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sg-separation axioms

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ABSTRACT

In this paper we discuss new separation axioms using sg-open sets.

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1. INTRODUCTION

Norman Levine introduced generalized closed sets in 1970. After him various Authors [1-18; 20-29] studied different versions of generalized sets and related topological properties. Recently V.K. Sharma studied separation axioms for g-open. Following V.K. Sharma we are going to study further properties of sg-separation axioms.Throughout the paper a space X means a topological space (X,τ). For any subset A of X its complement, interior, closure, sg-interior, sg-closure are denoted respectively by the symbols A^c, A^o, cl(A), sg-int(A) and sg-cl(A).

1.1. Definition 1.1

$A\subset X$ is called
(i) g-closed[resp: sg-closed] if $cl(A) \subseteq U$[resp: $scl(A) \subseteq U$] whenever $A \subseteq U$ and U is open[resp: semi-open] in X.
(ii) g-open[resp: sg-open] if its complement is (i) g-closed[resp: sg-closed].
Note 1: The class of regular open sets, open sets, g-open sets and sg-open sets are denoted by RO(X), τ(X), GO(X) and SGO(X) respectively. Clearly $RO(X)\subset\tau(X)\subset GO(X)\subset SGO(X)$.
Note 2: $A\in SGO(X, x)$ means A is a semipro generalized-open neighborhood of X containing x.

1.2. Definition 1.2

$A\subset X$ is called clopen[resp: nearly-clopen; semi-clopen; g-clopen; sg-clopen] if it is both open[resp: regular-open; semi-open; g-open; sg-open] and closed[resp: regular-closed; semi-closed; g- closed; sg-closed]

1.3. Definition 1.3

A function $f: X \to Y$ is said to be
(i) Continuous [resp: nearly continuous, semi-continuous] if inverse image of open set is open[resp: regular-open, semi-open]
(ii) g-continuous [resp: sg-continuous] if inverse image of closed set is g-closed [resp: sg-closed]
(iii) irresolute [resp: nearly irresolute, sg-irresolute] if inverse image of semi-open [resp: regular-open, sg-open] set is semi-open [resp: regular-open, sg-open]
(iv) g-irresolute [resp: sg-irresolute; sg-irresolute] if inverse image of g-closed [resp: sg-closed, sg-closed] set is g-closed [resp: sg-closed; sg-closed]
(v) open [resp: nearly open, semi-open] if the image of open set is open [resp: regular-open, semi-open]
(vi) g-open [resp: sg-open] if the image of open set is g-open [resp: sg-open]
(vii)homeomorphism [resp: nearly homeomorphism, semi-homeomorphism] if *f* is bijective, continuous [resp: nearly-continuous, semi-continuous] and f^{-1} is continuous[resp: nearly-continuous, semi-continuous]
(viii)rc-homeomorphism [resp: sc-homeomorphism] if *f* is bijective r-irresolute [resp: irresolute] and f^{1} is r-irresolute [resp: irresolute]
(ix) g-homeomorphism [resp: sg-homeomorphism] if *f* is bijective g-continuous [resp: sg-continuous] and f^{-1} is g-continuous [resp: sg-continuous]
(x) gc-homeomorphism [resp: sgc-homeomorphism] if *f* is bijective g-irresolute [resp: sg-irresolute] and f^{-1} is g-irresolute[resp: sg-irresolute]

1.4. Definition 1.4

X is said to be
(i) compact [nearly compact, semi-compact, g-compact, sg-compact] if every open[regular-open, semi-open, g-open, sg-open] cover has a finite sub cover.
(ii) T_0 [rT_0, sT_0, g_0] space if for each $x \neq y\in X$ $\exists$ $U\in\tau(X)$[RO(X); SO(X); GO(X)] containing either x or y.
(iii) T_1 [rT_1, sT_1, g_1] space if for each $x \neq y\in X$ $\exists$ U, $V\in\tau(X)$[RO(X); SO(X); GO(X)] such that $x\in U-V$ and $y\in V - U$.
(iv) T_2[rT_2, sT_2, g_2] space if for each $x \neq y\in X$ $\exists$ U, $V\in\tau(X)$[RO(X); SO(X); GO(X)] such that $x\in U$; $y\in V$ and $U\cap V = \phi$.
(v) $T_{1/2}$ [$rT_{1/2}$, $pT_{1/2}$] if every g-closed[rg-closed, pg-closed] set is closed [r-closed, pre-closed]

2. SG-CONTINUITY AND PRODUCT SPACES

2.1. Theorem 2.1

Let Y and $\{X_\alpha:\alpha\in I\}$ be Topological Spaces. Let $f: Y\to \Pi X_\alpha$ be a function. If f is sg-continuous, then $\pi_\alpha\bullet f: Y\to X_\alpha$ is sg-continuous.

Proof: Suppose f is sg-continuous and $\pi_\alpha:\Pi X_\beta\to X_\alpha$ is continuous for each $\alpha\in I$, $\pi_\alpha\bullet f$ is sg-continuous.

Converse of the above theorem is not true in general.

Example 2.1: Let X = {p, q, r, s}; $\tau_X = \{\phi, \{q\}, \{p, q\}, \{q, r\}, \{p, q, r\}, X\}$, $Y_1 = Y_2 = \{a, b\}$; $\tau_{Y1} = \{\phi, \{a\}, Y_1\}$; $\tau_{Y2} = \{\phi, \{a\}, Y_2\}$; $Y = Y_1\times Y_2$ = {(a, a), (a, b), (b, a), (b, b)} and $\tau_Y = \{\phi, \{(a, a)\}, \{(a, a), (a, b)\}, \{(a, a), (b, a)\}, \{(a, a), (a, b), (b, a)\}, Y_1\times Y_2\}$. Define f by f(p) = (a, a), f(q) = (b, b), f(r) = (a, b), f(s) = (b, a). It is easy to see that $\pi_1\bullet f$ and $\pi_2\bullet f$ are sg-continuous. However {(b, b)} is closed in Y but $f^{-1}(\{(b, b)\}) = \{q\}$ is not sg-closed in X. Therefore f is not sg-continuous.

2.2. Theorem 2.2

If Y is $sT_{1/2}$ and $\{X_\alpha:\alpha\in I\}$ be Topological Spaces. Let $f: Y\to \Pi X_\alpha$ be a function, then f is sg-continuous iff $\pi_\alpha\bullet f: Y\to X_\alpha$ is sg-continuous.

2.3. Corollary 2.3

Let $f_\alpha: X_\alpha\to Y_\alpha$ be a function and let $f: \Pi X_\alpha\to \Pi Y_\alpha$ be defined by $f(x_\alpha)_{\alpha\in I} = (f_\alpha(x_\alpha))_{\alpha\in I}$. If f is sg-continuous then each f_α is sg-continuous.

2.4. Corollary 2.4

For each α, let X_α be $sT_{1/2}$ and let $f_\alpha: X_\alpha\to Y_\alpha$ be a function and let $f: \Pi X_\alpha\to \Pi Y_\alpha$ be defined by $f(x_\alpha)_{\alpha\in I} = (f_\alpha(x_\alpha))_{\alpha\in I}$, then f is sg-continuous iff each f_α is sg-continuous.

3. SG_I SPACES i = 0, 1, 2

3.1. Definition 3.1

X is said to be

(i) a sg_0 space if for each pair of distinct points x, y of X, there exists a sg-open set G containing either of the point x or y.

(ii) a sg_1 space if for each pair of distinct points x, y of X there exists a sg-open set G containing x but not y and a sg-open set H containing y but not x.

(iii)a sg_2 space if for each pair of distinct points x, y of X there exists disjoint sg-open sets G and H such that G containing x but not y and H containing y but not x.

Note 2: X is $sg_2\to$ X is $sg_1\to$ X is sg_0.

Example 3.1: Let X = {a, b, c} and $\tau = \{\phi, \{a, c\}, X\}$ then X is sg_0 but not rT_0 and T_0, i = 0, 1, 2.

(ii) $\tau = \{\phi, \{a\}, \{a, c\}, X\}$ then X is not sg_i for i = 0, 1, 2.

Example 3.2: Let X = {a, b, c, d} and

(i) $\tau = \{\phi, \{a\}, \{c\}, \{a, c\}, X\}$ then X is sg_i ; i = 0, 1, 2.

(ii) $\tau = \{\phi, \{a\}, \{a, b\}, \{c, d\}, \{a, c, d\}, X\}$ then X is not sg_i for i = 0, 1, 2.

Remark 3.1: If X is $sT_{1/2}$ then sT_i and sg_i are one and the same for i = 0,1,2.

3.2. Theorem 3.1

(i) Every [resp: regular open] open subspace of sg_i space is sg_i for i = 0, 1, 2.

(ii) [25]The product of sg_i spaces is again sg_i for i = 0, 1, 2.

(iii) sg-continuous image of T_i[resp: rT_i] spaces is sg_i for i = 0, 1, 2.

3.3. Theorem 3.2

(i) X is sg_0 iff $\forall$ x$\in$ X, $\exists$ U$\in$ SGO(X) containing x such that the subspace U is sg_0.

(ii)X is sg_0 iff distinct points of X have disjoint sg-closures.

3.4. Theorem 3.3

The following are equivalent:

(i) X is sg_1.

(ii) Each one point set is sg-closed.

(iii)Each subset of X is the intersection of all sg-open sets containing it.

(iv) For any x$\in$ X, the intersection of all sg-open sets containing the point is the set {x}.

3.5. Theorem 3.4

If X is sg_1 then distinct points of X have disjoint sg-closures.

3.6. Theorem 3.5

Suppose x is a sg-limit point of a subset of A of a sg_1 space X. Then every neighborhood of x contains infinitely many distinct points of A.

3.7. Theorem 3.6

X is sg_2 iff the intersection of all sg-closed, sg-neighborhoods of each point of the space is reduced to that point.

Proof: Let X be sg_2 and x$\in$X, for each $y\neq x$ in X, $\exists$ U, V$\in$SGO(X) such that x$\in$U, y$\in$V and $U\cap V = \phi$. Since x$\in$U-V, hence X-V is a sg-closed, sg-neighborhood of x to which y does not belong. Consequently, the intersection of all sg-closed, sg-neighborhoods of x is reduced to {x}.

Conversely let $y\neq x$ in X, by hypothesis $\exists$ a sg-closed, sg-neighborhood U of x such that $y\notin U$. Now $\exists$ G$\in$SGO(X) such that $x\in G\subset U$. Thus G and X-U are disjoint sg-open sets containing x and y respectively. Hence X is sg_2.

3.8. Theorem 3.7

If to each x$\in$X, there exist a sg-closed, sg-open subset of X containing x which is also a sg_2 subspace of X, then X is sg_2.

Proof: Let x$\in$X, U a sg-closed, sg-open subset of X containing x and which is also a sg_2 subspace of X, then the intersection of all sg-closed, sg-neighborhoods of x in U is reduced to {x}. U being sg-closed, sg-open, these are sg-closed, sg-neighborhoods of x in X. Thus the intersection of all sg-closed, sg-neighborhoods of x is reduced to {x}. Hence by Theorem 3.6, X is sg_2.

3.9. Theorem 3.9

If X is sg_2 then the diagonal Δ in $X\times X$ is sg-closed.

Proof: Let (x, y)$\in$X$\times$X-Δ, then $x \neq y$. Since X is sg_2 $\exists$ U; V$\in$SGO(X) s.t. x$\in$U; y$\in$V and U$\cap$V = ϕ, implies (U$\times$V)$\cap\Delta$ = ϕ and therefore (x, y)$\in$(U$\times$V)$\subset$X$\times$X-$\Delta\in$SGO(X$\times$X). Hence Δ is sg-closed.

3.10. Theorem 3.9

In sg_2-space, sg-limits of sequences, if exists, are unique.

3.11. Theorem 3.10

In a sg_2 space, a point and disjoint sg-compact subspace can be separated by disjoint sg-open sets.

Proof: Let X be a sg_2 space, x$\in$X and C a sg-compact subspace of X not containing x. Let y$\in$C then for $x \neq y$ in X, $\exists$ disjoint sg-open nbds G_x and H_y. Allowing this for each y in C, we obtain a class {H_y} whose union covers C; and since C is sg-compact, some finite subclass {H_i, i = 1 to n} covers C. If G_i is sg-nbd of x corresponding to H_i, we put G = $\cup_{i=1\text{-}n}G_i$ and H = $\cap_{i=1\text{-}n}H_i$, satisfying the required properties.

3.12. Corollary 3.1

(i) In a T_1 [resp: rT_1; g_1] space, each singleton set is sg-closed.
(ii) If X is T_1 [resp: rT_1; g_1] then distinct points of X have disjoint sg-closures.
(iii)If X is T_2 [resp: rT_2; g_2] then the diagonal Δin X$\times$X is sg-closed.
(iv) Show that in a T_2 [resp: rT_2; g_2] space, a point and disjoint compact[resp: nearly-compact; g-compact] subspace can be separated by disjoint sg-open sets

3.13. Theorem 3.11

Every sg-compact subspace of a sg_2 space is sg-closed.

Proof: Let C be sg-compact subspace of a sg_2 space. If x$\in C^c$, by above Theorem x has a sg-nbd G s.t x$\in$G $\subset C^c$. Thus C^c is the union of sg-open sets and therefore C^c is sg-open. Thus C is sg-closed.

3.14. Corollary 3.2

Every compact [resp: nearly-compact; g-compact] subspace of a T_2 [resp: rT_2; g_2] space is sg-closed.

3.15. Theorem 3.12

If *f:* X$\to$ Y is injective, sg-irresolute and Y is sg_i then X is sg_i, i = 0, 1, 2.

Proof: Let $x \neq y\in$X, then $\exists$ a sg-open set $V_x\subset$Y such that *f*(x)$\in V_x$ and *f*(y)$\notin V_x$ and $\exists$ a sg-open set $V_y\subset$ Y such that *f*(y)$\in V_y$ and *f*(x)$\notin V_y$ with *f*(x) $\neq$ *f*(y). By sg-irresoluteness of *f*, $f^{-1}(V_x)$ is sg-open in X such that x$\in f^{-1}(V_x)$; y$\notin f^{-1}(V_x)$ and $f^{-1}(V_y)$ is sg-open in X such that y$\in f^{-1}(V_y)$; x$\notin f^{-1}(V_y)$. Hence X is sg_2

Similarly one can prove the remaining part of the Theorem.

3.16. Corollary 3.3

(i) If *f:* X$\to$ Y is injective, sg-continuous and Y is T_i then X is sg_i, i = 0, 1, 2.
(ii) If *f:* X$\to$ Y is injective, r-irresolute[r-continuous] and Y is rT_i then X is sg_i, i = 0, 1, 2.
(iii)The property of being a space is sg_0 is a sg-Topological property.
(iv) Let *f:* X $\to$ Y is a sgc-homeomorphism, then X is sg_i if Y is sg_i, i = 0, 1, 2.

3.17. Theorem 3.13

Let X be T_1 and *f*: X $\to$ Y be sg-closed surjection. Then X is sg_1.

3.18. Theorem 3.14

Every sg-irresolute map from a sg-compact space into a sg_2 space is sg-closed.

Proof: If *f*: X $\to$ Y is sg-irresolute where X is sg-compact and Y is sg_2. Let C$\subset$X be closed, then C$\subset$X is sg-closed and hence C is sg-compact and so *f*(C) is sg-compact. But then *f*(C) is sg-closed in Y. Hence the image of any sg-closed set in X is sg-closed set in Y. Thus *f* is sg-closed.

3.19. Theorem 3.15

Any sg-irresolute bijection from a sg-compact space onto a sg_2 space is a sgc-homeomorphism.

Proof: Let *f* be a sg-irresolute bijection from a sg-compact space onto a sg_2 space. Let G$\in$SGO(X). Then X-G$\in$SGC(X) and hence *f*(X-G)$\in$SGC(Y). Since *f* is bijective *f*(X-G) = Y-*f*(G) and therefore *f*(G)$\in$SGO(Y). Hence *f* is M-sg-open. Thus *f* is sgc-homeomorphism.

3.20. Corollary 3.4

Any sg-continuous bijection from a sg-compact space onto a sg_2 space is a sg-homeomorphism.

3.21. Theorem 3.16

The following are equivalent:
(i) X is sg_2.
(ii) For each pair $x \neq y\in$ X $\exists$ a sg-open, sg-closed set V such that x$\in$V and y$\notin$V, and
(iii)For each pair $x \neq y\in$ X $\exists$ *f:* X$\to$ [0, 1] such that *f*(x) = 0 and *f*(y) = 1 and *f* is sg-continuous.

3.22. Theorem 3.17

If *f:* X$\to$ Y is sg-irresolute and Y is sg_2 then
(i) the set A = {(x_1, x_2): *f*(x_1) = *f*(x_2)} is sg-closed in X$\times$ X.
(ii)G(*f*), Graph of *f*, is sg-closed in X$\times$Y.

Proof: (i) Let A = {(x_1, x_2): *f*(x_1) = *f*(x_2)}. If (x_1, x_2)$\in$X$\times$X-A, then *f*(x_1) $\neq$ *f*(x_2) $\Rightarrow\exists$ disjoint V_1 and $V_2\in$SGO(Y) such that *f*(x_1)$\in V_1$ and *f*(x_2)$\in V_2$, then by sg-irresoluteness of *f*, $f^{-1}(V_j)\in$SGO(X, x_j) for each j. Thus (x_1, x_2)$\in$ $f^{-1}(V_1)\times f^{-1}(V_2) \subset$ X$\times$X-A $\in$SGO(X$\times$X). Hence A$\in$SGC(X$\times$X).
(ii) Let (x, y)$\notin$G(*f*) $\Rightarrow$ y $\neq$ *f*(x) $\Rightarrow\exists$ disjoint V; W$\in$SGO(Y) such that *f*(x)$\in$V and y$\in$W. Since *f* is sg-irresolute, $\exists$ U$\in$SGO(X) such that x$\in$U and *f*(U)$\subset$W. Therefore we obtain (x, y)$\in$U$\times$V$\subset$X$\times$Y-G(*f*). Hence X$\times$Y-G(*f*)$\in$SGO(X$\times$Y). Hence G(*f*)$\in$SGC(X$\times$Y).

3.23. Theorem 3.18

If f: X$\to$ Y is sg-open and A = $\{(x_1, x_2): f(x_1) = f(x_2)\}$ is closed in X$\times$X. Then Y is sg_2.

3.24. Theorem 3.19

Let Y and $\{X_\alpha:\alpha\in I\}$ be Topological Spaces. If f: Y$\to \Pi X_\alpha$ be a sg-continuous function and Y is $rT_{1/2}$, then ΠX_α and each X_α are sg_i, i = 0,1,2.

3.25. Theorem 3.20

Let X be an arbitrary space, R an equivalence relation in X and p: X $\to$ X/R the identification map. If R$\subset$ X$\times$X is sg-closed in X$\times$X and p is sg-open map, then X/R is sg_2.

Proof: Let p(X), $\neq p(y)\in$X/R. Since x and y are not related, R$\subset$X$\times$X is sg-closed in X$\times$X. There are sg-open sets U and V such that x$\in$U, y$\in$V and U$\times$V$\subset$ R^c. Thus $\{p(U), p(V)\}$ are disjoint and also sg-open in X/R since p is sg-open.

3.26. Theorem 3.21

The following four properties are equivalent:
(i) X is sg_2
(ii) Let x$\in$ X. For each y $\neq$ x, $\exists$ U$\in$SGO(X) such that x$\in$U and y$\notin$sgcl(U)
(iii)For each x$\in$ X, $\cap$\{sgcl(U)/U$\in$SGO(X) and x$\in$ U\} = \{x\}.
(iv) The diagonal Δ = \{(x, x)/x$\in$X\} is sg-closed in X$\times$ X.

Proof: (i) Ĝ **(ii)** Let x$\in$ X and y $\neq$ x. Then there are disjoint sg-open sets U and V such that x$\in$ U and y$\in$ V. Clearly V^c is sg-closed, sgcl(U) $\subset V^c$, y$\notin V^c$ and therefore y$\notin$ sgcl(U).
(ii) Ĝ **(iii)** If y $\neq$ x, then $\exists$ U$\in$SGO(X, x) and y$\notin$sgcl(U). So y$\notin\cap$\{sgcl(U)/U$\in$SGO(X) and x$\in$U\}.
(iii) Ĝ **(iv)** We prove Δ^c is sg-open. Let (x, y) $\notin\Delta$. Then y $\neq$ x and $\cap$\{sgcl(U)/U$\in$SGO(X) and x$\in$U\} = \{x\} there is some U$\in$SGO(X) with x$\in$U and y$\notin$ sgcl(U). Since U$\cap$(sgcl(U))^c = ϕ, U$\times$(sgcl(U))^c is a sg-open set such that (x, y)$\in$U$\times$(sgcl(U))$^c\subset\Delta^c$.
(iv) Ĝ **(i)** y $\neq$ x, then (x, y)$\notin\Delta$ and thus there exist sg-open sets U and V such that (x, y)$\in$U$\times$V and (U$\times$V)$\cap\Delta$ = ϕ. Clearly, for the sg-open sets U and V we have; x$\in$U, y$\in$V and U$\cap$V = ϕ.

4. SGG_3 AND SGG_4 SPACES

4.1. Definition 4.1

X is said to be
(i) a sg_3 space if for every sg-closed sets F and a point x$\notin$F $\exists$ disjoint U, V$\in$SPO(X)such that F$\subseteq$U; x$\in$V
(ii) a sgg_3 space if for every sg-closed sets F and x$\notin$ F $\exists$ disjoint U, V$\in$SGO(X)such that F$\subseteq$U; x$\in$V
(iii)a sg_4 space if for each pair of disjoint F; H$\in$SGC(X), $\exists$ disjoint U, V$\in$SPO(X) s.t. F$\subseteq$U; H$\subseteq$V
(iv) a sgg_4 space if for each pair of disjoint F; H$\in$SGC(X), $\exists$ disjoint U, V$\in$SGO(X) s.t. F$\subseteq$U; H$\subseteq$V
Note: $rT_i \to sg_i \to sgg_i$, i = 3, 4. but the converse is not true in general.
Example 4.1: Let X = \{a, b, c\} and
(i) τ = \{ϕ, \{a\}, \{b, c\}, X\} then X is sgg_i.
(ii) τ = \{ϕ, \{a\}, X\} then X is not sgg_i, sg_i and rT_i for i = 3, 4.

4.2. Lemma 4.1

X is sg-regular iff X is nearly-regular and $rT_{1/2}$.
Proof: X is sg-regular, then obviously X is nearly-regular. Let A$\subseteq$X be sg-closed. For each x$\notin$A $\exists$ $V_x\in$ SGO(X, x) such that $V_x\cap$A = ϕ. If V = Ć\{V_x:x$\notin$ A\}, then V is sg-open and V = X-A. Hence A is sg-closed implies X is $rT_{1/2}$.

4.3. Theorem 4.1

If X is sg_3. Then for each x$\in$X and each U$\in$SGO(X, x) $\exists$ a sg-neighborhood V of x such that sgcl(A)$\subset$U.
Proof: Let x$\in$X and U a sg-neighborhood of x. Let B = X - U, then B is sg-closed and by sg-regularity of X, $\exists$ disjoint V, W$\in$SGO(X) such that x$\in$V and B$\subseteq$W. Then sgcl(V)$\cap$B = $\phi \Rightarrow$ sgcl(V)$\subseteq$X - B.

4.4. Theorem 4.2

The following are equivalent:
(i) X is sg_3.
(ii) For every point x$\in$X and for every G$\in$SGO(X, x), $\exists$ U$\in$SGO(X) such that x$\in$U$\subseteq$ sgcl(U) $\subseteq$ G.
(iii)For every sg-closed set F, the intersection of al sg-closed sg-neighborhoods of F is exactly F.
(iv) For every set A and B$\in$SGO(X) such that A$\cap$B $\neq\phi$, $\exists$ G$\in$SGO(X)such that A$\cap$G $\neq\phi$ and sgcl(G)$\subseteq$B.
(v) For every A $\neq\phi$ and B$\in$SGC(X) with A$\cap$B = ϕ, $\exists$ disjoint G; H$\in$SGO(X) such that A$\subseteq$G and B$\subseteq$H.

4.5. Theorem 4.3

If X is sgg_3. Then for each x$\in$X and each U$\in$SGO(X, x), $\exists$ V$\in$SGO(X, x) such that sgcl(A)$\subset$U.
Proof: Let x$\in$ X and U a sg-nbd of x. Let B = X - U, then B is sg-closed and by sgg-regularity of X, $\exists$ disjoint V, W$\in$SGO(X) such that x$\in$V and B$\subseteq$W. Then sgcl(V)$\cap$B = $\phi \Rightarrow$ sgcl(V)$\subseteq$X - B.

4.6. Corollary 4.1

If X is T_3 [resp: rT_3; g_3]. Then for each x$\in$ X and each sg-open neighborhood U of x there exists a sg-open neighborhood V of x such that sgcl(A)$\subset$U.

4.7. Theorem 4.4

If f: X$\to$ Y is sg-closed, sg-irresolute bijection. Then X is sgg_3 iff Y is sgg_3.
Proof: Let F be closed set in X and x$\notin$F, then $f(x)\notin f(F)$ and f(F) is sg-closed in Y. By sgg_3 of Y, $\exists$ V; W$\in$ SGO(y) such that f(X)$\in$V and f(F)$\subseteq$W. Hence x$\in f^{-1}$(V) and F$\subseteq f^{-1}$(W), where f^{-1}(V) and f^{-1}(W) are disjoint sg-open sets in X (by sg-irresoluteness of f). Hence X is sgg_3.
Conversely, X be sgg_3 and K any sg-closed in Y with y$\notin$K, then f^{-1}(K) is sg-closed in X such that $f^{-1}(y) \notin f^{-1}(K)$. By sgg_3 of X, $\exists$ disjoint V, W$\in$SGO(X) such that f^{-1}(y)$\in$V and f^{-1}(K)$\subseteq$W. Hence y$\in$ f(V) and K$\subseteq$ f(W) such that f(V) and f(W) are disjoint sg-open sets in X. Thus Y is sgg_3

4.8. Theorem 4.5

X is sg-normal iff for every sg-closed set F and a sg-open set G containing A, there exists a sg-open set V such that F$\subseteq$ V$\subseteq$ sgcl(V) $\subseteq$ G

4.9. Theorem 4.6

X is sg-normal iff for every pair of disjoint sg-closed sets A and B, there exist disjoint sg-open sets U and V such that $A\subseteq U$ and $B\subseteq V$.
Proof: Necessity: Follows from the fact that every sg-open set is sg-open.
Sufficiency: Let A, B be are disjoint sg-closed sets and U, V are disjoint sg-open sets such that $A\subseteq U$ and $B\subseteq V$. Since U and V are sg-open sets, $A\subseteq U$ and $B\subseteq V \Rightarrow A\subseteq sg(U)^o$ and $B\subseteq sg(V)^o$. Hence $sg(U)^o$ and $sg(V)^o$ are disjoint sg-open sets satisfying the axiom of sg-normality.

4.10. Theorem 4.7

The following are equivalent:
(i) X is sg-normal
(ii) For any pair of disjoint closed sets A and B, $\exists$ disjoint U; $V\in SGO(X)$ such that $A\subseteq U$ and $B\subseteq V$
(iii)For every closed set A and an open B containing A, $\exists$ $U\in SGO(X)$ such that $A\subseteq U\subseteq sgcl(U)\subseteq B$
(iv) For every closed set A and a sg-open B containing A, $\exists$ $U\in SGO(X)$ such that $A\subseteq U\subseteq sgcl(U)\subseteq (B)^o$
(v) For every $A\in SGC(X)$, and every $B\in \tau(X,A)$, $\exists$ $U\in SGO(X)$ such that $A\subseteq sgcl(A)\subseteq U\subseteq sgcl(U) \subseteq B$.

4.11. Theorem 4.8

The following are equivalent:
(i) X is sg-normal
(ii) For every $A\in SGC(X)$ and every sg-open set containing A, $\exists$ a sg-clopen set V such that $A\subseteq V\subseteq U$.

4.12. Theorem 4.9

Let X be an almost normal space and $F\cap A = \phi$ where F is regularly closed and A is sg-closed, then $\exists$ disjoint U; $V\in\tau$ such that $F\subseteq U$; $B\subseteq V$.

4.13. Theorem 4.10

X is almost normal iff for every disjoint sets F and A where F is regular closed and A is closed, $\exists$ disjoint sg-open sets in X such that $F\subseteq U$; $B\subseteq V$.
Proof: Necessity: Follows from the fact that every open set is sg-open.
Sufficiency: Let F, A be disjoint s.t. $F\in RC(X)$ and A is closed, $\exists$ disjoint U; $V\in SGO(X)$ s.t. $F\subseteq U$; $B\subseteq V$. Hence $F\subseteq U^o$; $B\subseteq V^o$, where U^o and V^o are disjoint open sets. Hence X is almost regular.

5. SG-R_i SPACES; i = 0,1:.

5.1. Definition 5.1

Let $x\in X$. Then
(i) sg-kernel of x is defined and denoted by $Ker_{\{sg\}}\{x\} = \cap\{U:U\in SGO(X) \text{ and } x\in U\}$
(ii)$Ker_{\{sg\}}F = \cap\{U: U\in SGO(X) \text{ and } F\subset U\}$

5.2. Lemma 5.1

Let $A\subset X$, then $Ker_{\{sg\}}\{A\} = \{x\in X: sgcl\{x\}\cap A \neq \phi.\}$

5.3. Lemma 5.2

Let $x\in X$. Then $y\in Ker_{\{sg\}}\{x\}$ iff $x\in sgcl\{y\}$.
Proof: Suppose that $y\notin Ker_{\{sg\}}\{x\}$. Then $\exists$ $V\in SGO(X)$ containing x such that $y\notin V$. Therefore we have $x\notin sgcl\{y\}$. The proof of converse part can be done similarly.

5.4. Lemma 5.3

For any points $x \neq y\in X$, the following are equivalent:
(i) $Ker_{\{sg\}}\{x\} \neq Ker_{\{sg\}}\{y\}$; (ii) $sgcl\{x\} \neq sgcl\{y\}$.
Proof: (i) Ĝ **(ii):** Let $Ker_{\{sg\}}\{x\} \neq Ker_{\{sg\}}\{y\}$, then $\exists$ $z\in X$ such that $z\in Ker_{\{sg\}}\{x\}$ and $z\notin Ker_{\{sg\}}\{y\}$. From $z\in Ker_{\{sg\}}\{x\}$ it follows that $\{x\}\cap sgcl\{z\} \neq \phi \Rightarrow x\in sgcl\{z\}$. By $z\notin Ker_{\{sg\}}\{y\}$, we have $\{y\}\cap sgcl\{z\} = \phi$. Since $x\in sgcl\{z\}$, $sgcl\{x\}\subset sgcl\{z\}$ and $\{y\}\cap sgcl\{x\} = \phi$. Therefore $sgcl\{x\} \neq sgcl\{y\}$. Now $Ker_{\{sg\}}\{x\} \neq Ker_{\{sg\}}\{y\} \Rightarrow sgcl\{x\} \neq sgcl\{y\}$.
(ii) Ĝ **(i):** If $sgcl\{x\} \neq sgcl\{y\}$. Then $\exists$ $z\in X$ such that $z\in sgcl\{x\}$ and $z\notin sgcl\{y\}$. Then $\exists$ a sg-open set containing z and therefore containing x but not y, namely, $y\notin Ker_{\{sg\}}\{x\}$. Hence $Ker_{\{sg\}}\{x\} \neq Ker_{\{sg\}}\{y\}$.

5.5. Definition 5.2

X is said to be
(i) sg-R_0 iff $sgcl\{x\}\subseteq G$ whenever $x\in G\in SGO(X)$.
(ii) weakly sg-R_0 iff $\cap\, sgcl\{x\} = \phi$.
(iii) sg-R_1 iff for $x,y\in X$ such that $sgcl\{x\} \neq sgcl\{y\}$ $\exists$ disjoint U; $V\in SGO(X)$ such that $sgcl\{x\}\subseteq U$ and $sgcl\{y\}\subseteq V$.
Example 5.1: Let X = {a, b, c, d} and
(i) $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}$ then X is sgR_0 and weakly sgR_0.
(ii) $\tau = \{\phi, \{a, b\}, \{a, b, c\}, X\}$ then X is sgR_1.
Remark 5.1:
(i) $r\text{-}R_i \Rightarrow R_i \Rightarrow g\,R_i \Rightarrow sgR_i$, i = 0, 1.
(ii) Every weakly-R_0 space is weakly sg R_0.

5.6. Lemma 5.1

Every sgR_0 space is weakly sgR_0.
Converse of the above Theorem is not true in general by the following Examples.
Example 5.2:
(i) Let X = {a, b, c, d} and $\tau = \{\phi, \{a\}, \{a, b\}, \{c, d\}, \{a, c, d\}, X\}$, then X is weakly sgR_0 but not sgR_0.
(ii) Let X = {a, b, c} and $\tau = \{\phi, \{a, b\}, X\}$, then X is sgR_0 and R_0.

5.7. Theorem 5.1

Every sg-regular space X is sg_2 and sg-R_0.

Proof: Let X be sg-regular and let $x \neq y \in X$. By Lemma 4.1, {x} is either sg-open or sg-closed. If {x} is sg-open, {x} is sg-open and hence sg-clopen. Thus {x} and X - {x} are separating sg-open sets. Similar argument, for {x} is sg-closed gives {x} and X - {x} are separating sg-closed sets. Thus X is sg_2 and sg-R_0.

5.8. Theorem 5.2

X is sg-R_0 iff given $x \neq y \in X$; sgcl{x} $\neq$ sgcl{y}.

Proof: Let X be sg-R_0 and let let x, $\neq y \in$ X. Suppose U is a sg-open set containing x but not y, then $y \in$ sgcl{y}$\subset$ X-U and so $x \notin$ sgcl{y}. Hence sgcl{x} $\neq$ sgcl{y}.

Conversely, let x, $\neq y \in$ X such that sgcl{x} $\neq$ sgcl{y}$\Rightarrow$ sgcl{x}$\subset$ X-sgcl{y} = U(say) a sg-open set in X. This is true for every sgcl{x}. Thus $\cap$ sgcl{x}$\subseteq$ U where $x \in$ sgcl{x}$\subseteq$ U$\in$ SGO(X), which in turn implies $\cap$ sgcl{x}$\subseteq$ U where $x \in U \in$ SGO(X). Hence X is sgR_0.

5.9. Theorem 5.3

X is weakly sgR_0 iff $Ker_{\{sg\}}\{x\} \neq X$ for any $x \in X$.

Proof: Let $x_0 \in X$ such that $ker_{\{sg\}}\{x_0\} = X$. This means that x_0 is not contained in any proper sg-open subset of X. Thus $x_0 \in$ sgcl{x} of every singleton set. Hence $x_0 \in \cap$sgcl{x}, a contradiction.

Conversely assume $Ker_{\{sg\}}\{x\} \neq X$ for any $x \in X$. If there is an $x_0 \in X$ such that $x_0 \in \cap${sgcl{x}}, then every sg-open set containing x_0 must contain every point of X. Therefore, the unique sg-open set containing x_0 is X. Hence $Ker_{\{sg\}}\{x_0\} = X$, which is a contradiction. Thus X is weakly sg-R_0.

5.10. Theorem 5.4

The following statements are equivalent:

(i) X is sg-R_0 space.

(ii) For each $x \in X$, sgcl{x}$\subset Ker_{\{sg\}}\{x\}$

(iii)For any sg-closed set F and a point $x \notin F$, $\exists$ U$\in$ SGO(X) such that $x \notin U$ and $F \subset U$.

(iv) Each sg-closed set F can be expressed as F = $\cap${G: G is sg-open and $F \subset G$}.

(v) Each sg-open set G can be expressed as G = $\cup${A: A is sg-closed and $A \subset G$}.

(vi) For each sg-closed set F, $x \notin F$ implies sg-cl{x}$\cap$ F = ϕ.

Proof: **(i) Ĝ (ii)** For any $x \in X$, we have $Ker_{\{sg\}}\{x\} = \cap${U: U$\in$SGO(X) and $x \in U$}. Since X is sg-R_0, each sg-open set containing x contains sgcl{x}. Hence sgcl{x}$\subset Ker_{\{sg\}}\{x\}$.

(ii) Ĝ (iii) Let $x \notin F \in$SGC(X). Then for any $y \in F$; sgcl{y}$\subset$F and so $x \notin$sgcl{y}$\Rightarrow$ $y \notin$sgcl{x} that is $\exists$ $U_y \in$SGO(X) such that $y \in U_y$ and $x \notin U_y$ $\forall$ $y \in F$. Let U = $\cup${U_y: $U_y \in$SGO(X, y) and $x \notin U_y$}. Then U$\in$SGO(X) such that $x \notin U$ and $F \subset U$.

(iii) Ĝ (iv) Let F$\in$SGO(X) and N = $\cap${G: G is sg-open and F$\subset$ G}. Then F$\subset$N $\rightarrow$ (1).

Let $x \notin$ F, then by (iii) $\exists$ G$\in$SGO(X) such that $x \notin G$ and F$\subset$G.

Hence $x \notin N$ which implies $x \in N \Rightarrow x \in F$. Hence N$\subset$F $\rightarrow$ (2).

Therefore from (1) and (2), each sg-closed set F = $\cap${G: G is sg-open and F$\subset$G}

(iv) Ĝ (v) obvious.

(v) Ĝ (vi) Let $x \notin F \in$SGC(X). Then X-F = G$\in$SGO(X, x). Then by (v), G = $\cup${A: A is sg-closed and A$\subset$G}, and so $\exists$ M$\in$SGC(X) such that $x \in M \subset G$; and hence sgcl{x}$\subset$G which implies sgcl{x}$\cap$F = ϕ.

(vi) Ĝ (i) Let $x \in G \in$SGO(X). Then $x \notin$(X-G), which is a sg-closed set. Therefore by (vi) sgcl{x}$\cap$(X-G) = ϕ, which implies that sgcl{x}$\subseteq$ G. Thus X is sgR_0 space.

5.11. Theorem 5.5

Let *f*: X $\rightarrow$ Y be a sg-closed one-one function. If X is weakly sg-R_0, then so is Y.

5.12. Theorem 5.6

If X is weakly sg-R_0, then for every space Y, X$\times$Y is weakly sg-R_0.

Proof: $\cap$sgcl{(x,y)}$\subseteq\cap${sgcl{x}$\times$sgcl{y}} = $\cap$[sgcl{x}]$\times$[sgcl{y}]$\subseteq\phi\times$Y = ϕ. Hence X$\times$ Y is sgR_0.

5.13. Corollary 5.1

(i) If X and Y are weakly sgR_0, then X$\times$ Y is weakly sgR_0.

(ii) If X and Y are sgR_0, then X$\times$ Y is weakly sgR_0.

(iii) If X is sgR_0 and Y are weakly R_0, then X$\times$ Y is weakly sgR_0.

5.14. Theorem 5.7

X is sgR_0 iff for any x, $y \in X$, sgcl{x} $\neq$ sgcl{y}$\Rightarrow$ sgcl{x}$\cap$ sgcl{y} = ϕ.

Proof: Let X is sgR_0 and x, $y \in X$ such that sgcl{x} $\neq$ sgcl{y} .Then $\exists$ $z \in$sgcl{x} such that $z \notin$sgcl{y} (or $z \in$sgcl{y}) such that $z \notin$sgcl{x}. There exists V$\in$SGO(X) such that $y \notin V$ and $z \in V$; hence $x \in V$. Therefore, $x \notin$sgcl{y}. Thus $x \in$[sgcl{y}]$^c \in$SGO(X), which implies sgcl{x}$\subset$[sgcl{y}]^c and sgcl{x}$\cap$ sgcl{y} = ϕ. The proof for otherwise is similar.

Sufficiency: Let $x \in V \in$SGO(X). Let $y \notin V$, i.e., $y \in V^c$. Then $x \neq y$ and $x \notin$sgcl{y}. Hence sgcl{x} $\neq$ sgcl{y}. But sgcl{x}$\cap$sgcl{y} = ϕ. Hence $y \notin$sgcl{x}. Therefore sgcl{x}$\subset$V.

5.15. Theorem 5.8

X is sgR_0 iff for any x, $y \in X$, $Ker_{\{sg\}}\{x\} \neq Ker_{\{sg\}}\{y\} \Rightarrow Ker_{\{sg\}}\{x\} \cap Ker_{\{sg\}}\{y\} = \phi$.

Proof: If X is sgR_0, by Lemma 5.3 for any x, $y \in X$ if $Ker_{\{sg\}}\{x\} \neq Ker_{\{sg\}}\{y\}$ then sgcl{x} $\neq$ sgcl{y}. Assume that $z \in Ker_{\{sg\}}\{x\} \cap Ker_{\{sg\}}\{y\}$. By $z \in Ker_{\{sg\}}\{x\}$ and Lemma 5.2, it follows that $x \in$sgcl{z}. Since $x \in$sgcl{z}, sgcl{x} = sgcl{z}. Similarly, we have sgcl{y} = sgcl{z} = sgcl{x}. This is a contradiction. Therefore, we have $Ker_{\{sg\}}\{x\} \cap Ker_{\{sg\}}\{y\} = \phi$.

Conversely, let x, $y \in X$, s.t. sgcl{x} $\neq$ sgcl{y}, then by Lemma 5.3, $Ker_{\{sg\}}\{x\} \neq Ker_{\{sg\}}\{y\}$. By hypothesis $Ker_{\{sg\}}\{x\} \cap Ker_{\{sg\}}\{y\} = \phi$ which implies sgcl{x}$\cap$sgcl{y} = ϕ. But $z \in$sgcl{x} implies that $x \in Ker_{\{sg\}}\{z\}$ and hence $Ker_{\{sg\}}\{x\} \cap Ker_{\{sg\}}\{z\} \neq \phi$ Therefore by Theorem 5.7 X is a sgR_0 space.

5.16. Theorem 5.9

The following properties are equivalent:

(i) X is a sg-R_0 space.

(ii) For any A $\neq \phi$ and G$\in$SGO(X) such that A$\cap$G $\neq \phi$ $\exists$ F$\in$SGC(X)such that A$\cap$F $\neq \phi$ and F$\subset$G.

Proof: (i) Ĝ **(ii):** Let A ≠ ϕ and G∈SGO(X) such that A∩G ≠ ϕ. There exists x∈A∩G. Since x∈G∈SGO(X), sgcl{x}⊂G. Set F = sgcl{x}, then F∈SGC(X), F⊂G and A∩F ≠ ϕ
(ii) Ĝ **(i):** Let G∈ SGO(X) and x∈ G. By (2), sgcl{x}⊂ G. Hence X is sg-R_0.

5.17. Theorem 5.10

The following properties are equivalent:
(i) X is a sg-R_0 space;
(ii) x∈ sgcl{y} iff y∈sgcl{x}, for any points x and y in X.
Proof: (i) Ĝ **(ii):** Assume X is sgR_0. Let x∈sgcl{y} and D be any sg-open set such that y∈D. Now by hypothesis, x∈D. Therefore, every sg-open set which contain y contains x. Hence y∈sgcl{x}.
(ii) Ĝ **(i):** Let U be a sg-open set and x∈U. If y∉ U, then x∉sgcl{y} and hence y∉sgcl{x}. This implies that sgcl{x}⊂U. Hence X is sgR_0.

5.18. Theorem 5.11

The following properties are equivalent:
(i) X is a sgR_0 space;
(ii) If F is sg-closed, then F = $Ker_{\{sg\}}$(F);
(iii) If F is sg-closed and x∈ F, then $Ker_{\{sg\}}${x}⊆F;
(iv) If x∈ X, then $Ker_{\{sg\}}${x}⊂sgcl{x}.
Proof: (i) Ĝ **(ii):** Let x∉F∈SGC(X) ⇒ (X-F)∈SGO(X, x). For X is sgR_0, sgcl({x})⊂(X-F). Thus sgcl({x})∩ F = ϕ and x∉$Ker_{\{sg\}}$(F). Hence $Ker_{\{sg\}}$(F) = F.
(ii) Ĝ **(iii):** A⊂B ⇒ $Ker_{\{sg\}}$(A)⊂$Ker_{\{sg\}}$(B). Therefore, by (2) $Ker_{\{sg\}}${x}⊂ $Ker_{\{sg\}}$(F) = F.
(iii) Ĝ **(iv):** Since x∈sgcl{x} and sgcl{x} is sg-closed, by (3) $Ker_{\{sg\}}${x}⊂ sgcl{x}.
(iv) Ĝ **(i):** Let x∈sgcl{y}. Then by Lemma 5.2 y∈$Ker_{\{sg\}}${x}. Since x∈sgcl{x} and sgcl{x} is sg-closed, by (iv) we obtain y∈$Ker_{\{sg\}}${x}⊆sgcl{x}. Therefore x∈sgcl{y} implies y∈sgcl{x}. The converse is obvious and X is sgR_0.

5.19. Corollary 5.2

The following properties are equivalent:
(i) X is sgR_0. (ii) sgcl{x} = $Ker_{\{sg\}}${x}∀ x∈ X.
Proof: Straight forward from Theorems 5.4 and 5.11.

Recall that a filterbase F is called sg-convergent to a point x in X, if for any sg-open set U of X containing x, there exists B∈ F such that B⊂ U.

5.20. Lemma 5.4

Let x and y be any two points in X such that every net in X sg-converging to y sg-converges to x. Then x∈ sgcl{y}.
Proof: Suppose that x_n = y for each n∈N. Then $\{x_n\}_{n\in N}$ is a net in sgcl{({y})}. Since $\{x_n\}_{n\in N}$ sg-converges to y, then $\{x_n\}_{n\in N}$ sg-converges to x and this implies that x∈sgcl{y}.

5.21. Theorem 5.12

The following statements are equivalent:
(i) X is a sgR_0 space;
(ii) If x, y∈X, then y∈sgcl{x} iff every net in X sg-converging to y sg-converges to x.
Proof: (i) Ĝ **(ii):** Let x, y∈X such that y∈sgcl{x}. Suppose that $\{x_\alpha\}_{\alpha\in I}$ is a net in X such that $\{x_\alpha\}_{\alpha\in I}$ sg-converges to y. Since y∈sgcl{x}, by Thm. 5.7 we have sgcl{x} = sgcl{y}. Therefore x∈sgcl{y}. This means that $\{x_\alpha\}_{\alpha\in I}$ sg-converges to x.
Conversely, let x, y∈ X such that every net in X sg-converging to y sg-converges to x. Then x∈ sg-cl{y}[by 5.4]. By Thm. 5.7, we have sgcl{x} = sgcl{y}. Therefore y∈sgcl{x}.
(ii) Ĝ **(i):** Let x, y∈X such that sgcl{x}∩sgcl{y} ≠ ϕ. Let z∈sgcl{x}∩ sgcl{y}. So ∃ a net $\{x_\alpha\}_{\alpha\in I}$ in sgcl{x} such that $\{x_\alpha\}_{\alpha\in I}$ sg-converges to z. Since z∈sgcl{y}, then $\{x_\alpha\}_{\alpha\in I}$ sg-converges to y. It follows that y∈sgcl{x}. Similarly we obtain x∈sgcl{y}. Thus sgcl{x} = sgcl{y}. Hence X is sgR_0.

5.22. Theorem 5.13

(i) Every subspace of sgR_1 space is again sgR_1.
(ii) Product of any two sgR_1 spaces is again sgR_1.
(iii) X is sgR_1 iff given x ≠ y∈ X, sgcl{x} ≠ sgcl{y}.
(iv) Every sg_2 space is sgR_1.
(v) If X is sg-R_1, then X is sg-R_0.
(vi)X is sg-R_1 iff for x, y∈ X, $Ker_{\{sg\}}${x} ≠ $Ker_{\{sg\}}${y}, ∃ disjoint U; V∈SGO(X) such that sgcl{x}⊂U and sgcl{y}⊂V.

The converse of (iv) is not true. However, we have the following result.

5.23. Theorem 5.14

Every sg_1 and sgR_1 space is sg_2.
Proof: Let x ≠ y∈X. Since X is sg_1; {x} and {y} are sg-closed sets such that sgcl{x} ≠ sgcl{y}. Since X is sgR_1, there exists disjoint sg-open sets U and V such that x∈U; y∈V. Hence X is sg_2.

5.24. Corollary 5.3

X is sg_2 iff it is sgR_1 and sg_1.

Theorem 5.15: The following are equivalent
(i) X is sg-R_1. (ii) ∩sgcl{x} = {x}. (iii)For any x∈X, ∩sg[nbds{x}] = {x}.
Proof: (i) Ĝ **(ii)** Let y ≠ x∈X such that y∈sgcl{x}. Since X is sgR_1, ∃ U∈SGO(X) such that y∈U, x∉U or x∈U, y∉U. In either case y∉sgcl{x}. Hence ∩sgcl{x} = {x}.
(ii) Ĝ **(iii)** If y ≠ x∈X, then x∉∩sgcl{y}, so there is a sg-open set containing x but not y. Therefore y∉∩sg[nbds{x}]. Hence ∩sg[nbds{x}] = {x}.
(iii) Ĝ **(i)** Let x ≠ y∈X. by hypothesis, y∉∩sg[nbds{x}] and x∉∩sg[nbds{y}], which implies sgcl{x} ≠ sgcl{y}. Hence X is sg-R_1.

5.25. Theorem 5.16

The following are equivalent:

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(i) X is sg-R_1.
(ii) For each pair x, y$\in$X with sgcl{x} $\neq$ sgcl{y}, $\exists$ a sg-open, sg-closed set V s.t. x$\in$V and y$\notin$V, and
(iii)For each x $\neq$ y$\in$X with sgcl{x} $\neq$ sgcl{y}, $\exists$ a sg-continuous function f: X$\rightarrow$[0, 1] s.t. f(x) = 0 and f(y) = 1.
Proof: (i) Ĝ **(ii)** Let x, y$\in$X with sgcl{x} $\neq$ sgcl{y}, $\exists$ disjoint U; W$\in$SGO(X) such that sgcl{x}$\subset$U and sgcl{y}$\subset$W and V = sgcl(U) is sg-open and sg-closed such that x$\in$V and y$\notin$V.
(ii) Ĝ **(iii)** Let x, y$\in$ X with sgcl{x} $\neq$ sgcl{y}, and let V be sg-open and sg-closed such that x$\in$V and y$\notin$V. Then f: X$\rightarrow$ [0, 1] defined by f(z) = 0 if z$\in$V and f(z) = 1 if z$\notin$V satisfied the desired properties.
(iii) Ĝ **(i)** Let x, y$\in$ X such that sgcl{x} $\neq$ sgcl{y}, let f: X$\rightarrow$ [0, 1] such that f is sg-continuous, f(x) = 0 and f(y) = 1. Then U = f^{-1}([0, 1/2)) and V = f^{-1}((1/2, 1]) are disjoint sg-open and sg-closed sets in X, such that sgcl{x}$\subset$U and sgcl{y}$\subset$V.

6. SG-C_i AND SG-D_i SPACES, i = 0,1,2:

6.1. Definition 6.1

X is said to be a
(i) sg-C_0 space if for each pair of distinct points x, y of X there exists a sg-open set G whose closure contains either of the point x or y.
(ii) sg-C_1 space if for each pair of distinct points x, y of X there exists a sg-open set G whose closure containing x but not y and a sg-open set H whose closure containing y but not x.
(iii)sg-C_2 space if for each pair of distinct points x, y of X there exists disjoint sg-open sets G and H such that G containing x but not y and H containing y but not x.
Note: sg-C_2 $\Rightarrow$ sg-C_1 $\Rightarrow$ sg-C_0. Converse need not be true in general as shown by.
Example 6.1: Let X = {a, b, c, d} and
(i) τ = {ϕ, {a}, {b}, {a, b}, X} then X is sg-C_i, i = 1, 2.
(ii) τ = {ϕ, {a}, {a, b}, {c, d}, {a, c, d}, X} then X is not sg-C_i, i = 0, 1, 2.

6.2. Theorem 6.1

(i) Every subspace of sg-C_i space is sg-C_i.
(ii) Every sg$_i$ spaces is sg-C_i.
(iii)Product of sg-C_i spaces are sg-C_i.

6.3. Theorem 6.2

Let (X, τ) be any sg-C_i space and A $\neq$ ϕ $\subseteq$ X then A is sg-C_i iff (A, $\tau_{/A}$) is sg-C_i.

6.4. Theorem 6.3

(i) If X is sg-C_1 then each singleton set is sg-closed.
(ii)In an sg-C_1 space disjoint points of X has disjoint sg- closures.

6.5. Definition 6.2

A$\subset$ X is called a sg-Difference(Shortly sgD-set) set if there are two U, V$\in$ SGO(X) such that U $\neq$ X and A = U-V.

Clearly every sg-open set U different from X is a sgD-set if A = U and V = ϕ.

6.6. Definition 6.3

X is said to be a
(i) sg-D_0 if for any pair of distinct points x and y of X there exist a sgD-set in X containing x but not y or a sgD set in X containing y but not x.
(ii) sg-D_1 if for any pair of distinct points x and y in X there exist a sgD-set of X containing x but not y and a sgD-set in X containing y but not x.
(iii)sg-D_2 if for any pair of distinct points x and y of X there exists disjoint sgD-sets G and H in X containing x and y respectively.
Example 6.2: Let X = {a, b, c} and τ = {ϕ, {b}, {a, c}, X} then X is sgD_i, i = 0, 1, 2.
Remark 6.2: (i) If X is rT_i, then it is sg$_i$, i = 0, 1, 2 and converse is false.
(ii) If X is sg$_i$, then it is sg$_{\{i-1\}}$, i = 1, 2.
(iii) If X is sg$_i$, then it is sg-D_i , i = 0, 1, 2.
(iv) If X is sg-D_i, then it is sg-$D_{\{i-1\}}$, i = 1, 2.

6.7. Theorem 6.4

The following statements are true:
(i) X is sg-D_0 iff it is sg$_0$.
(ii) X is sg-D_1 iff it is sg-D_2.

Corollary 6.1: If X is sg-D_1, then it is sg$_0$.
Proof: Remark 6.1(iv) and Theorem 6.2(i)

6.8. Definition 6.4

A point x$\in$X which has X as the unique sg-neighborhood is called sg.c.c point.

6.9. Theorem 6.5

For an sg$_0$ space X the following are equivalent:
(i) X is sg-D_1; (ii) X has no sg.c.c point.
Proof: (i) Ĝ **(ii)** Since X is sg-D_1, then each point x of X is contained in a sgD-set O = U - V and thus in U. By Definition U $\neq$ X. This implies that x is not a sg.c.c point.
(ii) Ĝ **(i)** If X is sg$_0$, for each x $\neq$ y$\in$ X, at least one of them, x (say) has a sg-nbd U containing x but not y. Thus U $\neq$ X is a sgD-set. If X has no sg.c.c point, y is not a sg.c.c point, so there exists a sg-nbd V of y such that V $\neq$ X. Thus y$\in$(V-U) but not x and V-U is a sgD-set. Thus X is sg-D_1.

6.10. Corollary 6.2

A sg$_0$ space X is sg-D_1 iff there is a unique sg.c.c point in X.
Proof: Only uniqueness is sufficient to prove. If x_0 and y_0 are two sg.c.c points in X then since X is sg$_0$, at least one of x_0 and y_0 say x_0, has a sg-neighborhood U such that $x_0\in$ U and $y_0\notin$ U, hence U $\neq$ X, x_0 is not a sg.c.c point, a contradiction.

6.11. Definition 6.5

X is sg-symmetric if for x and y in X, $x \in$ sgcl{y} implies $y \in$ sgcl{x}.

6.12. Theorem 6.6

X is sg-symmetric iff {x} is sgg-closed for each $x \in X$.

Proof: Assume that $x \in$sgcl{y} but $y \notin$sgcl{x}. This means that $[\text{sgcl}\{x\}]^c$ contains y. This implies that sgcl{y}$\subset [\text{sgcl}\{x\}]^c$. Now $[\text{sgcl}\{x\}]^c$ contains x which is a contradiction.

Conversely, If $\{x\} \subset E \in$SGO(X) but sgcl{x}$\not\subset$E, then sgcl{x} and E^c are not disjoint. Let y belongs to their intersection. Now we have $x \in$sgcl{y}$\subset E^c$ and $x \notin E$. But this is a contradiction.

6.13. Corollary 6.3

If X is a sg_1, then it is sg-symmetric.

Proof: Follows from theorem 2.2(ii), Remark 6.2 and theorem 6.6.

6.14. Corollary 6.4

The following are equivalent:

(i) X is sg-symmetric and sg_0

(ii) X is sg_1.

Proof: By Corollary 6.3 and Remark 6.1 it suffices to prove only (i) $\Rightarrow$ (ii). Let $x \neq y$ and by sg_0, we may assume that $x \in G_1 \subset \{y\}^c$ for some $G_1 \in$SGO(X).Then $x \notin$sgcl{y} and hence $y \notin$sgcl{x}. There exists a $G_2 \in$SGO(X) such that $y \in G_2 \subset \{x\}^c$ and X is a sg_1 space.

6.15. Theorem 6.7

For a sg-symmetric space X the following are equivalent:

(i) X is sg_0; (ii) X is sg-D_1; (iii) X is sg_1.

Proof: (i) Ĝ **(iii)** Corollary 6.4 and **(iii)** Ĝ **(ii)** Ĝ **(i)** Remark 6.1.

6.16. Theorem 6.8

(i) If *f*:X$\to$ Y is a sg-irresolute surjective function and $E \in$SGD(Y), $f^{-1}(E) \in$SGD(X).

(ii) If Y is sg-D_1 and *f*: X $\to$ Y is sg-irresolute and bijective, then X is sg-D_1.

6.18. Theorem 6.9

X is sg-D_1 iff for each pair of distinct points x , y in X there exist a sg-irresolute surjective function *f*: X$\to$ Y, where Y is a sg-D_1 space such that *f*(x) and *f*(y) are distinct.

Proof: Necessity. For every $x \neq y \in X$, it suffices to take the identity function on X.

Sufficiency. Let $x \neq y \in X$. By hypothesis, $\exists$ a sg-irresolute, surjective function *f*: X onto a sg-D_1 space Y s.t. $f(x) \neq f(y)$. Therefore, $\exists$ disjoint G_x; $G_y \in$SGD(Y) s.t. $f(x) \in G_x$ and $f(y) \in G_y$. Then by Theorem 6.8(i), $f^{-1}(G_x)$ and $f^{1}(G_y) \in$SGD(X) containing x and y respectively. Therefore X is sg-D_1 space.

6.19. Corollary 6.5

Let $\{X_\alpha/\alpha \in I\}$ be any family of topological spaces. If X_α is sg-D_1 for each $\alpha \in I$, so is ΠX_α. **Proof:** Let $(x_\alpha) \neq (y_\alpha) \in \Pi X_\alpha$. Then there exists an index $\beta \in I$ s.t. $x_\beta \neq y_\beta$. The natural projection $P_\beta: \Pi X_\alpha \to X_\beta$ is almost continuous and almost open and $P_\beta((x_\alpha)) = P_\beta((y_\alpha))$. Since X_β is sg-D_1, ΠX_α is sg-D_1.

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