



Common natural waves in dissipative inhomogeneous plane bodies

Safarov Ismail Ibrahimovich¹, Teshayev Mukhsin Khudoyberdiyevich², Akhmedov Maqsud Sharipovich³, Boltaev Zafar Ihterovich⁴

1. Doctor of physical and mathematical sciences, professor., Bukhara Technological- Institute of Engineering, 15 K. Murtazoyev Street, Republic of Uzbekistan; Email: safarov54@mail.ru (+998 93) 625-08-15
2. Scientific - Associate Professor of Physical - Mathematical Sciences. Bukhara Technological- Institute of Engineering, 15 K. Murtazoyev Street, Republic of Uzbekistan, Email: Muhsin_5@mail.ru; (+99893) 681-04-31
3. Researcher, Bukhara Technological- Institute of Engineering, 15 K. Murtazoyev Street, Republic of Uzbekistan, Email: Maqsud.axmedov.1985@mail.ru; (+99890) 612-01-02
4. Scientific – researcher, Bukhara Technological- Institute of Engineering, 15 K. Murtazoyev Street, Republic of Uzbekistan, (+99891) 401-01-51

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General Note



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ABSTRACT

In this paper the distribution of natural waves in dissipative neodnorodnonyh flat bodies. Wave motion is described by linear integral-differential equations. Solving this problem, we obtain the relationship between the velocity of the wave and its length. The problem of this kind is of great interest to geophysicists in the field of engineering and construction.

Keywords: layer viscoelastic half-layer equation, phase velocity.

1. Introduction

Many construction and engineering design work in dynamic conditions, is made up of deformable bodies having different viscoelastic properties [1,2,3]. In addition, an important role in the processes of wave elastic bodies due to play in signal processing tasks, especially in connection with the creation of mechanical resonators and filters [4,5,6]. The mechanisms by which the energy of elastic waves is converted into heat, are not entirely clear. Proposed various loss mechanisms [7 - 11], but not one of them does not fully meet all the requirements. Probably the most important mechanisms are the internal friction in the form of sliding friction (or attachment, and then slip) and viscous losses in the pore fluids; The latter mechanism is the most significant in a highly permeable rocks. Other effects are likely, in general have less importance, are the loss of the heat generated in the compression phase of the wave motion by conduction, piezoelectric and thermoelectric effects and the energy for the formation of new surfaces (which plays a significant role only near the source). Therefore, development of a common methodology and algorithm for calculating wave fields dissipative inhomogeneous layered bodies, is an urgent task mechanics of deformable solids [12-13].

2. Formulation of the problem

Suppose that in a Cartesian (x, y, z) coordinate system, the beginning and the OZ axis, a sequence of parallel planes (fig.1)

$$Z=0, Z=h_1, Z=h_1+h_2, \dots, Z=h_1+h_2+\dots+h_n$$

The plane $Z = h_1 + h_2 + \dots + h_n$, ($n = 2$) is called the n - m horizon. Assume that the space between said planes filled with isotropic elastic medium, forming parallel layers. Layers $0 < z < h_1$, characterized by a constant λ_0, μ_0, ρ_0 , we call zero. Wednesday is $h_1 + h_2 + \dots h_n < z < h_1 + h_2 + \dots h_n + h_{n+1}$ fills the space between the n -mand the $n+1$ -mhorizons, characterized by a constant λ_1, μ_1, ρ_1 , will be called the n - m layer. This will always be assumed that the adjacent layers differ from each other by at least one of the permanent λ, μ and ρ . In the theoretical study of the processes described, we assume that within each layer of wave propagation is described by the usual equations of the theory of elasticity. As for the conditions at the interfaces of adjacent layers,

it will be assumed that when passing through them are continuous components of the vector of elastic displacement and stress tensor. Such contact is called hard. The paper investigates the dynamics of dissipative inhomogeneous planar bilayer structures.

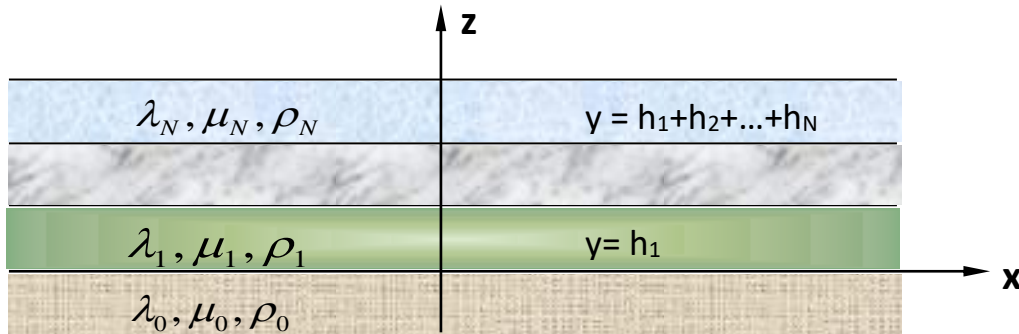


Fig.1. Design scheme: the body in the half.

Accounting internal friction caused by the scattering of energy in construction materials, is more difficult. Soft layers of multilayer structures (aggregates), as a rule, are made of materials that have developed rheological properties. Therefore, the dissipation of energy in the first place to be considered for soft layers, because it is mainly occurs during deformation of these layers. Mechanical systems, for which the viscoelastic properties of the elements are identical is **called dissipative** homogeneous system with different rheological characteristics - **dissipative inhomogeneous** [1,8,9].

The equations of motion of the deformable layer in the absence of mass forces are [1]:

$$\tilde{\mu}_j \nabla^2 \vec{u} + (\tilde{\lambda}_j + \tilde{\mu}_j) \text{grad div} \vec{u} = \rho_j \frac{\partial^2 \vec{u}}{\partial t^2}, \quad (j = 1, 2, 3..) \quad (1)$$

Here $\vec{u}(u_x, u_y, u_z)$ - displacement vector points of the medium; ρ_j - material density; u_i – moving parts; ν_j - Poisson's ratio;

$$\tilde{\lambda}_j = \frac{\nu_j \tilde{E}_j}{(1 + \nu_j)(1 - 2\nu_j)}; \quad \tilde{\mu}_j = \frac{\nu_j \tilde{E}_j}{2(1 + \nu_j)}, \text{ where}$$

\tilde{E} – modulus operator, which have the form [9,13]:

$$\tilde{E}_j \varphi(t) = E_{0j} \left[\varphi(t) - \int_0^t R_{Ej}(t - \tau) \varphi(\tau) d\tau \right] \quad (2)$$

$\varphi(t)$ – arbitrary function of time; $R_{Ej}(t - \tau)$ – relaxation kernel; E_{0j} – instantaneous modulus of elasticity; we accept the integral terms in (2) small, then the function $\varphi(t) = \psi(t) e^{-i\omega_R t}$, where $\psi(t)$ – a slowly varying function of time, ω_R – real constant. Next, using the freezing procedure [9], we note the relation (2) approximate species

$$\bar{E}_j \varphi = E_{0j} [1 - \Gamma_j^C(\omega_R) - i\Gamma_j^S(\omega_R)] \varphi,$$

where $\Gamma_j^C(\omega_R) = \int_0^\infty R_j(\tau) \cos \omega_R \tau d\tau$, $\Gamma_j^S(\omega_R) = \int_0^\infty R_j(\tau) \sin \omega_R \tau d\tau$, respectively, the cosine and sine

Fourier transform of the relaxation of the core material. As an example, the viscoelastic material take three parametric relaxation nucleus $R_j(t) = A_j e^{-\beta_j t} / t^{1-\alpha_j}$. On the effect of the function $R_j(t - \tau)$ superimposed usual requirements integrability, continuity (except $t = \tau$), signs – certainty and monotony:

$$R > 0, \quad \frac{dR(t)}{dt} \leq 0, \quad 0 < \int_0^\infty R(t) dt < 1.$$

\vec{u} – medium displacement vector j-th layer.

On the border of the two bodies can specify two types of conditions:

1. In the case of hard contact in the interface is put the condition of continuity of the relevant components of the stress tensor and displacement vector, i.e.

$$\begin{aligned} \sigma_{yy}^{(1)} &= \sigma_{yy}^{(2)}; \quad \sigma_{xy}^{(1)} = \sigma_{xy}^{(2)}; \\ u_x^{(1)} &= u_x^{(2)}; \quad u_y^{(1)} = u_y^{(2)}. \end{aligned} \quad (3a)$$

If the interface is no friction, the

$$\sigma_{yy}^{(1)} = \sigma_{yy}^{(2)}; \quad \sigma_{xy}^{(1)} = \sigma_{xy}^{(2)} = 0; \quad u_y^{(1)} = u_y^{(2)}; \quad (2,6)$$

2. On the free surface is placed a condition of freedom from stress,

$$\text{i.e.} \quad \sigma_{yy}^{(1)} = 0; \quad \sigma_{yx}^{(1)} = 0, \quad (2,c)$$

where

$$\sigma_{xx}^{(j)} = \lambda_j \theta_j + 2\mu_j \frac{\partial u_j}{\partial x}; \quad \sigma_{xy}^{(j)} = \mu_j \left(\frac{\partial u_j}{\partial y} + \frac{\partial g_j}{\partial x} \right).$$

$$\sigma_{yy}^{(j)} = \lambda_j \theta_j + 2\mu_j \frac{\partial g_j}{\partial y} \theta_j = \frac{\partial u_j}{\partial x} + \frac{\partial g_j}{\partial y}.$$

3. Solving methods

Now consider the solution of the differential equation (1) - (2) for a single layer. The equation of motion of the displacement is reduced to the following form:

$$\begin{aligned} \bar{\mu}_n \left(\frac{\partial^2 u_n}{\partial x^2} + \frac{\partial^2 u_n}{\partial y^2} \right) + (\bar{\lambda}_n + \bar{\mu}_n) \frac{\partial}{\partial x} \left(\frac{\partial u_n}{\partial x} + \frac{\partial g_n}{\partial y} \right) - \rho_n \frac{\partial^2 u_n}{\partial t^2} &= 0; \\ \bar{\mu}_n \left(\frac{\partial^2 g_n}{\partial x^2} + \frac{\partial^2 g_n}{\partial y^2} \right) + (\bar{\lambda}_n + \bar{\mu}_n) \frac{\partial}{\partial y} \left(\frac{\partial u_n}{\partial x} + \frac{\partial g_n}{\partial y} \right) - \rho_n \frac{\partial^2 g_n}{\partial t^2} &= 0; \end{aligned} \quad (3)$$

where ρ_n - material density. The solution is found in the form of:

$$u_n = U_n(y) e^{k(x-ct)}; \quad g_n = V_n(y) e^{k(x-ct)}; \quad n = 1, 2, \dots, N \quad (4)$$

where $U_n(y)$ and $V_n(y)$ - amplitude integrated vector - function; k - wave number; $C = C_R + iC_i$ - complex phase velocity; and ω - complex frequency.

To clarify their physical meaning, consider two cases:

1) $k = k_R$; $C = C_R + iC_i$, then the decision (4) has the form of a sine wave in x , whose amplitude decays over time;

2) $k = k_R + ik_I$; $C = C_R$, then at each point x fluctuations established, but x decay.

In both cases, the imaginary part k_I or C_I characterized by the intensity of the dissipative processes. Substituting (4) into (3) we obtain:

$$\bar{\mu}_n (U_n'' - k^2 U_n) + (\bar{\lambda}_n + \bar{\mu}_n) ik (ik U_n + V_n') + \rho_n k^2 C^2 U_n = 0; \quad (5)$$

$$\bar{\mu}_n (U_n'' - k^2 U_n) + (\bar{\lambda}_n + \bar{\mu}_n) ik (ik U_n + V_n') + \rho_n k^2 C^2 U_n = 0.$$

Thus, we have the equation (5) second-order two regions each. The problem is solved directly, without reducing the equation to the fourth order equation. All the arguments are for the layer.

Private solutions of the system (5) is in the form

$$\begin{pmatrix} U_n \\ V_n \end{pmatrix} = \begin{pmatrix} A_n \\ B_n \end{pmatrix} e^{r_n y},$$

Where r_n – constant. A homogeneous algebraic system relative A_n and B_n it has a nontrivial solution if its determinant is equal to zero

$$\begin{vmatrix} \bar{C}_{Tn}^2 (r_n^2 - k^2) - (\bar{C}_{Ln}^2 - \bar{C}_{Tn}^2) k^2 + \bar{C}^2 k^2 & i(\bar{C}_{Ln}^2 - \bar{C}_{Tn}^2) k r_n \\ i(\bar{C}_{Ln}^2 - \bar{C}_{Tn}^2) k r_n & \bar{C}_{Tn}^2 (r_n^2 - k^2) + (\bar{C}_{Ln}^2 - \bar{C}_{Tn}^2) r_n^2 + C^2 k^2 \end{vmatrix} = 0, \quad (6)$$

$\bar{C}_{Ln}^2 = (\bar{\lambda}_n + 2\bar{\mu}_n) / \rho_n$. $\bar{C}_{Tn}^2 = \bar{\mu}_n / \rho_n$. at $\eta = 0$ value \bar{C}_{Ln}^2 and \bar{C}_{Tn}^2 are respectively the speed of compression and shear waves in an elastic medium [4]. Equation (6) may have four roots

$$(r_n)_{1,3} = \pm k \sqrt{1 - C^2 / \bar{C}_{Ln}^2}; \quad (r_n)_{2,4} = \pm k \sqrt{1 - C^2 / \bar{C}_{Tn}^2}; \quad n = 0, 1; \quad i = 1, \dots, 4.$$

As a result, we find four particular solutions of the form

$$\begin{pmatrix} U_n \\ V_n \end{pmatrix} = \sum_{i=1}^4 C_{ni} \begin{pmatrix} A_{ni} \\ B_{ni} \end{pmatrix} e^{(r_n)i^y}, \quad n = 0, 1. \quad (7)$$

Substituting values (r_n) at (7) find A_{ni}, B_{ni} at $r_n = (r_n)$ i.

Expressions for the displacement are as follows:

$$\begin{pmatrix} U_1 \\ V_1 \end{pmatrix} = C_{11} \begin{pmatrix} ik \\ -k\bar{q}_1 \end{pmatrix} e^{-k\bar{q}_1 y} + C_{12} \begin{pmatrix} ik \\ k\bar{q}_1 \end{pmatrix} e^{k\bar{q}_1 y} + \\ + C_{13} \begin{pmatrix} -k\bar{S}_1 \\ -ik \end{pmatrix} e^{-kS_{11} y} + C_{14} \begin{pmatrix} -k\bar{S}_1 \\ -ik \end{pmatrix} e^{+kS_{11} y};$$

$$\begin{pmatrix} U \\ V \end{pmatrix} = C_{22} \begin{pmatrix} ik \\ -k\bar{q}_1 \end{pmatrix} e^{k\bar{q}_1 y} + C_{24} \begin{pmatrix} k\bar{S} \\ -ik \end{pmatrix} e^{k\bar{S} y}.$$

Consequently, both the hard and to obtain a sliding contact set six boundary conditions which lead to six homogeneous equations with six unknowns $C_{11}, C_{12}, C_{13}, C_{14}, C_{22}, C_{24}$. To such a system of equations have a nontrivial solution, the determinant of the coefficients must be zero. This equation gives the dispersion equation for dissipative systems, where

$$\bar{S}_n = (1 - C^2 / \bar{C}^2_n)^{1/2}; \quad \bar{q}_n = (1 - C^2 / \bar{C}^2_{Ln}), \quad n = 0, 1$$

As an example, consider the problem of distribution of natural waves in the viscoelastic layer at the half.

Hard contact. Dispersion equation has the form

$$\begin{vmatrix} (1 + \bar{S}_1^2) e^{-\bar{\xi} q_1} & (1 + \bar{S}_1^2) e^{\bar{\xi} q_1} & -2e^{\bar{\xi} q_1} & \dots & \dots & 2e^{\bar{\xi} q_1} & \dots & \dots & 0 & \dots & \dots & 0 \\ -2\bar{q}_1 e^{\bar{\xi} q_1} & \dots & \dots & 2\bar{q}_1 e^{\bar{\xi} q_1} & \dots & \left(\bar{s}_1 + \frac{1}{\bar{s}_1}\right) e^{-\bar{\xi} q_1} & \dots & \left(\bar{s}_1 + \frac{1}{\bar{s}_1}\right) e^{\bar{\xi} q_1} & \dots & \dots & 0 & \dots & \dots & 0 \\ (1 + \bar{S}_1^2) e^{\bar{\xi} q_1} & \dots & (1 + \bar{S}_1^2) e^{-\bar{\xi} q_1} & -2e^{\bar{\xi} q_1} & \dots & 2e^{-\bar{\xi} q_1} & \dots & (1 + \bar{s}^2) \gamma_1 & \dots & -2/\gamma_1 & \dots & \dots & \dots & \dots \\ -2\bar{q}_1 e^{\bar{\xi} q_1} & \dots & 2\bar{q}_1 e^{\bar{\xi} q_1} & \dots & \dots & \left(\bar{s}_1 + \frac{1}{\bar{s}_1}\right) e^{\bar{\xi} q_1} & \dots & \left(\bar{s}_1 + \frac{1}{\bar{s}_1}\right) e^{-\bar{\xi} q_1} & \dots & \frac{2\bar{q}_1}{\gamma_1} & \dots & \left(\bar{s} + \frac{1}{\bar{s}}\right) / \gamma & \dots & \dots \\ e^{\bar{\xi} q_1} & \dots & e^{\bar{\xi} q_1} & \dots & \dots & e^{\bar{\xi} q_1} & \dots & e^{\bar{\xi} q_1} & \dots & \dots & \dots & -1 & \dots & -1 \\ \bar{q} e^{\bar{\xi} q_1} & \dots & \bar{q}_1 e^{-\bar{\xi} q_1} & \dots & \dots & -\frac{1}{\bar{s}} e^{\bar{\xi} \bar{s}} & \dots & -\frac{1}{\bar{s}} e^{\bar{\xi} \bar{s}} & \dots & -\bar{q} & \dots & \dots & \dots & \frac{1}{\bar{s}} \end{vmatrix} = 0 \quad (8)$$

where ζ –dimensionless wave number $\zeta = kh, \gamma_1 = \bar{\mu}_1 / \bar{\mu}$ or $\lambda + 2\mu =$

$= 2(1-\nu)/(1-2\nu)$. As a relaxation kernel viscoelastic material will take a three-parameter kernel

$R(t) = \frac{A e^{-\beta t}}{t^{1-\alpha}}$ Rizhanitsena -Koltunova [13], has a weak singularity, where A, α, β -parameters of materials [13]. Assume the following parameters: $A = 0,048$; $\beta = 0,05$; $\alpha = 0,1$. Using the complex representation of the modulus of elasticity as described previously.

The roots of the frequency equation is solved by Muller, at each iteration of the method applied by Muller Gaussian with the release of the main element. Thus, the solution of equation (8) does not require disclosure of the determinant. As an initial approximation we choose the phase velocity of the waves of the elastic system. For free waves in $R_j = 0$ phase velocity and wave number are valid values. In the calculations we take the following values:

$$\theta = \rho_1/\rho_2 = 0,75; \beta = 10^{-4}; n = 1.$$

Consider two options for a dissipative system. In the first embodiment, the dissipative system is structurally homogeneous.

Wave number ξ varies 0 – 3. The calculation results are shown in fig.2.a. The dependence of the frequency and damping of the dimensionless wave number ξ it was monotonous, and the character according to the same frequency and damping coefficients. In the second embodiment,

the dissipative system is structurally non-uniform: half-considered, the equation (8) and elastic parameters coincide with those adopted above. The calculation results are shown in fig.2.b. Frequency Dependence ξ It is the same as for the homogeneous system: corresponding curves coincide up to 5%. Dependencies damping coefficients of ξ non-monotonic.

Of particular interest is the minimum value ξ_a fixed damping coefficient:

$$\delta = \min(-\omega_{I_k}), \quad k = 1, 2, \dots, K, \quad (9)$$

here δ – coefficient determining the damping properties of the system (let's call it a global damping coefficient).

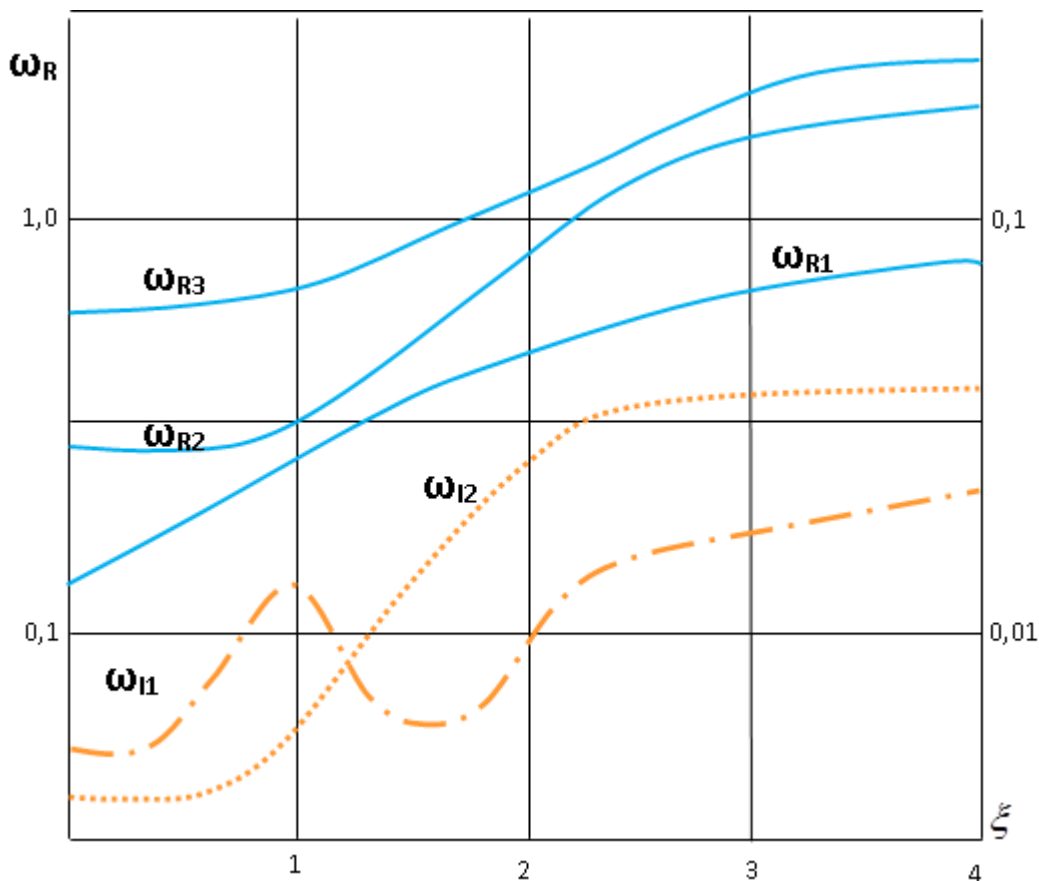


Fig. 2.a. Changing the complex natural frequencies of the wave number .a) Dissipative homogeneous mechanical system.

For a homogeneous system coefficient δ is entirely determined by the imaginary part of the first modulo complex frequency. For heterogeneous systems as coefficient δ can act imaginary parts of both the first and the second frequency depending on their values. "Turn the Tables" occurs when the characteristic value of ξ , the closest at this value of the real parts of the first and second frequencies. δ ratio at a specified characteristic value has a pronounced maximum.

The sliding contact. Dispersion equation is similar in form to equation (8). All parameter values coincide with those adopted above. Figure 3a and b shows the dependence of the frequency and damping coefficients of the wave number ξ , respectively, for structural and homogeneous and heterogeneous systems. These results confirm earlier findings. Changing the parameter from so that essentially dependent coefficient δ , can be achieved by varying the geometric dimensions of the members without altering their mechanical properties.

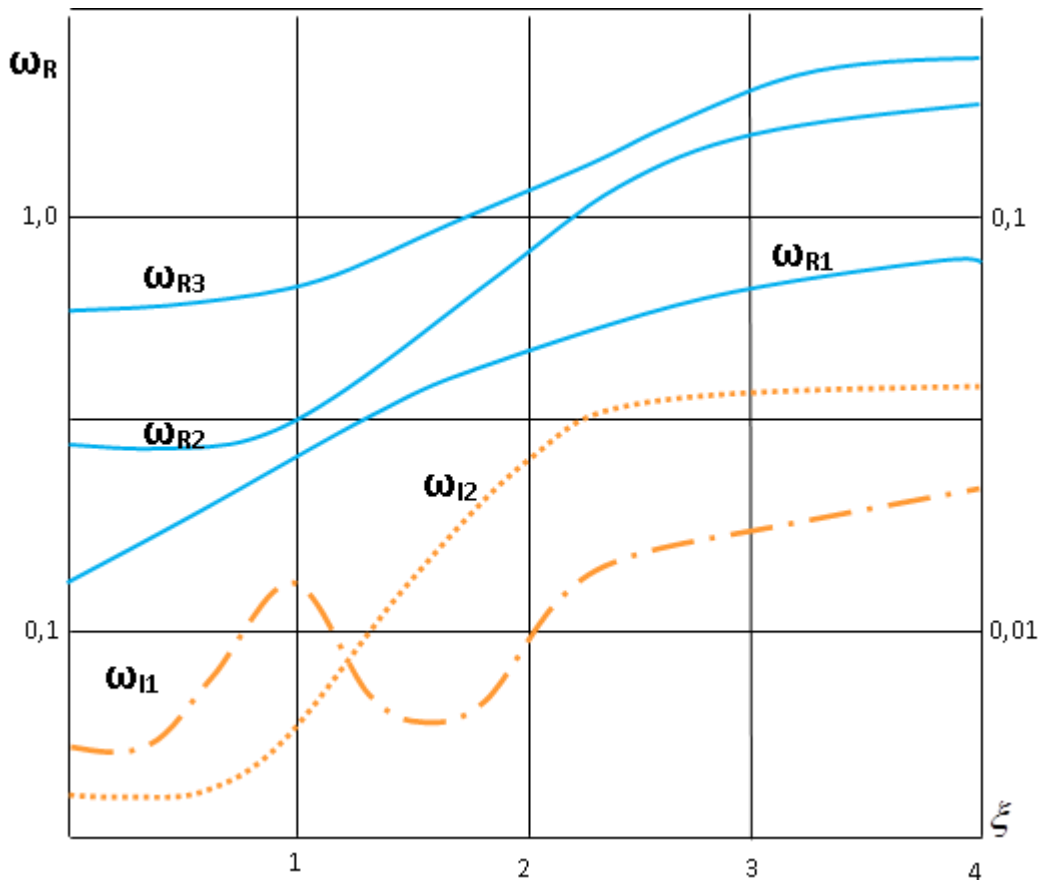
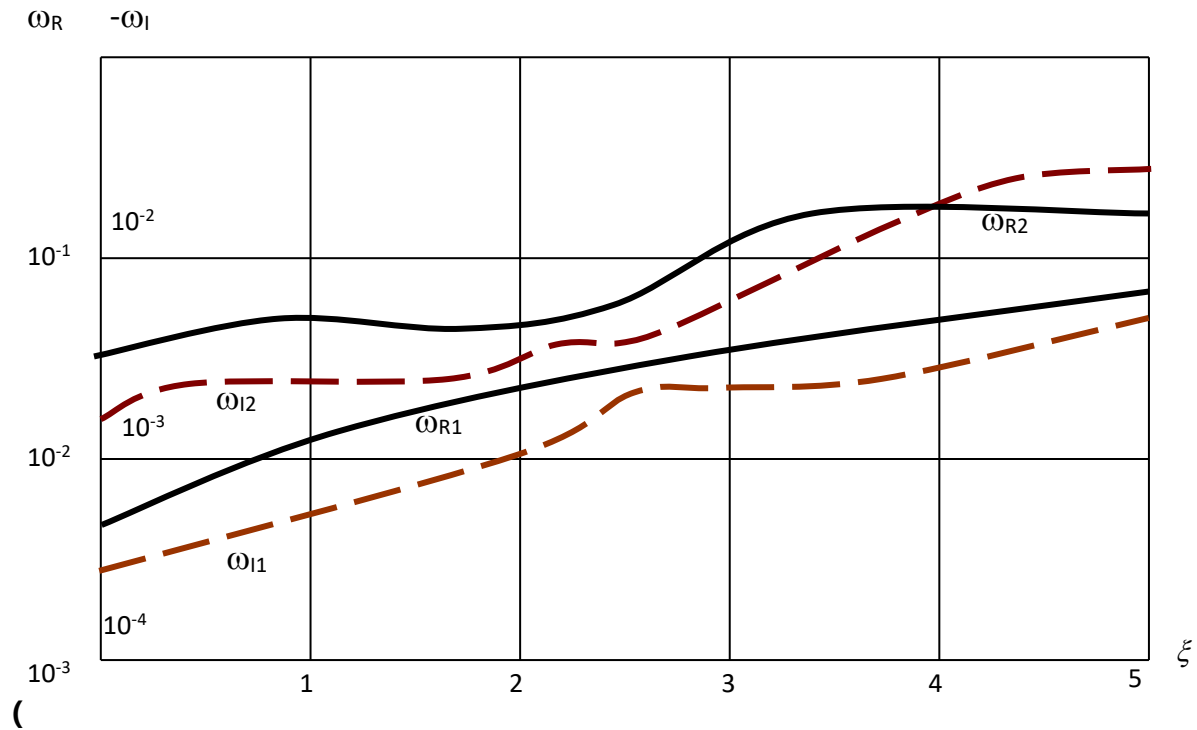


Fig.2. b Change of complex natural frequencies of the wave number.
b) Dissipative heterogeneous mechanical system;



a) Homogeneous dissipative mechanical system.

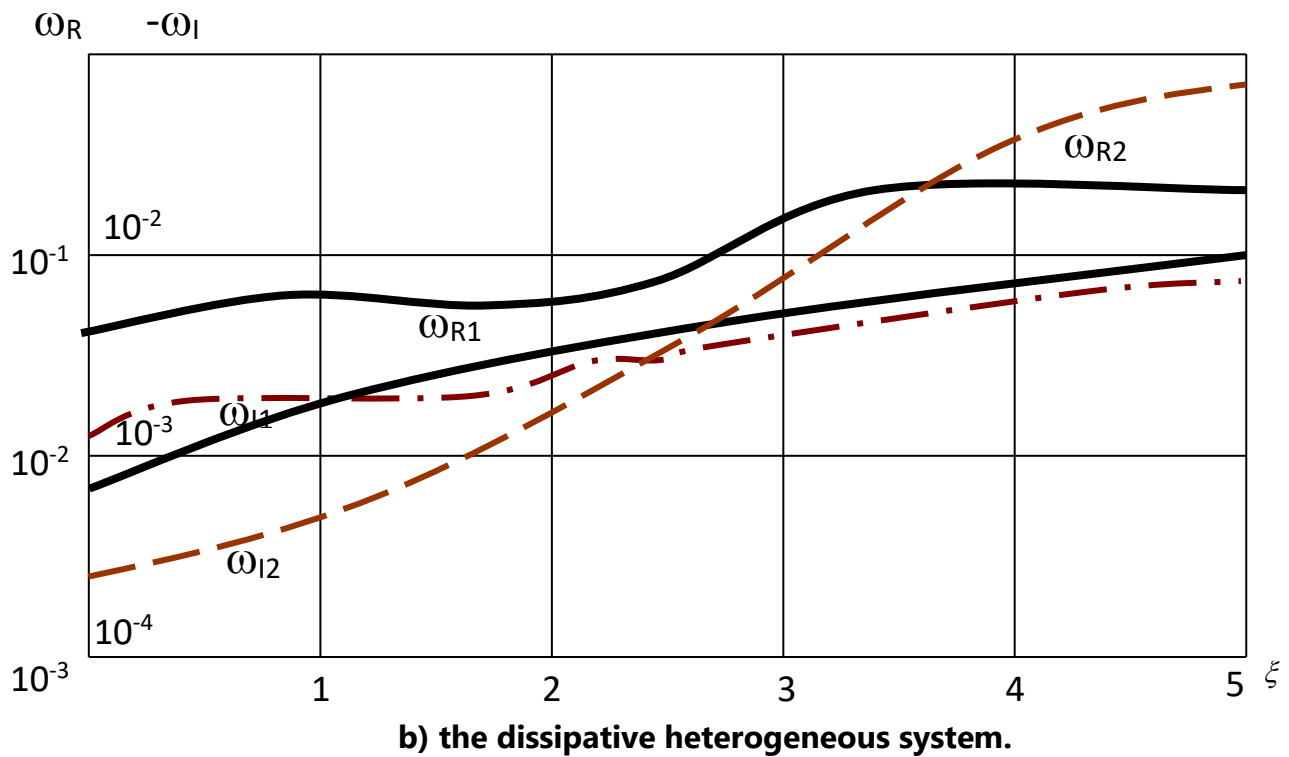


Fig.3. Changing the complex natural frequencies of the wave number.

This opens a promising possibility of effective control damping characteristics of heterogeneous viscoelastic systems by changing their inhomogeneous systems with close frequencies.

Analysis of Stoneley waves and Rayleigh. Considering the half-layer and with different coefficients of Poisson ($\nu_1=0.3$; $\nu_2=0.35$; rheological properties and other parameters coincide with those adopted above), we see that the Rayleigh wave velocity and attenuation wave velocity for a layer less than Stoneley wave velocity at the boundary layer section - half-wave if the latter exist (fig.4).

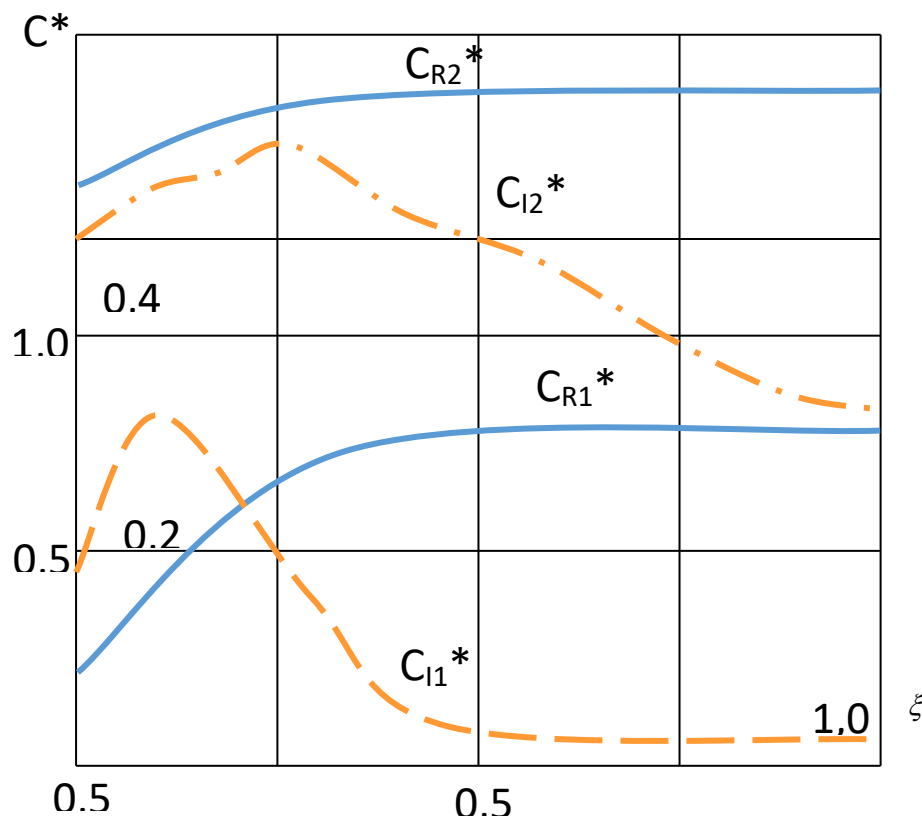


Fig. 4 Change C^* and ξ the wave number.

Results of the study of structurally homogeneous and heterogeneous systems are practically the same. At $\gamma/\theta > 1$ The speed of the shear waves layer above the speed of shear waves in the half, then there is only one form of vibrations. The results of calculations for the hard and sliding contact with the original data $\theta = 1.44$; $\gamma = 8$; $\nu_1 = \nu_2 = 0.25$ (Other parameters coincide with those adopted above) are shown in fig. 5.

Analysis of Love waves. Consider the problem of the distribution of the Love waves in two-layer media, taking

$$u_n = [u(y), w_n(y)] e^{i(\alpha y + \omega t)}. \quad (10)$$

The wave equation (1) takes the form:

$$\bar{\mu}_n \left[\frac{\partial^2 w_n}{\partial x^2} + \frac{\partial^2 w_n}{\partial y^2} \right] - \rho_n \partial^2 w_n = 0; \quad n = 1, 2 \quad (11)$$

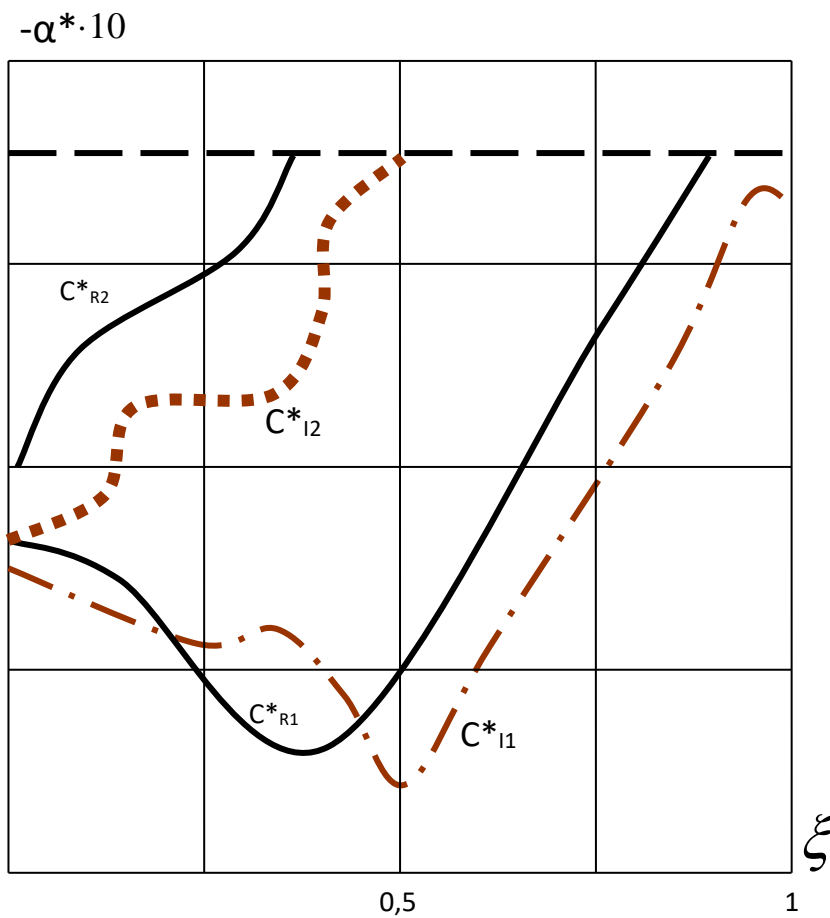


Fig. 5. Change C_R^* and C_I^* the wave number ξ .

Suppose that the boundary = hfree of loads. The solution of equation (9) in the form:

$$u=0, \quad v=0, \quad w_n = W(y) e^{i(\alpha x + \omega t)}, \quad (12)$$

where $W_1 = A \sin(\beta_1 y) + B \cos(\beta_1 y)$, $0 < y < h$

$$W_3 = C \exp(\beta_2 y), \quad y < 0;$$

$$\beta_1^2 = \frac{\omega^2}{(C_T)_1^2} - \alpha^2; \quad \beta_2 = \frac{\omega^2}{(C_T)_2^2} - \alpha^2.$$

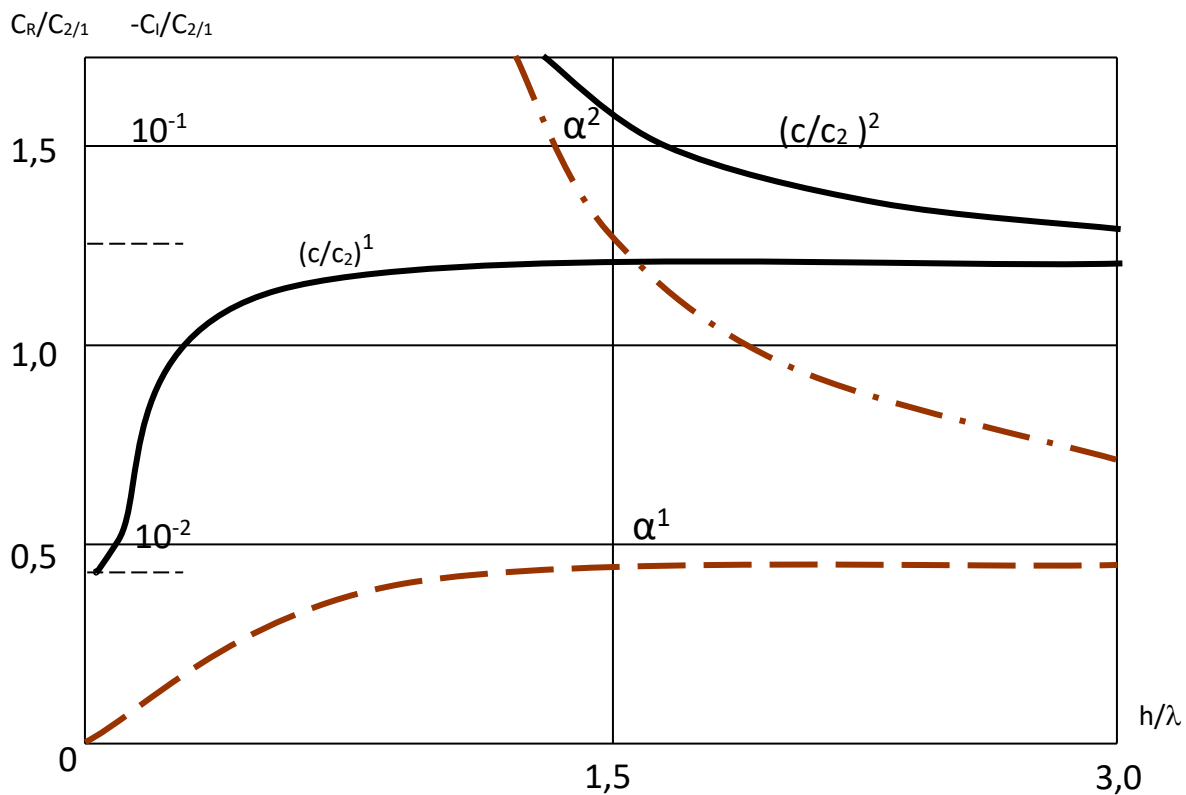


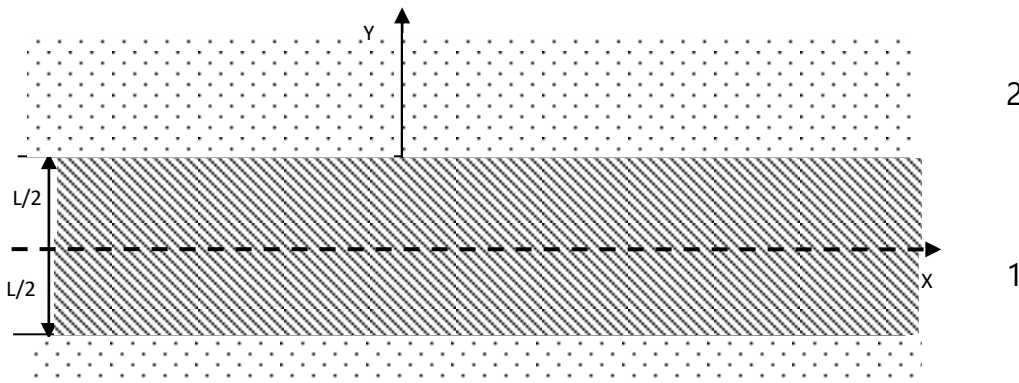
Fig.6. Dispersion curves Lava stress waves in a layer of visco elastic half-space($\alpha^* = \alpha H$; $C^* = C/C_0$).

The solution of equations (10) selected from the condition limits at infinity: $R_e = (\beta_2) \geq 0$. The integration constants A, B, C associated with the boundary conditions in the plane $y = h$ and the conditions of continuity of stresses and displacements in the plane $y = 0$.

To satisfy the boundary conditions must be equal

$$\begin{vmatrix} \cos(\beta_1 h) & \sin(\beta_1 h) & 0 \\ 0 & 1 & -1 \\ \mu_1 \beta & 0 & \mu_2 \beta_2 \end{vmatrix} = 0.$$

In the numerical calculation will take $\mu_1/\mu_2 = 0,1$; $\theta = 0.75$, other parameters coincide with those adopted above. The calculation results are shown in Fig. 6. The results show that when there Stoneley waves or Love waves, the observed effect does not occur.



As a second example, consider the dissemination of its own waves in a flat layer being in a deformable (viscoelastic) medium (Figure 7).

to the phase velocity and the damping rate of the geometric and physical - mechanical parameters of the system turned out to be non-monotonic;

- On the basis of the numerical results obtained revealed that the possibility of separation of thin-walled structures of the soft layer and the effect of the resonance to speed on the size of the site contacts. Also taking into account the material of viscous properties of 15 - 10% increase in the value of the phase velocity;
- Detected that higher phase waveforms and torsional expansion exceeds the highest possible speed (S) waves in an infinite medium, the group velocity never exceeds S. also found that the group velocity of a nondispersive medium 10 - 15% higher than the comparison dispersion medium. In other words, the shape of the pulses as they propagate not remains unchanged as homogeneous elastic bodies.

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