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On sg-Separation Axioms

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ABSTRACT

In this paper we define almost sg-normality and mild sg-normality, continue the study of further properties of sg-normality. We show that these three axioms are regular open hereditary. Also define the class of almost sg-irresolute mappings and show that sg-normality is invariant under almost sg-irresolute M-sg-open continuous surjection.

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Key words and Phrases: sg-open, almost normal, midly normal, M-sg-closed, M-sg-open, rc-continuous.

1. INTRODUCTION

In 1967, A. Wilansky has introduced the concept of US spaces. In 1968, C.E. Aull studied some separation axioms between the T_1 and T_2 spaces, namely, S_1 and S_2. Next, in 1982, S.P. Arya et al have introduced and studied the concept of semi-US spaces and also they made study of s-convergence, sequentially semi-closed sets, sequentially s-compact notions. G.B. Navlagi studied P-Normal Almost-P-Normal, Mildly-P-Normal and Pre-US spaces. Recently S. Balasubramanian and P.Aruna Swathi Vyjayanthi studied *v*-Normal Almost- *v*-Normal, Mildly-*v*-Normal and *v*-US spaces. Inspired with these we introduce sg-Normal Almost- sg-Normal, Mildly- sg-Normal, sg-US, sg-S_1 and sg-S_2. Also we examine sg-convergence, sequentially sg-compact, sequentially sg-continuous maps, and sequentially sub sg-continuous maps in the context of these new concepts. All notions and symbols which are not defined in this paper may be found in the appropriate references. Throughout the paper X and Y denote Topological spaces on which no separation axioms are assumed explicitly stated.

2. PRELIMINARIES

2.1. Definition 2.1

$A\subset X$ is called (i) g-closed if cl $A\subseteq U$ whenever $A\subseteq U$ and U is open in X.
(ii) sg-closed if *s*cl(A) $\subseteq$ U whenever $A\subseteq U$ and U is semiopen in X.

2.2. Definition 2.2

A space X is said to be
(i) T_1 (T_2) if for any $x \neq y$ in X, there exist (disjoint) open sets U; V in X such that $x\in U$ and $y\in V$.
(ii) Weakly Hausdorff if each point of X is the intersection of regular closed sets of X.
(iii) Normal [resp: mildly normal] if for any pair of disjoint [resp: regular-closed]closed sets F_1 and F_2 , there exist disjoint open sets U and V such that $F_1 \subset U$ and $F_2 \subset V$.
(iv) Almost normal if for each closed set A and each regular closed set B such that $A\cap B = \phi$, there exist disjoint open sets U and V such that $A\subset U$ and $B\subset V$.
(v) Weakly regular if for each pair consisting of a regular closed set A and a point x such that $A \cap \{x\} = \phi$, there exist disjoint open sets U and V such that $x \in U$ and $A\subset V$.
(vi) A subset A of a space X is S-closed relative to X if every cover of A by semiopen sets of X has a finite subfamily whose closures cover A.
(vii) R_0 if for any point x and a closed set F with $x\notin F$ in X, there exists a open set G containing F but not x.
(viii) R_1 iff for x, $y \in X$ with $cl\{x\} \neq cl\{y\}$, there exist disjoint open sets U and V such that $cl\{x\}\subset U$, $cl\{y\}\subset V$.
(ix) US-space if every convergent sequence has exactly one limit point to which it converges.
(x) pre-US space if every pre-convergent sequence has exactly one limit point to which it converges.
(xi) pre-S_1 if it is pre-US and every sequence $<x_n>$ pre-converges with subsequence of $<x_n>$ pre-side points.
(xii) pre-S_2 if it is pre-US and every sequence $<x_n>$ in X pre-converges which has no pre-side point.
(xiii) is weakly countable compact if every infinite subset of X has a limit point in X.
(xiv) Baire space if for any countable collection of closed sets with empty interior in X, their union also has empty interior in X.

2.3. Definition 2.3

Let $A\dot{C}$ *X*. Then a point x is said to be a
(i) limit point of *A* if each open set containing x contains some point y of *A* such that $x \neq y$.
(ii) T_0–limit point of *A* if each open set containing x contains some point y of *A* such that $cl\{x\} \neq cl\{y\}$, or equivalently, such that they are topologically distinct.

(iii) *pre*-T_0–limit point of *A* if each open set containing x contains some point y of *A* such that pcl{x} ≠ pcl{y}, or equivalently, such that they are topologically distinct.

Note 1: Recall that two points are topologically distinguishable or distinct if there exists an open set containing one of the points but not the other; equivalently if they have disjoint closures. In fact, the T_0–axiom is precisely to ensure that any two distinct points are topologically distinct.

Example 1: Let X = {a, b, c, d} and τ = {{a}, {b, c}, {a, b, c}, X, ϕ}. Then b and c are the limit points but not the T_0–limit points of the set {b, c}. Further d is a T_0–limit point of {b, c}.

Example 2: Let X = (0, 1) and τ = {ϕ, X, and U_n = (0, 1–1⁄n), n = 2, 3, 4,. . . }. Then every point of X is a limit point of X. Every point of X~U_2 is a T_0–limit point of X, but no point of U_2 is a T_0–limit point of X.

2.4. Definition 2.4

A set *A* together with all its T_0–limit points will be denoted by T_0–cl*A*.

Note 2: i. Every T_0–limit point of a set *A* is a limit point of the set but the converse is not true in general.
ii. In T_0–space both are same.

Note 3: R_0–axiom is weaker than T_1–axiom. It is independent of the T_0–axiom. However $T_1 = R_0+T_0$

Note 4: Every countable compact space is weakly countable compact but converse is not true in general. However, a T_1–space is weakly countable compact iff it is countable compact.

3. sg-T_0 LIMIT POINT

3.1. Definition 3.01

In X, a point x is said to be a sg-T_0–limit point of *A* if each sg-open set containing x contains some point y of *A* such that *sgcl*{x} ≠ *sgcl*{y}, or equivalently; such that they are topologically distinct with respect to sg-open sets.

Note 5:regular open set ⇒ open set ⇒ semi-open set ⇒ sg-open set we have
r-T_0–limit point ⇒ T_0–limit point ⇒ s-T_0–limit point ⇒ sg-T_0–limit point

Example 3: Let X = {a, b, c} and τ = {ϕ, {b}, {a, b}, {b, c}, X}. For A = {a, b}, a is *sg-T_0*–limit point.

3.2. Definition 3.02

A set A together with all its sg-T_0–limit points is denoted by T_0-*sgcl*(A)

3.3. Lemma 3.01

If x is a sg-T_0–limit point of a set A then x is sg-limit point of A.

3.4. Lemma 3.02

(i) If X is sg-T_0–space then every sg-T_0–limit point and every sg-limit point are equivalent.
(ii) If X is r-T_0–space then every sg-T_0–limit point and every sg-limit point are equivalent.

3.5. Theorem 3.03

For x ≠ y ∈X,

(i) x is a sg-T_0–limit point of {y} iff x∉sgcl{y} and y∈sgcl{x}.
(ii) x is not a sg-T_0–limit point of {y} iff either x∈sgcl{y}or sgcl{x} = sgcl{y}.
(iii) x is not a sg-T_0–limit point of {y} iff either x∈sgcl{y}or y∈sgcl{x}.

3.6. Corollary 3.04

(i) If x is a sg-T_0–limit point of {y}, then y cannot be a sg-limit point of {x}.
(ii) If sgcl{x} = sgcl{y}, then neither x is a sg-T_0–limit point of {y} nor y is a sg-T_0–limit point of {x}.
(iii) If a singleton set A has no sg-T_0–limit point in X, then sgclA = sgcl{x} for all x∈ sgcl{A}.

3.7. Lemma 3.05

In X, if x is a sg-limit point of a set A, then in each of the following cases x becomes sg-T_0–limit point of A ({x} ≠ A).

(i) sgcl{x} ≠ sgcl{y} for y∈A, x ≠ y.
(ii) sgcl{x} = {x}
(iii) X is a sg-T_0–space.
(iv) A~{x} is sg-open

4. sg-T_0 AND sg-R_i AXIOMS, i = 0,1

In view of Lemma 3.6(iii), *sg*-T_0–axiom implies the equivalence of the concept of limit point of a set with that of *sg*-T_0–limit point of the set. But for the converse, if x∈ *sgcl*{y} then *sgcl*{x} ≠ *sgcl*{y} in general, but if x is a *sg*-T_0–limit point of {y}, then *sgcl*{x} = *sgcl*{y}.

4.1. Lemma 4.01

In a space X, a limit point x of {y} is a sg-T_0–limit point of {y} iff sgcl{x} ≠ sgcl{y}.
This lemma leads to characterize the equivalence of *sg*-T_0–limit point and *sg*-limit point of a set as *sg*-T_0–axiom.

4.2. Theorem 4.02

The following conditions are equivalent:

(i) X is a sg-T_0 space
(ii) Every sg-limit point of a set A is a sg-T_0–limit point of A
(iii) Every r-limit point of a singleton set {x} is a sg-T_0–limit point of {x}
(iv) For any x, y in X, x ≠ y if x∈ sgcl{y}, then x is a sg-T_0–limit point of {y}

Note 6: In a *sg*-T_0–space X if every point of X is a r-limit point of X, then every point of X is *sg*-T_0–limit point of X. But a space X in which each point is a *sg*-T_0–limit point of X is not necessarily a *sg*-T_0–space

4.3. Theorem 4.03

The following conditions are equivalent:

(i) X is a sg-R_0 space
(ii) For any x, y in X, if x∈ sgcl{y}, then x is not a sg-T_0–limit point of {y}
(iii) A point sg-closure set has no sg-T_0–limit point in X

(iv) A singleton set has no sg-T_0–limit point in X.

4.4. Theorem 4.04

In a sg-R_0 space X, a point x is sg-T_0–limit point of A iff every sg-open set containing x contains infinitely many points of A with each of which x is topologically distinct

4.5. Theorem 4.05

X is sg-R_0 space iff a set A of the form $A = \cup\, sgcl\{x_{i\, i=1\ to\ n}\}$ a finite union of point closure sets has no sg-T_0–limit point.
If *sg*-R_0 space is replaced by rR_0 space in the above theorem, we have the following corollaries:

4.6. Corollary 4.06

The following conditions are equivalent:
(i) X is a r-R_0 space
(ii) For any x, y in X, if $x \in sgcl\{y\}$, then x is not a sg-T_0–limit point of {y}
(iii) A point sg-closure set has no sg-T_0–limit point in X
(iv) A singleton set has no sg-T_0–limit point in X.

4.7. Corollary 4.07

In an rR_0–space X,
(i) If a point x is rT_0–limit point of a set then every sg-open set containing x contains infinitely many points of A with each of which x is topologically distinct.
(ii) If a point x is sg-T_0–limit point of a set then every sg-open set containing x contains infinitely many points of A with each of which x is topologically distinct.
(iii) If $A = \cup\, sgcl\{x_{i,\, i=1\ to\ n}\}$ a finite union of point closure sets has no sg-T_0–limit point.
(iv) If $X = \cup\, sgcl\{x_{i,\, i=1\ to\ n}\}$ then X has no sg-T_0–limit point.
Various characteristic properties of *sg*-T_0–limit points studied so far is enlisted in the following theorem.

4.8. Theorem 4.08

In a sg-R_0–space, we have the following:
(i) A singleton set has no sg-T_0–limit point in X.
(ii) A finite set has no sg-T_0–limit point in X.
(iii) A point sg-closure has no set sg-T_0–limit point in X
(iv) A finite union point sg-closure sets have no set sg-T_0–limit point in X.
(v) For x, $y \in X$, $x \in T_0$– sgcl{y} iff x = y.
(vi) For any x, $y \in X$, $x \neq y$ iff neither x is sg-T_0–limit point of {y}nor y is sg-T_0–limit point of {x}
(vii) For any x, $y \in X$, $x \neq y$ iff T_0– sgcl{x} $\cap T_0$– sgcl{y} = ϕ.
(viii) Any point $x \in X$ is a sg-T_0–limit point of a set A in X iff every sg-open set containing x contains infinitely many points of A with each which x is topologically distinct.

4.9. Theorem 4.09

X is sg-R_1 iff for any sg-open set U in X and points x, y such that $x \in X \sim U$, $y \in U$, there exists a sg-open set V in X such that $y \in V \subset U$, $x \notin V$.

4.10. Lemma 4.10

In sg-R_1 space X, if x is a sg-T_0–limit point of X, then for any non empty sg-open set U, there exists a non empty sg-open set V such that $V \subset U$, $x \notin$ sgcl(V).

4.11. Lemma 4.11

In a sg- regular space X, if x is a sg-T_0–limit point of X, then for any non empty sg-open set U, there exists a non empty sg-open set V such that sgcl(V)$\subset$U, $x \notin$ sgcl(V).

4.12. Corollary 4.12

In a regular space X,
(i) If x is a sg-T_0–limit point of X, then for any non empty sg-open set U, there exists a non empty sg-open set V such that sgcl(V)$\subset$U, $x \notin$ sgcl(V).
(ii) If x is a T_0–limit point of X, then for any non empty sg-open set U, there exists a non empty sg-open set V such that sgcl(V)$\subset$U, $x \notin$ sgcl(V).

4.13. Theorem 4.13

If X is a sg-compact sg-R_1-space, then X is a Baire Space.

Proof: Let $\{A_n\}$ be a countable collection of *sg*-closed sets of X, each A_n having empty interior in X. Take A_1, since A_1 has empty interior, A_1 does not contain any *sg*-open set say U_0. Therefore we can choose a point $y \in U_0$ such that $y \notin A_1$. For X is *sg*-regular, and $y \in (X \sim A_1) \cap U_0$, a *sg*-open set, we can find a *sg*-open set U_1 in X such that $y \in U_1$, *sgcl*$(U_1) \subset (X \sim A_1) \cap U_0$. Hence U_1 is a non empty *sg*-open set in X such that *sgcl*$(U_1) \subset U_0$ and *sg*cl$(U_1) \cap A_1 = \phi$. Continuing this process, in general, for given non empty *sg*-open set U_{n-1}, we can choose a point of U_{n-1} which is not in the *sg*-closed set A_n and a *sg*-open set U_n containing this point such that *sgcl*$(U_n) \subset U_{n-1}$ and *sgcl*$(U_n) \cap A_n = \phi$. Thus we get a sequence of nested non empty *sg*-closed sets which satisfies the finite intersection property. Therefore $\cap$ *sgcl*$(U_n) \neq \phi$. Then some $x \in \cap$ *sgcl*(U_n) which in turn implies that $x \in U_{n-1}$ as *sgcl*$(U_n) \subset U_{n-1}$ and $x \notin A_n$ for each n.

4.14. Corollary 4.14

If X is a compact sg-R_1-space, then X is a Baire Space.

4.15. Corollary 4.15

Let X be a sg-compact sg-R_1-space. If $\{A_n\}$ is a countable collection of sg-closed sets in X, each A_n having non-empty sg-interior in X, then there is a point of X which is not in any of the A_n.

4.16. Corollary 4.16

Let X be a sg-compact R_1-space. If $\{A_n\}$ is a countable collection of sg-closed sets in X, each A_n having non-empty sg- interior in X, then there is a point of X which is not in any of the A_n.

4.17. Theorem 4.17

Let X be a non empty compact sg-R_1-space. If every point of X is a sg-T_0–limit point of X then X is uncountable.

Proof: Since X is non empty and every point is a *sg*-T_0-limit point of X, X must be infinite. If X is countable, we construct a sequence of *sg*- open sets $\{V_n\}$ in X as follows:

Let X = V_1, then for x_1 is a *sg*-T_0-limit point of X, we can choose a non empty *sg*-open set V_2 in X such that $V_2 \subset V_1$ and $x_1 \notin$ *sgcl*V_2. Next for x_2 and non empty *sg*-open set V_2, we can choose a non empty *sg*-open set V_3 in X such that $V_3 \subset V_2$ and $x_2 \notin$ *sgcl*V_3. Continuing this process for each x_n and a non empty *sg*-open set V_n, we can choose a non empty *sg*-open set V_{n+1} in X such that $V_{n+1} \subset V_n$ and $x_n \notin$ *sgcl*V_{n+1}.

Now consider the nested sequence of *sg*-closed sets *sgcl*$V_1 \supset$ *sgcl*$V_2 \supset$ *sgcl*$V_3 \supset$………$\supset$ *sgcl*$V_n \supset$. . . Since X is *sg*-compact and $\{$*sgcl*$V_n\}$ the sequence of *sg*-closed sets satisfies finite intersection property. By Cantors intersection theorem, there exists an x in X such that $x \in$ *sgcl*V_n. Further $x \in X$ and $x \in V_1$, which is not equal to any of the points of X. Hence X is uncountable.

4.18. Corollary 4.18

Let X be a non empty sg-compact sg-R_1-space. If every point of X is a sg-T_0–limit point of X then X is uncountable

5. *sg*–T_0-IDENTIFICATION SPACES AND *sg*–SEPARATION AXIOMS

5.1. Definition 5.01

Let *(X, ^{2})* be a topological space and let $\Re$ be the equivalence relation on X defined by $x \Re y$ iff *sgcl*{x} = *sgcl*{y}

5.2. Problem 5.02

Show that $x \Re y$ iff *sgcl*{x} = *sgcl*{y} is an equivalence relation

5.3. Definition 5.03

The space (X_0, Q(X_0)) is called the sg-T_0–identification space of *(X, ^{2})*, where X_0 is the set of equivalence classes of $\Re$ and Q(X_0) is the decomposition topology on X_0.

Let P_X: *(X, ^{2})* ì *(X_0, Q(X_0))* denote the natural map

5.4. Lemma 5.04

If $x \in X$ and $A \subset X$, then $x \in$ sgclA iff every sg-open set containing x intersects A.

5.5. Theorem 5.05

The natural map P_X:(X,τ)→ (X_0, Q(X_0)) is closed, open and $P_X^{-1}(P_X(O))$ = O for all O∈PO(X,τ) and (X_0, Q(X_0)) is sg-T_0

Proof: Let O∈ *PO(X, ^{2})* and let C∈ P_X(O). Then there exists x∈O such that P_X(x) = C. If y∈C, then *sgcl*{y} = *sgcl*{x}, which, by lemma, implies y∈O. Since ^{2} Ċ P*O(X, ^{2}),* then $P_X^{-1}(P_X(U))$ = U for all U∈^{2}, which implies P_X is closed and open.

Let G, H∈*X_0* such that G ≠ H; let x∈G and y∈H. Then *sgcl*{x} ≠ *sgcl*{y}, which implies x∉*sgcl*{y} or y∉*sgcl*{x}, say x∉*sgcl*{y}. Since P_X is continuous and open, then G∈A = P_X\{X~*sgcl*{y}\}∉PO*(X_0, Q(X_0))* and H∉A

5.6. Theorem 5.06

The following are equivalent:

(i) X is sgR_0 (ii) X_0 = {sgcl{x}: x∈X} and (iii) (X_0, Q(X_0)) is sgT_1

Proof: (i) ⇒ (ii) Let x∈C∈X_0. If y∈C, then y∈sgcl{y} = sgcl{x}, which implies C∈sgcl{x}. If y∈sgcl{x}, then x∈sgcl{y}, since, otherwise, x∈X~sgcl{y}∈PO(X,τ) which implies sgcl{x}⊂X~sgcl{y}, which is a contradiction. Thus, if y∈sgcl{x}, then x∈sgcl{y}, which implies sgcl{y} = sgcl{x} and y∈C. Hence X_0 = {sgcl{x}: x∈X}

(ii)⇒(iii) Let A ≠ B∈*X_0*. Then there exists x, y∈X such that A = *sgcl*{x}; B = *sgcl*{y}, and *sgcl*{x}∩*sgcl*{y} = ϕ. Then A∈C = P_X (X~*sgcl*{y})∈PO*(X_0, Q(X_0))* and B∉C. Thus *(X_0, Q(X_0))* is sg-T_1

(iii) ⇒ (i) Let x∈U∈*SGO(X)*. Let y∉U and C_x, C_y ∈*X_0* containing x and y respectively. Then x∉ *sgcl*{y}, which implies $C_x \neq C_y$ and there exists sg-open set A such that C_x∈A and C_y∉A. Since P_X is continuous and open, then y∈B = P_X^{-1}(A)∈ x∈*SGO(X)* and x∉B, which implies y∉*sgcl*{x}. Thus *sgcl*{x}Ċ *U.* This is true for all *sgcl*{x} implies ∩*sgcl*{x}Ċ *U.* Hence *X* is sg-R_0

5.7. Theorem 5.07

(X, τ) is sg-R_1 iff (X_0, Q(X_0)) is sg-T_2

The proof is straight forward using theorems 5.05 and 5.06 and is omitted

5.8. Theorem 5.08

X is sg-T_i; i = 0,1,2. iff there exists a sg-continuous, almost–open, 1–1 function from (X, τ) into a sg-T_i space ; i = 0,1,2. respectively.

5.9. Theorem 5.09

If f:(X,τ)→ (Y, σ) is sg-continuous, sg-open, and x, y∈X such that sgcl{x} = sgcl{y}, then sgcl{f(x)} = sgcl{f(y)}.

5.10. Theorem 5.10

The following are equivalent

(i) (X, τ) is sg-T_0

(ii) Elements of X_0 are singleton sets and

(iii)There exists a sg-continuous, sg-open, 1–1 function f: (X, τ)→ (Y, σ), where (Y, σ) is sg-T_0

Proof: (i) is equivalent to (ii) and (i) ⇒ (iii) are straight forward and is omitted.

(iii) ⇒ (i) Let x, y∈X such that f(x) ≠ f(y), which implies *sgcl*{f(x)} ≠ *sgcl*{f(y)}. Then by theorem 5.09, *sgcl*{x} ≠ *sgcl*{y}. Hence *(X, ^{2})* is sg-T_0

5.11. Corollary 5.11

A space (X, τ) is sg-T_i ; i = 1,2 iff (X, τ) is sg-T_{i-1} ; i = 1,2, respectively, and there exists a sg-continuous , sg-open, 1–1 function f: (X, τ) into a sg-T_0 space.

5.12. Definition 5.04

f is point–sg-closure 1–1 iff for x, y∈X such that *sgcl*{x} ≠ *sgcl*{y}, *sgcl*{f(x)} ≠ *sgcl*{f(y)}.

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5.13. Theorem 5.12

(i)If f: (X, τ)→ (Y, σ) is point– sg-closure 1–1 and (X, τ) is sg-T_0 , then f is 1–1
(ii)If f: (X, τ)→ (Y, σ), where (X, τ)and (Y, σ) are sg-T_0 then f is point– sg-closure 1–1 iff f is 1–1
The following result can be obtained by combining results for sg-T_0– identification spaces, sg-induced functions and sg-T_i spaces; i = 1,2.

5.14. Theorem 5.13

X is sg-R_i ; i = 0,1 iff there exists a sg-continuous , almost–open point– sg-closure 1–1 function f: (X, τ) into a sg-R_i space; i = 0,1 respectively.

6. sg-Normal; Almost sg-normal and Mildly sg-normal spaces

6.1. Definition 6.1

A space X is said to be sg-normal if for any pair of disjoint closed sets F_1 and F_2 , there exist disjoint sg-open sets U and V such that $F_1 \subset U$ and $F_2 \subset V$.
Example 4: Let X = {a, b, c} and τ = {ϕ, {a}, {b, c}, X}. Then X is sg-normal.
Example 5: Let X = {a, b, c, d} with τ = {ϕ, {a}, {b}, {d}, {a, b}, {a, d}, {b, d}, {a, b, c}, {a, b, d}, X} is *sg*-normal, normal and almost normal.
We have the following characterization of sg-normality.

6.2. Theorem 6.1

For a space X the following are equivalent:
(i) *X* is sg-normal.
(ii) For every pair of open sets *U* and *V* whose union is *X*, there exist sg-closed sets *A* and *B* such that *AĊU, BĊV* and *AĆB = X*.
(iii) For every closed set *F* and every open set *G* containing *F*, there exists a sg-open set *U* such that *FĊUĊsgcl(U)ĊG*.
Proof: (i)⇒(ii): Let *U* and *V* be a pair of open sets in a sg-normal space *X* such that *X = UĆV*. Then *X–U, X–V* are disjoint closed sets. Since X is sg-normal there exist disjoint sg-open sets *U_1* and *V_1* such that *X–UĊU_1* and *X-VĊV_1*. Let *A = X–U_1, B = X–V_1*. Then *A* and *B* are sg-closed sets such that *AĊU, BĊV* and *AĆB = X*.
(ii) ⇒(iii): Let *F* be a closed set and *G* be an open set containing *F*. Then *X–F* and *G* are open sets whose union is *X*. Then by (b), there exist sg-closed sets *W_1* and *W_2* such that *W_1Ċ X–F* and *W_2Ċ G* and *W_1ĆW_2 = X*. Then *FĊ X–W_1, X–G Ċ X–W_2* and *(X–W_1)ą(X–W_2) = ¤*. Let *U = X–W_1* and *V= X–W_2*. Then *U* and *V* are disjoint sg-open sets such that *FĊUĊX–VĊG*. As *X–V* is sg-closed set, we have *sgcl(U) ĊX–V* and *FĊUĊsgcl(U)ĊG*.
(iii) ⇒ (i): Let *F_1* and *F_2* be any two disjoint closed sets of *X*. Put *G = X–F_2*, then *F_1ąG = ¤*. *F_1ĊG* where *G* is an open set. Then by (c), there exists a sg-open set *U* of *X* such that *F_1 Ċ U Ċ sgcl(U) ĊG*. It follows that *F_2 Ċ X–sgcl(U) = V*, say, then *V* is sg-open and *UąV = ¤*. Hence *F_1* and *F_2* are separated by sg-open sets *U* and *V*. Therefore *X* is sg-normal.

6.3. Theorem 6.2

A regular open subspace of a sg-normal space is sg-normal.
Example 6: Let X = {a, b, c, d} with τ = {ϕ, {a}, {b}, {d}, {a, b}, {a, d}, {b, d}, {a, b, c}, {a, b, d}, X} is sg-normal and sg-regular.
However we observe that every sg-normal sg-R_0 space is sg-regular.

6.4. Definition 6.2

A function *f*:X→Y is said to be almost–sg-irresolute if for each x in X and each sg-neighborhood V of *f*(x), *sgcl*(f^{-1}(V)) is a sg-neighborhood of x.
Clearly every sg-irresolute map is almost sg-irresolute.
The Proof of the following lemma is straightforward and hence omitted.

6.5. Lemma 6.1

f is almost sg-irresolute iff f^{-1}(V) ⊂ sg-int(*sgcl*(f^{-1}(V))) for every V∈*SGO*(Y).

6.6. Lemma 6.2

f is almost sg-irresolute iff *f*(*sgcl*(U)) ⊂ *sgcl*(*f*(U)) for every U∈*SGO*(X).
Proof: Let U∈*SGO*(X).Suppose y∉ *sgcl*(*f*(U)). Then there exists V∈ *sg O*(y) such that V∩*f*(U) = ϕ. Hence f^{-1}(V)∩U= ϕ. Since U∈*SGO*(X), we have sg-int(*sgcl*(f^{-1}(V))) ∩ *sgcl*(U) = ϕ. By lemma 6.1, f^{-1}(V)∩ *sgcl*(U) = ϕ and hence V∩*f*(*sgcl*(U)) = ϕ. This implies that y∉*f*(*sgcl*(U)).
Conversely, if V∈*SGO*(Y), then W = X- *sgcl*(f^{-1}(V)))∈ *sgO*(X). By hypothesis, *f*(*sgcl*(W))⊂ *sgcl* (*f*(W))) and hence X- sg-int(*sgcl*(f^{-1}(V))) = *sgcl*(W)⊂f^{-1}(*sgcl*(*f*(W)))⊂*f*(*sgcl*[*f*(X-f^{-1}(V))])⊂f^{-1}[*sgcl*(Y-V)] = f^{-1}(Y-V) = X-f^{-1}(V). Therefore, f^{-1}(V)⊂ sg-int(*sgcl*(f^{-1}(V))). By lemma 6.1, *f* is almost sg-irresolute.

6.7. Theorem 6.3

If *f*:X→Y is M-sg-open continuous almost sg-irresolute, X is sg-normal, then Y is sg-normal.
Proof: Let A be a closed subset of Y and B be an open set containing A. Then by continuity of *f*, f^{-1}(A) is closed and f^{-1}(B) is an open set of X such that f^{-1}(A) ⊂ f^{-1}(B). As X is sg-normal, there exists a sg-open set U in X such that f^{-1}(A) ⊂ U ⊂ *sgcl*(U)⊂ f^{-1}(B). Then *f*(f^{-1}(A))⊂ *f*(U) ⊂ *f*(*sgcl*(U)) ⊂ *f*(f^{-1}(B)). Since *f* is M-sg-open almost sg-irresolute surjection, we obtain A⊂ *f*(U) ⊂ *sgcl*(*f*(U)) ⊂ B. Then again by Theorem 6.1 the space Y is sg-normal.

6.8. Lemma 6.3

A mapping *f* is M-sg-closed if and only if for each subset B in Y and for each sg-open set U in X containing f^{-1}(B), there exists a sg-open set V containing B such that f^{-1}(V)⊂U.

6.9. Theorem 6.4

If *f*:X→Y is M-sg-closed continuous, X is sg-normal space, then Y is sg-normal.
Proof of the theorem is routine and hence omitted.
Now in view of lemma 2.2 [9] and lemma 6.3, we prove that the following result.

6.10. Theorem 6.5

If *f* is an M-sg-closed map from a weakly Hausdorff sg-normal space X onto a space Y such that f^{-1}(y) is S-closed relative to X for each y∈Y , then Y is sg-T_2.
Proof: Let $y_1 \neq y_2$∈Y. Since X is weakly Hausdorff, $f^{-1}(y_1)$ and $f^{-1}(y_2)$ are disjoint closed subsets of X by lemma 2.2 [9]. As X is sg-normal, there exist disjoint V_i ∈SGO(X) such that $f^{-1}(y_i) \subset V_i$, for i = 1,2. Since *f* is M-sg-closed, there exist disjoint U_i∈SGO(Y, y_i) and $f^{-1}(U_i) \subset V_i$ for i = 1,2. Hence Y is sg-T_2.

6.11. Theorem 6.6

For a space *X* we have the following:

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(a) If *X* is normal then for any disjoint closed sets A and B, there exist disjoint sg-open sets U, V such that A⊂U and B⊂V;
(b) If *X* is normal then for any closed set A and any open set V containing A, there exists an sg-open set U of X such that A⊂U⊂*sgcl*(U) ⊂V.

6.12. Definition 6.2

X is said to be almost sg-normal if for each closed set A and each regular closed set B such that A∩B = ϕ, there exist disjoint sg-open sets U and V such that A⊂U and B⊂V.
Clearly, every sg-normal space is almost sg-normal, but not conversely in general.

6.13. Theorem 6.7

For a space X the following statements are equivalent:
(i) X is almost sg-normal
(ii) For every pair of sets U and V , one of which is open and the other is regular open whose union is X, there exist sg-closed sets G and H such that G⊂U ,H⊂V and G∪H = X.
(iii) For every closed set A and every regular open set B containing A, there is a sg-open set V such that A⊂V⊂*sgcl*(V)⊂B.
Proof: (i)⇒(ii) Let U be an open set and V be a regular open set in an almost sg-normal space X such that U∪V = X. Then (X-U) is closed set and (X-V) is regular closed set with (X-U)∩(X-V) = ϕ. By almost sg-normality of X, there exist disjoint sg-open sets U_1 and V_1 such that X-U ⊂ U_1 and X-V ⊂ V_1. Let G = X- U_1 and H = X-V_1. Then G and H are sg-closed sets such that G⊂U, H⊂V and G∪H = X.
(ii) ⇒ (iii) and (iii) ⇒ (i) are obvious.
One can prove that almost sg-normality is also regular open hereditary.
Almost sg-normality does not imply almost sg-regularity in general. However, we observe that every almost sg-normal sg-R_0 space is almost sg-regular.

6.14. Theorem 6.8

Every almost regular, *sg*-compact space X is almost sg-normal.
Recall that a function *f* : X→ Y is called rc-continuous if inverse image of regular closed set is regular closed.
Now, we state the invariance of almost sg-normality in the following.

6.15. Theorem 6.9

If *f* is continuous M-sg-open rc-continuous and almost sg-irresolute surjection from an almost sg-normal space X onto a space Y, then Y is almost sg-normal.

6.16. Definition 6.3

A space X is said to be mildly sg-normal if for every pair of disjoint regular closed sets F_1 and F_2 of X, there exist disjoint sg-open sets U and V such that F_1 ⊂ U and F_2 ⊂ V.
Example 7: Let X = {a, b, c, d} with τ = {ϕ, {a}, {b}, {d}, {a, b}, {a, d}, {b, d}, {a, b, c}, {a, b, d}, X} is Mildly *sg*-normal.
We have the following characterization of mild sg-normality.

6.17. Theorem 6.10

For a space X the following are equivalent.
(i) X is mildly sg-normal.
(ii) For every pair of regular open sets U and V whose union is X, there exist sg-closed sets G and H such that G ⊂ U, H ⊂ V and G∪H = X.
(iii) For any regular closed set A and every regular open set B containing A, there exists a sg-open set U such that A⊂U⊂*sgcl*(U)⊂B.
(iv) For every pair of disjoint regular closed sets, there exist sg-open sets U and V such that A⊂U, B⊂V and *sgcl*(U)∩ *sgcl*(V) = ϕ.
This theorem may be proved by using the arguments similar to those of Theorem 6.7.
Also, we observe that mild sg-normality is regular open hereditary.

6.18. Definition 6.4

A space X is weakly sg-regular if for each point x and a regular open set U containing {x}, there is a sg-open set V such that x∈V ⊂ clV ⊂ U.
Example 8: Let X = {a, b, c} and τ = {ϕ, {b},{a, b},{b, c}, X}. Then X is weakly *sg*-regular.
Example 9: Let X = {a, b, c} and τ = {ϕ, {a},{b},{a, b}, X}. Then X is not weakly *sg*-regular.

6.19. Theorem 6.11

If *f* : X → Y is an M-sg-open rc-continuous and almost sg-irresolute function from a mildly sg-normal space X onto a space Y, then Y is mildly sg-normal.
Proof: Let A be a regular closed set and B be a regular open set containing A. Then by rc-continuity of *f*, f^{-1}(A) is a regular closed set contained in the regular open set f^{-1}(B). Since X is mildly sg-normal, there exists a sg-open set V such that f^{-1}(A) ⊂V⊂ *sgcl*(V) ⊂ f^{-1}(B) by Theorem 6.10. As *f* is M-sg-open and almost sg-irresolute surjection, *f*(V)∈*SGO*(Y) and A⊂ *f*(V) ⊂ *sgcl*(*f*(V))⊂ B. Hence Y is mildly sg-normal.

6.20. Theorem 6.12

If *f*:X→Y is rc-continuous, M-sg-closed map and X is mildly sg-normal space, then Y is mildly sg-normal.

7. sg-US SPACES

7.1. Definition 7.1

A sequence <x_n> is said to be *sg*-converges to a point x of X, written as <x_n> $\to^{sg}$ x if <x_n> is eventually in every *sg*-open set containing x.
Clearly, if a sequence <x_n> *r*-converges to a point x of X, then <x_n> *sg*-converges to x.

7.2. Definition 7.2

X is said to be *sg*-US if every sequence <x_n> in X *sg*-converges to a unique point.

7.3. Definition 7.3

A set F is sequentially *sg*-closed if every sequence in F *sg*-converges to a point in F.

7.4. Definition 7.4

A subset G of a space X is said to be sequentially *sg*-compact if every sequence in G has a subsequence which *sg*-converges to a point in G.

7.5. Definition 7.5

A point y is a *sg*-cluster point of sequence <x_n> iff <x_n> is frequently in every *sg*-open set containing x. The set of all *sg*-cluster points of <x_n> will be denoted by *sg*-cl(x_n).

7.6. Definition 7.6
A point y is *sg*-side point of a sequence <x_n> if y is a *sg*-cluster point of <x_n> but no subsequence of <x_n> *sg*-converges to y.

7.7. Definition 7.7
A space X is said to be
(i) *sg*-S_1 if it is *sg*-US and every sequence <x_n> *sg*-converges with subsequence of <x_n> *sg*-side points.
(ii) *sg*-S_2 if it is *sg*-US and every sequence <x_n> in X *sg*-converges which has no *sg*-side point.
Using sequentially continuous functions, we define sequentially *sg*-continuous functions.

7.8. Definition 7.8
A function *f* is said to be sequentially *sg*-continuous at $x \in X$ if $f(x_n) \to^{sg} f(x)$ whenever <x_n> $\to^{sg}$ x. If *f* is sequentially *sg*-continuous at all $x \in X$, then *f* is said to be sequentially *sg*-continuous.

7.9. Theorem 7.1
We have the following:
(i) Every *sg*-T_2 space is *sg*-US.
(ii) Every *sg*-US space is *sg*-T_1.
(iii) X is *sg*-US iff the diagonal set is a sequentially *sg*-closed subset of X x X.
(iv) X is *sg*-T_2 iff it is both *sg*-R_1 and *sg*-US.
(v) Every regular open subset of a *sg*-US space is *sg*-US.
(vi) Product of arbitrary family of *sg*-US spaces is *sg*-US.
(vii) Every *sg*-S_2 space is *sg*-S_1 and Every *sg*-S_1 space is *sg*-US.

7.10. Theorem 7.2
In a *sg*-US space every sequentially *sg*-compact set is sequentially *sg*-closed.
Proof: Let X be *sg*-US space. Let Y be a sequentially *sg*-compact subset of X. Let <x_n> be a sequence in Y. Suppose that <x_n> *sg*-converges to a point in X-Y. Let <x_{np}> be subsequence of <x_n> that *sg*-converges to a point $y \in Y$ since Y is sequentially *sg*-compact. Also, let a subsequence <x_{np}> of <x_n> *sg*-converge to $x \in X$-Y. Since <x_{np}> is a sequence in the *sg*-US space X, x = y. Thus, Y is sequentially *sg*-closed set.

7.11. Theorem 7.3
If *f* and *g* are sequentially *sg*-continuous and Y is *sg*-US, then the set A = {x | *f*(x) = *g*(x)} is sequentially *sg*-closed.
Proof: Let Y be *sg*-US. If there is a sequence <x_n> in A *sg*-converging to $x \in X$. Since *f* and *g* are sequentially *sg*-continuous, $f(x_n) \to^{sg} f(x)$ and $g(x_n) \to^{sg} g(x)$. Hence *f*(x) = *g*(x) and $x \in A$. Therefore, A is sequentially *sg*-closed.

8. SEQUENTIALLY SUB-sg-CONTINUITY
In this section we introduce and study the concepts of sequentially sub-sg-continuity, sequentially nearly sg-continuity and sequentially sg-compact preserving functions and study their relations and the property of sg-US spaces.

8.1. Definition 8.1
A function *f* is said to be
(i) sequentially nearly sg-continuous if for each point $x \in X$ and each sequence <x_n> $\to^{sg}$ x in X, there exists a subsequence <x_{nk}> of <x_n> such that <$f(x_{nk})$>$\to^{sg} f(x)$.
(ii) sequentially sub-sg-continuous if for each point $x \in X$ and each sequence <x_n> $\to^{sg}$ x in X, there exists a subsequence <x_{nk}> of <x_n> and a point $y \in Y$ such that <$f(x_{nk})$> $\to^{sg}$ y.
(iii) sequentially sg-compact preserving if *f*(K) is sequentially sg-compact in Y for every sequentially sg-compact set K of X.

8.2. Lemma 8.1
Every function *f* is sequentially sub-sg-continuous if Y is a sequentially sg-compact.
Proof: Let <x_n> $\to^{sg}$ x in X. Since Y is sequentially sg-compact, there exists a subsequence {$f(x_{nk})$} of {$f(x_n)$} sg-converging to a point $y \in Y$. Hence *f* is sequentially sub-sg-continuous.

8.3. Theorem 8.1
Every sequentially nearly sg-continuous function is sequentially sg-compact preserving.
Proof: Assume *f* is sequentially nearly sg-continuous and K any sequentially sg-compact subset of X. Let <y_n> be any sequence in *f* (K). Then for each positive integer n, there exists a point $x_n \in K$ such that $f(x_n) = y_n$. Since <x_n> is a sequence in the sequentially sg-compact set K, there exists a subsequence <x_{nk}> of <x_n> sg-converging to a point $x \in K$. By hypothesis, *f* is sequentially nearly sg-continuous and hence there exists a subsequence <x_j> of <x_{nk}> such that $f(x_j) \to^{sg} f(x)$. Thus, there exists a subsequence <y_j> of <y_n> sg-converging to $f(x) \in f(K)$. This shows that *f*(K) is sequentially sg-compact set in Y.

8.4. Theorem 8.2
Every sequentially *s*-continuous function is sequentially sg-continuous.
Proof: Let *f* be a sequentially *s*-continuous and <x_n> $\to^{s}$ $x \in X$. Then <x_n> $\to^{s}$ x. Since *f* is sequentially *s*-continuous, $f(x_n) \to^{s} f(x)$. But we know that <x_n>$\to^{s}$ x implies <x_n> $\to^{sg}$ x and hence $f(x_n) \to^{sg} f(x)$ implies *f* is sequentially sg-continuous.

8.5. Theorem 8.3
Every sequentially sg-compact preserving function is sequentially sub-sg-continuous.
Proof: Suppose *f* is a sequentially sg-compact preserving function. Let x be any point of X and <x_n> any sequence in X sg-converging to x. We shall denote the set {x_n | n= 1,2,3, …} by A and K = A ∪ {x}. Then K is sequentially sg-compact since (x_n) $\to^{sg}$ x. By hypothesis, *f* is sequentially sg-compact preserving and hence *f*(K) is a sequentially sg-compact set of Y. Since {$f(x_n)$} is a sequence in *f*(K), there exists a subsequence {$f(x_{nk})$} of {$f(x_n)$} sg-converging to a point $y \in f(K)$. This implies that *f* is sequentially sub-sg-continuous.

8.6. Theorem 8.4

A function *f*: X→ Y is sequentially sg-compact preserving iff $f_{/K}$: K → *f*(K) is sequentially sub-sg-continuous for each sequentially sg-compact subset K of X.
Proof: Suppose *f* is a sequentially sg-compact preserving function. Then *f*(K) is sequentially sg-compact set in Y for each sequentially sg-compact set K of X. Therefore, by Lemma 8.1 above, $f_{/K}$: K→ *f*(K) is sequentially sg-continuous function.
Conversely, let K be any sequentially sg-compact set of X. Let $<y_n>$ be any sequence in *f*(K). Then for each positive integer n, there exists a point $x_n \in K$ such that $f(x_n) = y_n$. Since $<x_n>$ is a sequence in the sequentially sg-compact set K, there exists a subsequence $<x_{nk}>$ of $<x_n>$ sg-converging to a point x ∈ K. By hypothesis, $f_{/K}$: K→ *f*(K) is sequentially sub-sg-continuous and hence there exists a subsequence $<y_{nk}>$ of $<y_n>$ sg-converging to a point y∈ *f*(K).This implies that *f*(K) is sequentially sg-compact set in Y. Thus, *f* is sequentially sg-compact preserving function.
The following corollary gives a sufficient condition for a sequentially sub-sg-continuous function to be sequentially sg-compact preserving.

8.7. Corollary 8.1

If *f* is sequentially sub-sg-continuous and *f*(K) is sequentially sg-closed set in Y for each sequentially sg-compact set K of X, then *f* is sequentially sg-compact preserving function.

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