSSION

(1) In the quadrupole elastic energy model of neutron stars the total energy and moment of inertia connects with oblateness ε:

$$E = E_0 + \frac{1}{2} I\Omega^2 + A\varepsilon^2 + B(\varepsilon - \varepsilon_0)^2, \; I = I_0(1 + \varepsilon).$$

But in the magnetic dipole model we may not consider oblateness or $\varepsilon = 0$. We consider pulsars as spherical stars..

(2) The formula $\mu^2 = {\mu_0}^2 e^{-\xi t}$ cited in this paper is suitable to the magnetic inclination α, as a constant. Because

$\mu^2 = M^2 \sin^2 \alpha = {M_0}^2 e^{-\xi t} \sin^2 \alpha = ({M_0}^2 \sin^2 \alpha) e^{-\xi t} = {\mu_0}^2 e^{-\xi t} \quad {\mu_0}^2 = {M_0}^2 \sin^2 \alpha = {M_0}^2 \sin^2 \alpha_0$, i. e, $\alpha = \alpha_0 = const$, Hence the research of this paper includes the magnetic inclination as a constant. However the magnetic inclination of pulsars varies very small in a century. We can assume that the magnetic inclination α is a constant in a century (3). We may discuss mass-less as compared with magnetic decay. We use the formula (16) to calculate the variable rate of pulse period due to magnetic decay. Substitution of these data $\xi = 1.3\times10^{-6}/yr$ or $\xi = 3.54\times10^{-14}/s$ given by Qu et al (1976) into the formula (16), we obtain,

For PSR0355+54: $(\delta P)_{mag} = 0.00029(s/cent)$,

For PSR1930+22: $(\delta P)_{mag} = 0.00032(s/cent)$,

For PSR1800-21: $(\delta P)_{mag} = 0.00034(s/cent)$.

From these numerical results as compared with the results due to mass loss in Table 1 we can see that the values for magnetic decay are nearly the same order with that of mass-loss for three pulsars. Hence both results cannot be ignored.

6. CONCLUSIONS

(1) The change of moment inertia of pulsars can influences spin down of pulse period in the magnetic dipole model, and spin down connect with the exponential formulation.

(2) Pulsar loses its mass with the formulation of the magnetic dipole radiation. It can influences spin down of pulse period through the change of moment of inertia. The rate of spin down of pulse period is the order $10^{-3}(s/cent)$ for three pulsars. The rate of slow down due to mass-loss can be observed by using the recent astronomical instruments through a long time.

(3) It can be seen from Table 1 that the shorter the pulse period, the longer the rate of pulse period per century, such as PSR1800-21. ..

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