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Torch vibrations of a viscoelastic shell with a viscous liquid

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ABSTRACT
The article presents an effective numerical-analytical method for solving problems of hydro-viscose-elasticity and gives examples of calculating the natural oscillations of thin-walled cylindrical shells in contact with a viscous fluid by this method. The linear torsional vibrations of a cylindrical viscoelastic shell in contact with a viscous fluid are considered. The equation of motion of a viscoelastic shell is obtained on the basis of the moment theory of shells and satisfies the Kirchhoff Love and S.P. Timoshenko hypotheses. Fluid motion is described by the Navier Stokes equations. The problem is solved numerically by the methods of orthogonal sweep and Muller. The spectrum of complex Eigen frequencies and modes of oscillations is obtained depending on the wave number.

Keyword: shell, viscous fluid, orthogonal sweep, torsional vibrations, Kirchhoff-Love hypothesis, natural frequency

1. INTRODUCTION
Despite a wide-ranging study of the interaction of elastic systems, there are still issues to be solved. The question of the effect of wave formation and compressibility of a liquid has not been fully investigated. Calculation methods are often cumbersome and require simplification. The interaction of the viscoelastic systems themselves in fluid flows has been little studied. The phenomenon

of the propagation of a wavelike motion of a fluid in elastic cylindrical shells attracted the attention of researchers [1-6]. In these works devoted to wave processes, the system "elastic cylindrical shell - ideal fluid" uses classical and refined shell equations, considers the effect of radial and longitudinal inertial forces, takes into account the average density of a fluid or gas flow. In [7–9], an analysis of the laws of the wave process in an elastic shell with a viscous fluid is performed within the framework of the model of linearized hydrodynamic equations of a viscous compressible fluid. Unlike the others, the system "cylindrical shell (elastic or viscoelastic) and liquid" (more perfect or viscous) is considered as a dissipative non-uniform mechanical system [10-12].

2. PROBLEM STATEMENT

The natural oscillations of a cylindrical shell of radius infinite along the length of a deformable (viscoelastic) cylindrical shell are considered R_1 with constant thickness h_0, shell density ρ, E_0- instant modulus of viscoelastic shell, Poisson's ratio ν_0, filled into a viscous liquid with a density ρ_0 in equilibrium. Shell vibrations when exposed to internal pressure $\vec{p}(-p_1,-p_2,p_3)$ described by equations [8, 12, 19]:

$$L\vec{u}-\int_0^t LR_E(t-\tau)\vec{u}(\vec{r},\tau)d\tau=\frac{(1-\nu_0^2)}{E_0h_0}\vec{p}+\rho\frac{(1-\nu_0{}^2)}{E_0}\frac{\partial^2\vec{u}}{\partial t^2}. \quad (1)$$

Here E_0 - instant modulus, $\vec{u}=\vec{u}(u_r,u_\theta,u_z)$ - the vector of displacements of the points of the middle surface of the shell, and for Kirchgor – Love shells, it has dimension equal to three $(u_r=u;\quad u_\theta=v;\quad u_z=w)$, and for Tymoshenko-type shells, the dimension of the vector is five; here, in addition to the axial, circumferential, and normal displacements, the angles of rotation of the normal to the median surface in the axial and circumferential directions are added [12]; $R_E(t-\tau)$–core relaxation. The amplitudes of oscillations are considered small, which allows one to write down the basic relations in the framework of the linear theory. The system of linearized equations of motion of a viscous barotropic fluid can be represented as [12]

$$\frac{\partial\vec{\vartheta}}{\partial t}-\nu*\Delta\vec{\vartheta}+\frac{1}{\rho_0^*}grad\,P-\frac{\nu*}{3}grad\;div\,\vec{\vartheta}=0$$

$$\frac{1}{\rho_0^*}\frac{\partial\rho*}{\partial t}+div\,\vec{\vartheta}=0;\quad \frac{\partial P}{\partial\rho^*}=a_0^2\,,a_0=const.$$

$$\dot{u}_z=\vartheta_z\,,\dot{u}_r=\vartheta_r\,,\,\dot{u}_\theta=\vartheta_\theta\,,$$

$$q_z=-p_{rz}\,,q_r=-p_r\,,\,q_\theta=-p_{r\theta}\,.$$

$$p_{rz}=\mu^*\left(\frac{\partial\vartheta_z}{\partial r}+\frac{\partial\vartheta_r}{\partial z}\right);$$

$$p_{rr}=-p+\lambda^*\left(\frac{\partial\vartheta_r}{\partial r}+\frac{\partial\vartheta_z}{\partial z}+\frac{\vartheta_r}{r}\right)+2\mu^*\frac{\partial\vartheta_r}{\partial r}\;; \quad (2)$$

$$p_{r\theta}=\mu^*\left(\frac{1}{r}\frac{\partial\vartheta_z}{\partial\theta}+\frac{\partial\vartheta_\theta}{\partial r}-\frac{\vartheta_\theta}{r}\right).$$

Here in the equations (2) $\vec{\vartheta}$ = $\vec{\vartheta}(\vartheta_r,\vartheta_\theta,\vartheta_z)$ - velocity vector of fluid particles, ρ^* and P - density and pressure perturbations in the fluid, ρ_0^* and a_0 – density and speed of sound in a fluid at rest; ν^*, μ^* - kinematic and dynamic viscosity coefficients; for the second viscosity coefficient λ^* accepted ratio $\lambda^* = -\frac{2}{3}\mu^*$; $p_{rz}, p_{rr}, p_{r\theta}$ - components of the stress tensor in a liquid. Equations (1) are respectively kinematic and dynamic boundary conditions, which, by virtue of the thinness of the shell, will be satisfied on the middle surface (r = R). Relations (1) and (2) represent a closed system of hydro-viscoelastic relations for a cylindrical shell containing a viscous compressible fluid. So for shells that obey the Kirchhoff-Love hypothesis, joint shells and liquids that are harmonic in the axial coordinate z and exponentially damped in time, or harmonic in time and damped in z are subject to study. Expanding equations (2) and (3) in coordinate form, it is easy to see that relations (2) - (3) break up into independent boundary value problems:

Torsional vibrations

$$\frac{\partial p_{r\theta}}{\partial r} + \frac{2 p_{r\theta}}{r} + \frac{\partial p_{\theta z}}{\partial z} = \rho_0^* \ddot{\vartheta}_\theta,$$

$$p_{r\theta} = \eta\left(\frac{\partial \vartheta_\theta}{\partial r} - \frac{\vartheta_\theta}{r}\right), \quad p_{\theta z} = \eta \frac{\partial \vartheta_\theta}{\partial r}, \tag{5}$$

$$r = R: \quad Gh\frac{\partial^2 u_\theta}{\partial z^2} - (\rho_0 h \ddot{u}_\theta \pm p_{\theta r}) = 0, \qquad G = \frac{E_0\left[1 - \Gamma^c(\omega_R) - i\Gamma^s(\omega_R)\right]}{2(1+\nu_0)}.$$

$$r = 0: \quad p_{r\theta} = 0.$$

Longitudinal-transverse oscillations

$$\frac{\partial p_{rr}}{\partial r} + \frac{p_{rr} - p_{\theta\theta}}{r} + \frac{\partial p_{rz}}{\partial z} = \rho_0^* \ddot{\vartheta}_r$$

$$\frac{\partial p_{rz}}{\partial r} + \frac{p_{rz}}{r} + \frac{\partial p_{zz}}{\partial z} = \rho_0^* \ddot{\vartheta}_z$$

$$p_{rr} = -p + k_\eta div\, \dot{\vec{\vartheta}} + 2\eta \frac{\partial \vartheta_r}{\partial r},$$

$$p_{\theta\theta} = -p + k_\eta div\, \dot{\vec{\vartheta}} + 2\eta \frac{\vartheta_r}{r} \tag{6}$$

$$p_{zz} = -p + k_\eta div\, \dot{\vec{\vartheta}} + 2\eta \frac{\partial \vartheta_z}{\partial z}$$

$$\sigma_{xs} = \eta\left(\frac{\partial U_z}{\partial z} + \frac{\partial U_s}{\partial r}\right)$$

$$\dot{\rho} + \rho_0 div\, \dot{\vec{\vartheta}} = 0, \qquad div\, \vec{\vartheta} = \frac{\partial \vartheta_r}{\partial r} + \frac{\vartheta_r}{r} + \frac{\partial \vartheta_z}{\partial z}, \qquad \frac{\partial p}{\partial \rho} = C_0^2$$

$$r = R: \quad \frac{\partial^4 u_r}{\partial z^4} + \frac{C}{R}\left(\frac{u_r}{R} + \nu_0 \frac{\partial u_z}{\partial z}\right) + p_{rr} + \rho_0 h \ddot{u}_r = 0,$$

$$C\left(\frac{\partial^2 u_r}{\partial z^2} + \frac{\nu_0}{R}\frac{\partial u_r}{\partial R}\right) - (p_{rz} \pm \rho_0 h \ddot{u}_z) = 0, \quad C = \frac{E_0 h}{1-\nu_0^2}\left[1 - \Gamma^c(\omega_R) - i\Gamma^s(\omega_R)\right].$$

$$r = 0 \qquad p_{rz} = 0 \qquad u_r = 0.$$

Let the wave process be periodic in z and attenuate in time, then the real wave number k is given, and the complex frequency is the desired eigenvalue. The solutions of boundary value problems (2) - (6) for the main unknowns satisfying the constraints imposed above do not depend on time and the z coordinate should be sought in the form [14]

$$p_{rr} = \sum_{m=1}^{\infty} \sigma_{rm} \cos(m\varphi) e^{ikz - i\omega t},$$
$$p_{rz} = \sum_{m=1}^{\infty} \tau_{zm} \cos(m\varphi) e^{ikz - i\omega t},$$
$$p_{r\theta} = \sum_{m=1}^{\infty} \tau_{\varphi m} \sin(m\varphi) e^{ikz - i\omega t}, \qquad (7)$$
$$\vec{u} = \sum_{m=1}^{\infty} \vec{u}_m (U_m \cos(m\varphi), V_m \sin(m\varphi), W_m \cos(m\varphi)) e^{ikz - i\omega t},$$
$$\vec{\vartheta} = \sum_{m=1}^{\infty} \vec{\vartheta}_m (\vartheta_{rm} \cos(m\varphi), \vartheta_{\theta m} \sin(m\varphi), \vartheta_{zm} \cos(m\varphi)) e^{ikz - i\omega t}$$

where $\sigma_r, \tau_z, \tau_\varphi, U_m, V_m, W_m, \vartheta_r, \vartheta_\theta, \vartheta_z$ - amplitude complex vector - function; k is the wave number; C is the phase velocity; ω is the complex frequency; m is the circumferential wave number (the number of circumferential waves), taking the values m = 1, 2, 3 ... In the case of m = 0, axisymmetric oscillations. This approach will allow us to search for a solution to the problem for each fixed value of the circumferential wave number m independently. After performing in (5) the change of variables (7), the resolving relations describing the stationary torsional vibrations of the shell-liquid system are formulated as a spectral boundary problem for a system of two ordinary differential equations

$$\frac{d\tau_\varphi}{dr} = -(\rho_0 \omega^2 - i\eta^2 \kappa^2 \omega)\vartheta_\theta - \frac{2\tau_\varphi}{r}$$
$$\frac{d\vartheta_\theta}{dr} = \frac{\vartheta_\theta}{r} + \frac{i}{\omega\eta}\tau_\varphi \qquad (9)$$
$$r = R_1 : h(Gk^2 - k\rho\omega^2)V \pm \tau_\varphi = 0$$
$$r = 0 : \tau_\varphi = 0$$

We first investigate the oscillations of the fluid in the walls. Equations (9) can be converted to a single equation for displacement *v*

$$\frac{d^2\vartheta_\theta}{dr^2} + \frac{d\vartheta_\theta}{rdr} + (-k^2 + i\frac{\omega}{v^*} - \frac{1}{r^2})\vartheta_\theta = 0; \quad v^* = \frac{\eta}{\rho_0^*} \qquad (10)$$

The solution of equation (10) is

$$\vartheta_\theta = A_1 J_1 (r\sqrt{-k^2 + i\frac{\omega}{v^*}}) + B_1 J_1 (r\sqrt{-k^2 + i\frac{\omega}{v^*}}),$$

And limited at r = 0

$$\vartheta_\theta = A_1 J_1 (r\sqrt{-k^2 + i\frac{\omega}{v^*}}) = 0 \quad . \qquad (11)$$

Where $J_1$and N_1 Bessel and Neumann functions, respectively, of the first kind and first order; A_1 and B_1- arbitrary constants. Given the immobility of the shell, to obtain the dispersion equation

$$J_1(R_1\sqrt{-k^2+i\frac{\omega}{v^*}})=0 \qquad (12)$$

from where

$$\omega_n=-i(v^*k^2+\beta_m^2) \qquad (13)$$

Here through β_n marked by the roots of the Bessel function, referred to R. As can be seen from formulas (12), (13), the proper motions are always a periodic in time, and the nodal points are fixed (phase velocity C_o=0), at that time, the steady-state motion is oscillatory, and the nodal points move with the speed C_y, monotonically increasing from zero to infinity with increasing or viscosity v^* . These characteristic features of viscous medium motion will manifest themselves in the following more complex examples. We now consider relations (9) for the case of the internal location of the fluid. This problem can be solved in the same way, using special functions and we have the dispersion equation

$$-k^2+\frac{\omega^2}{a^2}+\frac{\omega v^*}{a^3\tilde{p}\tilde{h}R^2}+(z\frac{J_0(z)}{J_1(z)}-2)=0 \qquad (14)$$

Which was first obtained in the work of A. Guzya [7]. New notation introduced here.

$$\tilde{p}=\frac{\rho^*}{\rho_0};\tilde{h}=\frac{h}{R_1};z=R_1\sqrt{-k^2+i\frac{\omega}{v^*}};\ a=\sqrt{\frac{G}{\rho_0}}$$

Shell shear wave velocity: J_o-zero order Bessel function.

The direct solution of equation (14) faces certain difficulties caused by the necessity of calculating the Bessel function of the complex argument. Therefore, we study (14) using asymptotic representations of these functions for small and large arguments z. The smallness of z occurs at low-frequency oscillations. According to known expansions J_0 and J_1 power series

$$J_0=1-\frac{z^2}{4}=...;J_1(z)=\frac{z}{2}(1-\frac{z^2}{8}+...); \qquad (15)$$

Holding in the expansions (15) only the first terms, we get $-k^2+\frac{\omega}{a^2}=0$

Dispersion equation of torsional vibrations of a dry shell or filled with an ideal fluid, keeping in (14) the first two terms in each, we have the equation

$$-k^2+\frac{\omega^2}{a^2}+i\frac{\omega v^*}{4a^2\tilde{p}\tilde{h}}(k^2-i\frac{\omega}{v^*})=0 \qquad (16)$$

The root of which, for example, in the case of steady-state oscillations is determined by the expression

$$k=\frac{\omega}{a}\left[(1+\frac{1}{4\tilde{p}\tilde{h}})/(1-\frac{\omega v^*}{4a^2\tilde{p}\tilde{h}})\right]^{1/2}. \qquad (17)$$

A physical interpretation of equation (14) is provided below. We now consider the situation when z is large enough, which corresponds to high-frequency oscillations and low viscosity. Substituting (1), and additionally assuming smallness ν^* compared to $\frac{\omega}{k^2}$, we obtain an approximate dispersion equation, which is also given in [7]

$$-k^2+\frac{\omega^2}{a^3}(1+\sqrt{\frac{\nu^*}{\omega}}\frac{\tilde{p}}{\tilde{h}R}\frac{l+i}{1.41})=0 \qquad (18)$$

Hence, when aspiring viscosity ν^* to zero (as well as ω to infinity), we have a trivial result $\frac{\omega}{k}\to 0$, which was obtained at small ω from equation (16). Equation (18) is unacceptable at high viscosities. In this case, the phase velocity C increases unlimitedly with increasing ω. The considered example testifies to the inconsistency of various asymptotic estimates in the region of average oscillation frequencies. Thus, when analyzing wave processes by asymptotic methods in the first approximation, it is not possible to establish the limits of applicability of the formulas obtained, as well as to estimate the error of calculations. In this paper, for solving spectral problems, we use direct numerical integration of resolving relations of the type (9) using the orthogonal sweep method in complex arithmetic.

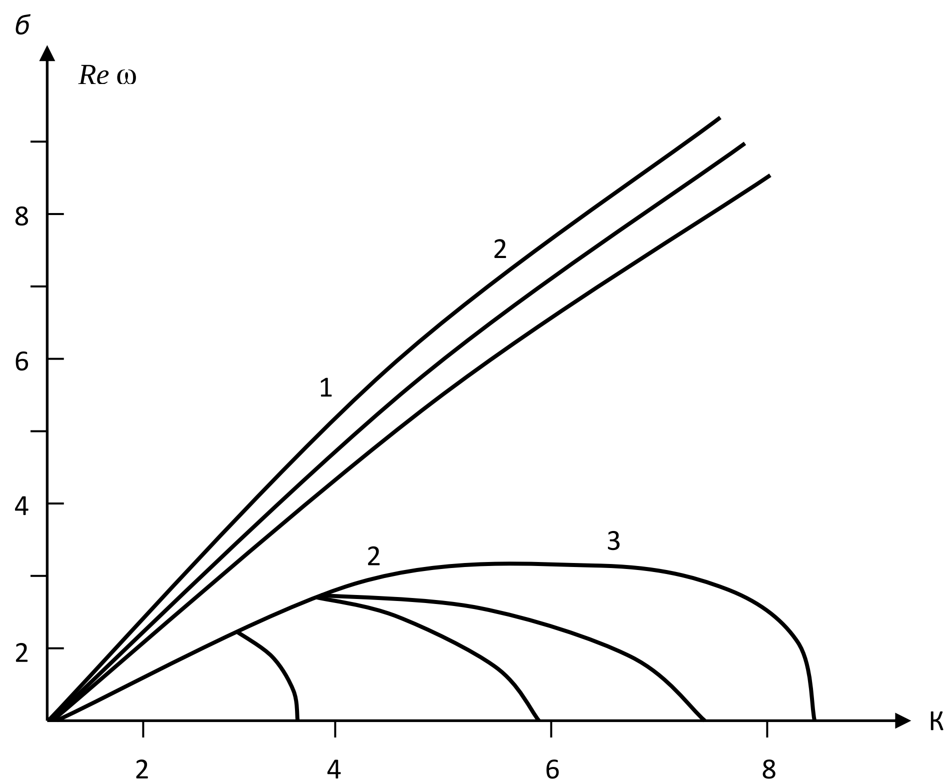

Figure 1 Dependency Re ω from wave number k

Such an approach makes it possible to avoid the above difficulties associated with the calculation of the Bessel functions of a complex argument. Another advantage is due to the specifics of the orthogonal sweep method, which, thanks to the ortho-normalization procedure, allows one to solve strongly rigid systems with a boundary layer. As a result of the conducted numerical study, it was found that the problem of natural oscillations (9) admits no more than one complex value ω corresponding to the oscillation of the shell together with the adjacent fluid layers. The remaining Eigen values found were purely imaginary. They correspond to periodic movements of a fluid with an almost motionless shell.

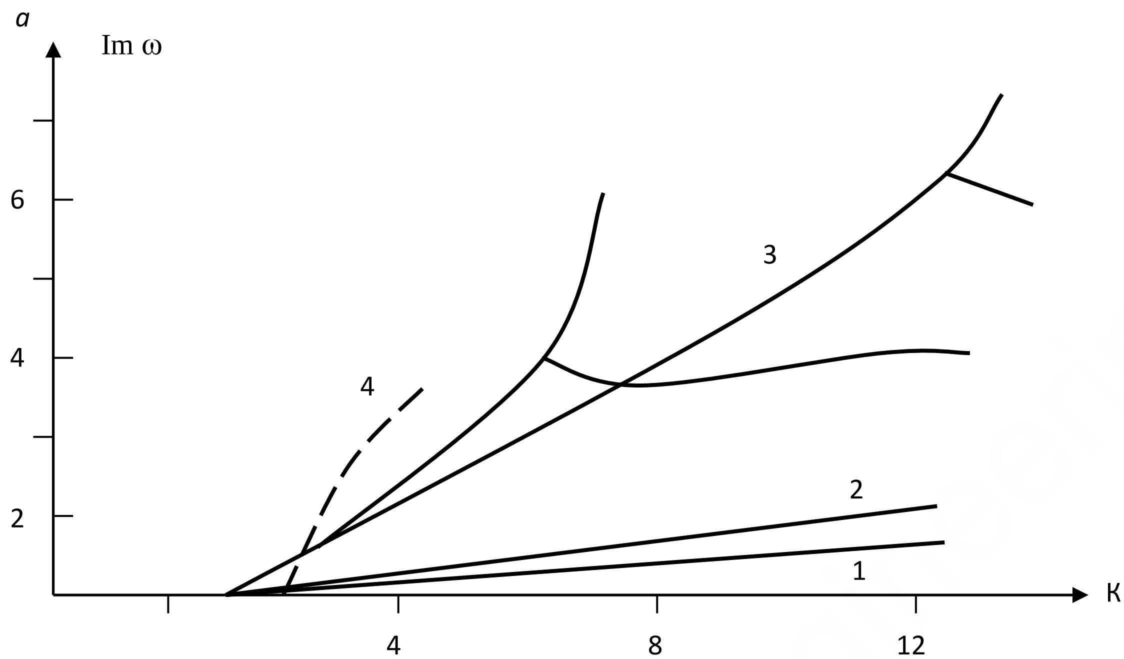

Figure 2 Addiction Im ω from wave number k

3. NUMERICAL RESULTS

Consider the option of natural oscillations, when the shell is filled with liquid. Fig. 1 and 2 shows, respectively, the dispersion curves in the dependence of Re ω, Im ω on the wave number of the k-first mode, whose damping coefficients are the smallest, and their Eigen values can be complex. In accordance with the numbering of the graphs, four different values of the coefficient η were set: 1) 0.0009: 2) 0.0018 3) 0.15 4) 0.018, with the remaining parameters according to (1). It follows that there is a minimum critical value of the viscosity coefficient η_k, above which, in the zone of high wave numbers of the first mode, appear as an a periodic wave number. As a result of a numerical experiment, it was found that the critical values of the viscosity coefficient η_k are in the interval $[0.0120 \quad 0.0125]$.

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